

College Prep Mathematics

Prerequisite: Secondary III

College Prep Math formalizes and reinforces concepts from the Secondary Mathematics series to provide students with the foundational skills and understanding prerequisite to College Algebra (1050). Students will reason abstractly and quantitatively while solving linear and quadratic equations and linear inequalities. They will efficiently use polynomial and rational expressions and functions, radicals and complex numbers, and exponential and logarithmic expressions and functions to model and solve mathematical problems. They will explore conic sections and represent parabolic data. Throughout this course, students will make sense of problems and persevere in solving them, use tools strategically, and attend to precision.

Standard I: Students will develop fluency with the language and operations of algebra to evaluate, analyze, and solve problems.

Objective 1: Perform operations and simplify expressions with rational, irrational, and complex numbers.

- a. Rewrite expressions involving radicals and rational exponents using the properties of exponents.
- b. Add, subtract, multiply, and divide radical expressions.
- c. Simplify expressions involving complex numbers and express them in standard form, $a + bi$.

Objective 2: Solve systems of equations and inequalities.

- a. Solve systems of linear equations in two variables algebraically and graphically.
- b. Solve a simple system consisting of a linear equation and a quadratic equation in two variables algebraically and graphically.
- c. Use matrices to solve systems of linear equations.*
- d. Model and solve problems arising from authentic situations using systems of equations and inequalities.

Objective 3: Solve and graph quadratic equations.

- a. Use the method of completing the square to transform any quadratic equation into an equation of the form $(x - p)^2 = q$, and use this to derive the quadratic formula.
- b. Solve quadratic equations by inspection, taking square roots, factoring, completing the square, and using the quadratic formula.
- c. Model and solve real-world situations using quadratic relationships in two variables.

Objective 4: Simplify rational and radical expressions and solve and graph rational and radical equations.

- a. Simplify and rewrite simple rational expressions in different forms using algebraic techniques, including long division.
- b. Solve rational equations.
- c. Solve radical equations in one variable, including those with extraneous solutions.
- d. Graph rational functions, identifying domain, range, zeros and asymptotes, and showing end behavior.

Mathematical Language and Symbols Students Should Use:

asymptote, completing the square, complex number, compound inequality, extraneous root, quadratic formula, radical, rational, irrational, system of equations, minimum, maximum, intercept

Standard II: Students will understand and represent functions and analyze function behavior.

Objective 1: Understand the concept of a function and use function notation.

- a. Understand that a function from one set (called the domain) to another set (called the range) assigns to each element of the domain exactly one element of the range.
- b. Determine when a relation is a function.
- c. Use function notation, evaluate functions for inputs in their domains, and interpret statements that use function notation in terms of a context.

Objective 2: Analyze functions using different representations.

- a. Graph linear, quadratic, square root, cube root, piecewise, and polynomial functions expressed symbolically, and show key features of the graphs—by hand in simple cases, and using technology for more complicated cases.
- b. Identify key features of functions from either graphs or equations.
- c. Write a function in different but equivalent forms to reveal and explain different properties of the function.
- d. Compare properties of two functions, each represented in a different way (algebraically, graphically, numerically in tables, or by verbal description).

Objective 3: Build new functions from existing functions.

- a. Combine standard function types using arithmetic operations.
- b. Compose functions.
- c. Find the inverse of a function when it exists.
- d. Produce an invertible function from a non-invertible function by restricting the domain.
- e. Graph functions using transformations of parent functions.

Objective 4: Construct exponential models and use them to solve problems.

- a. Understand that exponential functions grow by equal factors over equal intervals.
- b. Recognize and model situations in which a quantity grows or decays by a constant percent rate per unit interval relative to another.
- c. Construct exponential functions, including geometric sequences, when given a graph, a description of a relationship, or two input-output pairs.
- d. Graph exponential functions.
- e. Interpret the parameters in an exponential function in terms of a context.

Objective 5: Understand the inverse relationship between exponents and logarithms and use this relationship to solve problems.

- a. Simplify, expand, or condense simple logarithmic expressions.

- b. Connect the manipulation of logarithmic expressions with the laws of exponents.
- c. Convert logarithms between bases.
- d. Graph logarithmic functions.
- e. Model and solve authentic problem situations using logarithms.

Mathematical Language and Symbols Students Should Use:
 domain, exponential function, function, $f(x)$, interval notation, inverse, logarithm, parameter, parent function, piecewise function, range, relation, transformation, intercept, maximum, minimum

Standard III: Students will apply geometric concepts in modeling situations.

Objective 1: Use coordinate algebra to represent and analyze geometric situations.

- a. Prove the slope criteria for parallel and perpendicular lines and solve related problems.
- b. Use coordinates to compute perimeters of polygons and areas of triangles and rectangles using the distance formula.
- c. Use coordinates to model geometric problems involving distances and relationships and apply those models to contextual situations.*

Objective 2: Translate between the geometric descriptions and the equations for conic sections.

- a. Derive the equation of a circle, given center and radius, using the Pythagorean Theorem.
- b. Complete the square to find the center and radius of a circle when given the equation.
- c. Write an equation of a parabola in the form $y = a(x-h)^2 + k$ when given the graph or equation, or when given the focus and directrix.
- d. Identify the vertices, foci, and intercepts of ellipses and hyperbolas with centers at the origin.*
- e. Derive the equations of ellipses and hyperbolas given the foci, using the fact that the sum or difference of distances from the foci is constant.*

Mathematical Language and Symbols Students Should Use:
 eccentricity, ellipse, foci, hyperbola, parabola, vertex, parallel, perpendicular, Pythagorean Theorem, directrix

Standard IV: Students will apply statistical methods to make informed decisions.*

Objective 1: Formulate questions and answer these questions by organizing, summarizing, and analyzing data.

- a. Collect data, and classify the data as univariate or bivariate and categorical or quantitative.
- b. Summarize distributions using measures of center and variability, and communicate findings coherently using graphical representations and data summaries.
- c. Use the mean and standard deviation of normally distributed data to estimate population percentages.
- d. Fit a function to bivariate data.

Objective 2: Make inferences and justify conclusions using data.

- a. Understand statistics as a process for making inferences about population parameters based on a random sample from that population.
- b. Make inferences and justify conclusions based on data collection.
- c. Use data from a sample survey to estimate a population mean or proportion.

Objective 3: Use the rules of probability to compute probabilities, and use probabilities to interpret data.

- a. Understand and calculate probabilities of independent events.
- b. Understand and calculate conditional probabilities.
- c. Use permutations and combinations to compute probabilities of compound events and solve problems.
- d. Calculate expected values and use them to solve problems.

Mathematical Language and Symbols Students Should Use:

bivariate, categorical, cumulative frequency plot, interquartile range, measures of center, measures of variability (spread), normal distribution, quantitative data, univariate, regression, standard deviation

*May be eliminated in a semester course.