Utah State Board of Education
Elementary Mathematics Endorsement Course Framework
Mathematics for Teaching K-8 - Geometry and Measurement

Course Description: To provide practicing teachers a deeper understanding of the geometry and measurement content that exists in the state core and instructional strategies to facilitate the instruction of this content. Special attention in this course will be given to how children learn and connect the fundamental concepts of geometry and measurement, children's developmental trajectories in this mathematical content, how children construct their understanding of various geometric concepts, children's typical error patterns, problem solving strategies, interpreting and assessing students' work and learning, and integration of the NCTM process standards and the Utah Intended Learning Outcomes (ILOs).

Course Objectives:
During this course students will:

- Increase geometry and measurement content knowledge.
- Design worthwhile geometric and measurement tasks for elementary students.
- Orchestrate discourse related to geometric and measurement topics in an elementary classroom.
- Model and represent geometric and measurement concepts and relationships.
- Improve problem solving skills using geometry and measurement.
- Reason about, justify, and analyze geometric and measurement relationships.
- Communicate geometric and measurement ideas orally, visually, and in writing.
- Use a variety of tools including technology to enhance classroom instruction and increase student understanding of geometry and measurement.
- Address developmental issues associated with learning geometry and measurement.

Topics:

Measurement
- Understanding and using measurement concepts.
- Units for measurement including length, weight, capacity, and angles. Include metric, customary, and non-standard units of measurement. Conversion of units within and between different systems.

Line and Angle Relationships
- Parallel and perpendicular lines.
- Finding angles when parallel lines are cut by a transversal and showing lines are parallel given information about angles including corresponding angles, same side interior/exterior angles, and alternate interior/exterior angles.
- Angles that sum to a straight angle.
- Vertical angles.

Attributes of Geometric Figures
- Intersection properties of lines and circles with themselves and each other.

- Attributes of triangles including the sum of angles in a triangle, the sum of the non-right angles of a right triangle, the exterior angles of a triangle, the base angles of an isosceles triangle, and the angles of an equilateral triangle.
- Classifying triangles and symmetry of triangles.
- Attributes of quadrilaterals including the opposite angles in a parallelogram and the interior angles between two parallel sides in a trapezoid.
- The definition of a (regular) polygon and attributes of polygons including the sum of the angles of a polygon and the sum of exterior angles of a polygon.

Constructions

- Using appropriate tools to perform constructions to:
    - Copy line segments and angles.
    - Copy triangles given three sides, two sides, and an included angle, or two angles and the included side.
    - Construct a line parallel or perpendicular to a given line through a given point when possible.
    - Construct a circle that is circumscribed about a triangle or that is inscribed in a triangle by the use of perpendicular bisectors and angle bisectors.

Transformations and Tessellations

- Rotations, translations, reflections and their composites.
- Transformation of geometric figures including decontructing and recombining geometric figures.
- Tessellations of the plane by regular polygons and irregular shapes.

Definitions and Axiomatic Nature of Geometry

- Undefined and defined terms.
- Axioms for plane geometry including the parallel postulate.

Similarity and Congruence

- Similar triangles, scale factors, similarity tests for triangles.
- Congruence tests for triangles, proving facts about side-lengths and angles using congruent triangles.
- Similarity and congruence tests for quadrilaterals and how they differ from those for triangles.

Circles

- Definition of circle, radius, diameter, and circumference.
- The number π defined as the ratio of the circumference of a circle to the diameter and why it is the same for every circle.
- Central and inscribed angles, arc, and sector.
- Areas of circles and sectors, and π as the ratio of the area of a circle to the square of the radius.
- Demonstrations of the area formula for a circle and areas of sectors.

Perimeter and Area

- Square units, rectangles with whole number side lengths.
- Intersections and unions of regions and properties of area including finding area by deconstructing and recombining geometric figures.
- The relationship between area and perimeter in rectangles; i.e., among all rectangles, squares have the largest area with a fixed perimeter and the smallest perimeter with a fixed area.
- Derive area formulas for triangles, parallelograms, and trapezoids. Convert area units and show how scaling affects area.

Surface area and Volume

- Derive the surface area and volume formulas for a rectangular prism.
- Define prisms and cylinders including oblique prisms and cylinders and derive surface area and volume formulas for right prisms and cylinders.
- Use the slice (Cavalieri) principle for comparing areas and volumes.
- Define pyramids and cones and derive volume and surface area formulas. Define spheres and balls (the solid enclosed by a sphere) and derive volume and surface area formulas.
- Convert volume units and show how scaling affects area and volume.

Pythagorean Theorem

- Proof of the Pythagorean Theorem and its converse.
- Square roots and Pythagorean Triples.
- Special right triangles – isosceles right triangles and 30-60-90 triangles.

Applications of Geometry to Algebra

- Put coordinates on the line and the plane.
- Use a perpendicular coordinate system in the plane to define the slope of a line using two points on the line.
- Use similar triangles to show that the slope of a line is independent of the choice of points.
- Use geometry to demonstrate the relationship of slopes of perpendicular lines.
- Find the equation of a line given its slope and a point on the line.
- Use the Pythagorean Theorem to find a formula for the distance between two points in the coordinate plane.

Role of Proof and Logical Reasoning in Geometry

- Define *theorem* and *proof* and give formats for proofs.
- Identify known facts and derive new geometric facts from the previously-known facts using logical reasoning.
- Use auxiliary lines in proofs. Give proofs of basic theorems in geometry such as the area formulas for triangles, parallelograms, and trapezoids, the concurrence theorems for triangles (perpendicular bisectors, angle bisectors, altitudes, and medians) and the relationship in circles between central angles and inscribed angles with the same arc.

Pedagogy

The purpose of the Elementary Mathematics Endorsement courses is to ensure that practicing teachers gain the mathematical content knowledge needed to teach mathematical concepts to students in the elementary grades. Teachers must also, however, know how to transfer that content knowledge and the conceptual understandings inherent in the content to students. An understanding of sound pedagogical practice is essential to that transfer. Following are suggested pedagogical concepts and strategies that should be infused into the courses to aid teachers in student instruction. These concepts should never be taught in isolation, but should be modeled throughout the courses. It is not necessary nor intended that all the following concepts be infused in every course, but they should be covered in their entirety in the series.

Knowledge of mathematics learners:

- Employ prior knowledge, skills, dispositions, beliefs and conceptions, to clarify misconceptions and confusions around particular topics;
- Employ knowledge of cognition, child development, and learning theory;
- Use knowledge of learners culturally and socially to know what motivates and inspires them.

Knowledge of effective mathematics teaching practices for student and teacher learning:

- Design, select and adapt worthwhile tasks and sequences of examples that support a particular learning goal, including tasks to be solved by teachers themselves, and others for which they try to anticipate children's solutions, and design responses.
- Develop sensitivity with the careful use of technical language, attending to both mathematical integrity and usability by learners.
- Construct and evaluate multiple representations of a mathematical idea or process, and establish correspondences between representations.
- Use questions to effectively probe mathematical understanding and make productive use of responses.
- Develop learners' skills at clear and coherent public mathematical communication in a classroom setting, using a variety of resources and media.
- Model and cultivate mathematical practices – questioning, representing, communicating, problem solving, inquiring and conjecturing, making connections, reasoning and proving, self-assessing.
- Use various instructional applications of technology and be able to make judicious, mathematically and pedagogically grounded use of them.
- Construct and evaluate mathematical contexts or story problems for various mathematical situations and processes.
- Analyze and evaluate student work, and design appropriate responses.

- Develop skillful and flexible use of grouping strategies – whole group, small group, partner, and individual – in support of learning goals.
- Know the resources for managing the diversities – cultural, linguistic, gender, socioeconomic, developmental – of the classroom, and ways to skillfully deploy them in support of the learning of all students.
- Understand and address diversity in planning and implementing instruction and in helping other teachers foster mathematics classroom environments that support the learning of all students.
- Understand and use various cultural approaches to learning mathematics.

Knowledge of mathematics curriculum and assessment:

- Know common developmental paths (learning trajectories) related to foundational mathematical topics (e.g., place value, rational numbers and equivalence); understand these developmental paths, and use them to sequence activities to build learning environments that are developmentally appropriate and effective.
- Select, use, adapt and determine the suitability of mathematics curricula and teaching materials (e.g., textbooks, technology, manipulatives) for particular learning goals.
- Be cognizant of a formative assessment cycle (administering a formative assessment task, analyzing student work from the task, using that analysis to enhance teacher knowledge, and designing and teaching reengagement lessons) and resources.
- Provide appropriate interpretations of assessment results, and communicate results (in context) to specific individuals and groups (e.g., students, parents, caregivers, colleagues, administrators, policymakers, community members).

(Standards for Elementary Mathematics Specialists, Association of Mathematics Teacher Educators, 2009)

Possible Assignments:

- Solve challenging geometry problems.
- Analyze errors in student work.
- Solve problems including applications of geometry.
- Use appropriate measurement tools accurately.
- Write a lesson plan to teach a geometric topic.
- Create a project students can do addressing geometric topics.
- Analyze results of assessments in geometry to inform instruction.
- Content exams, performance tasks, and homework for determining participants' mathematical knowledge for teaching on the topic of geometry and measurement.
- Clinical interview: Design an interview to determine student understanding of geometry and measurement, including interview questions with extensions to press for student understanding. Videotape and conduct the interview. Analyze the video using a rubric that assesses both student understanding and the interview process.
- Differentiated lesson: Select one of the course objectives. Develop a lesson to meet the objective for the whole group. Then develop differentiation activities for at least

three populations (e.g., gifted students, English Language Learners, Students with Disabilities).

- Action research project: Action research is a particular approach to research that aims to improve practice by addressing real-world situations and needs through focused, reflective practice and the application of interventions or procedures that are designed for improving performance. Develop a classroom research project about teaching concepts of mathematics for your own classroom. Consult the mathematics education literature, citing at least three sources from peer-reviewed journals. Develop a research question which could be a new teaching, assessment, or curriculum strategy for rational numbers in your class. Collect data, and analyze the results.
- Case study: Write a case presenting the mathematical thinking of a student or group of students. Be sure to include details in your narrative, such as student dialogue, your questions, and what you were thinking as you listened to the students. Analyze the student thinking and discuss the questions that are raised for you in the students' mathematical thinking.
- Lesson study cycle: Work with a group to develop a task-based lesson including anticipated student responses, questions for classroom discourse, and formative assessment. One person in the group will teach the lesson while other members of the group observe student thinking. Group members refine the lesson in response to the observation data and then teach and analyze the lesson in their own classes.
- Reflections on your own mathematical understanding: Keep a reflections log that discusses: a mathematical idea from the session that is new or important for you, a question you have, and an application of the mathematical idea for your classroom.

Suggested Texts:

Parker, Thomas H. and Baldridge, Scott J. (2005). *Elementary Geometry for Teachers*. Portland, OR: Sefton-Ash Publishers.

Beckmann, Sybilla (2008). *Mathematics for Elementary Teachers (2nd ed.)*. Boston: Pearson. Chapters 8, 9, 10, 11

Henry, Patricia; Blackinton, Dixie; Mankus, Margo L.; Pugmire, Diane D.; and Wittmann, Pamela. *Describing our World Geometry.* Ogden, UT: Weber State University Press.

Supplementary Texts/Readings:

Schifter, Deborah, et. al., (2002). Examining Features of Shape: Casebook (Developing Mathematical Ideas: Geometry). Palo Alto, CA: Dale Seymour Publications.

Schifter, Deborah, et. al., (2002). Measuring Space in One, Two, and Three Dimensions: Casebook (Developing Mathematical Ideas). Palo Alto, CA: Dale Seymour Publications.

Ministry of Education, Singapore. *Singapore Mathematics Singapore Primary Mathematics Textbooks 4A, and 5A.* Oregon City, OR: Marshall Cavendish Education.

Ministry of Education, Singapore. *Singapore Mathematics Singapore Primary Mathematics Textbooks 3B, 5B, and 6B.* Oregon City, OR: Marshall Cavendish Education.

Ministry of Education, Singapore. *Singapore New Elementary Mathematics Textbook I (7th Grade).* Oregon City, OR: Marshall Cavendish Education.