Utah State Board of Education
Elementary Mathematics Endorsement Course Framework
Mathematics for Teaching K-8 - Numbers and Operations

Course Description: This course, designed for K-8 teachers, will cover the content of Number and Operations to develop a comprehensive understanding of our number system and relate its structure to computation, arithmetic, algebra, and problem solving. Course topics will include number, number sense, computation, and estimation through a coordinated program of activities that develop number concepts and skills. Special attention in this course will be given to how children learn and connect the fundamental concepts of number systems, children's developmental trajectories in the mathematical content of number and operations, how children construct their understanding of various number systems and arithmetic, children's typical error patterns, problem solving strategies, interpreting and assessing students' work and learning, and integration of the NCTM process standards and the Standards for Mathematical Practice.

Course Objectives:
During this course students will:

- Gain a comprehensive understanding of the number system and how its structure is related to computation, arithmetic, algebra, and problem solving.
- Use a variety of tools, technology, and mathematical representations to explore and model number and operations concepts.
- Communicate number and operations ideas orally, visually, and in writing, in order to facilitate effective discourse related to these topics in a positive mathematics learning environment.
- Understand how children learn the fundamentals of number systems, their problem-solving strategies, how they construct an understanding of various number systems and arithmetic, typical error patterns, and how to interpret and assess students' work and learning
- Integrate the NCTM process standards and the Standards for Mathematical Practice into lesson planning and student learning.

Topics:

Whole numbers (1.5 weeks)

- Understand place value in the base 10 number system.
- Use models for the place value system including concrete and pictorial (e.g. chips, base ten blocks, drawings).
- Understand other numeration systems (e.g., Roman, Egyptian, bases other than ten)
- Understand whole numbers as adjectives for counting how many (one-to-one correspondence).
- Identify the units that are being counted by whole numbers.

- Use the set model and the measurement model for counting and be able to identify whether questions about how many are using the set model or the measurement model.
- Use models to show part-part-whole relationships. (e.g. number bonds)

Addition and Subtraction (2.5 weeks)

- Define addition by bundling objects or pictures (set model) and using the number line (measurement model).
- Understand why addition can be computed by counting on.
- Understand and use the properties of addition (associative , commutative, and additive identity properties).
- Use strategies to compute addition facts to 10 + 10 for fluency such as:
  a. counting on; e.g., 7 + 2 = 9
  b. commutativity; e.g., 2 + 7 = 7 + 2 = 9
  c. derived facts; e.g. , 8 + 8 = 16, 8 + 9 = 8 + 8 + 1 = 17
  d. adding 10; e.g., 7 + 10 = 17
  e. combinations to ten; e.g., 5 + 5, 4 + 6, 3 + 7, 2 + 8, 1 + 9
  f. compensation; e.g., 8 + 7 = 10 + 5 = 15
- Use models to explain the standard algorithm as well as other algorithms for addition with and without regrouping.
- Relate addition and subtraction through a variety of interpretations (e.g., part-whole, take away, comparison).
- Use models to understand and use the standard algorithm for subtraction as well as other algorithms for subtraction with and without regrouping.
- Use mental math strategies for addition and subtraction.

Multiplication and Division (3.5 weeks)

- Understand and use models for multiplication including repeated addition, the set model, the measurement model using the number line, and the rectangular array model using both a rectangular array of points and a rectangular array of squares.
- Understand and use the properties of multiplication (associative, commutative, and multiplicative identity properties).
- Understand and use the distributive property of multiplication over addition.
- Do mental math for multiplication using the properties and develop strategies for learning single digit multiplication facts.
- Understand how the following properties are used in the standard algorithm for multiplication as well as other algorithms for multiplication.
  a. shifting place values when multiplying by 10
  b. single-digit multiplication facts
  c. regrouping as used in the addition algorithm
- Justify the standard multiplication algorithm for multiplication by a single-digit number, and for multi-digit numbers.
- Understand and use multiple algorithms for multiplication.
- Relate division to multiplication.

- Recognize and create problems using the partitive and quotitive interpretations of division.
- Understand the standard algorithm for division.
- Use strategies to estimate sums, differences, products, and quotients of multi-digit numbers.

Integers (2.5 weeks)

- Locate negative integers as points on the number line and define the set of integers.
- Understand and use the terms greater than, less than, absolute value, and opposite for integers.
- State and demonstrate the rules for integer arithmetic (e.g., a negative times a negative is a positive).
- Explain integer arithmetic using models (e.g. vector model (left arrows and right arrows on the number line), a chip model (positive and negative chips), and money (own or owe)).
- Develop the rules for integer arithmetic from axioms on numbers (addition, multiplication, identity, inverses, commutative and distributive laws).
- Define an order relationship on the integers and be familiar with the order properties (trichotomy, transitive, order is preserved by adding any number to both sides or multiplying both sides by a positive number).

Factors, Primes, and Proofs (2.5 weeks)

- Give and work with mathematical definitions (e.g., define an even and an odd number, define what it means for one number to be divisible by another).
- Understand the role of definitions in proofs.
- Give proofs understandable to children for simple numerical facts; (e.g., the sum of an even number and an odd number is odd).
- Use and justify divisibility tests for 2, 4, 6, 8, 5, 10, 3, 9, and 11.
- Classify whole numbers greater than 1 as prime or composite using the Sieve of Eratosthenes and by checking divisibility of primes less than or equal to the square root of the number.
- State the Fundamental Theorem of Arithmetic.
- Give the prime factorization of whole numbers greater than one.
- Define and find the greatest common factor of two or more numbers.
- Define and find the least common multiple of two or more numbers.

Pre-algebra (1 week)

- Use numerical and algebraic expressions and equations to solve problems.
- Relate properties of integers to algebraic properties.
- Give examples of algebraic identities and use them for mental math.
- Define, work with, and explain the rules for whole number exponents.

Pedagogy

The purpose of the Elementary Mathematics Endorsement courses is to ensure that practicing teachers gain the mathematical content knowledge needed to teach mathematical concepts to students in the elementary grades. Teachers must also, however, know how to transfer that content knowledge and the conceptual understandings inherent in the content to students. An understanding of sound pedagogical practice is essential to that transfer. Following are suggested pedagogical concepts and strategies that should be infused into the courses to aid teachers in student instruction. These concepts should never be taught in isolation, but should be modeled throughout the courses. It is not necessary nor intended that all the following concepts be infused in every course, but they should be covered in their entirety in the series.

Knowledge of mathematics learners:

- Employ prior knowledge, skills, dispositions, beliefs and conceptions, to clarify misconceptions and confusions around particular topics;
- Employ knowledge of cognition, child development, and learning theory;
- Use knowledge of learners culturally and socially to know what motivates and inspires them.

Knowledge of effective mathematics teaching practices for student and teacher learning:

- Design, select and adapt worthwhile tasks and sequences of examples that support a particular learning goal, including tasks to be solved by teachers themselves, and others for which they try to anticipate children's solutions, and design responses.
- Develop sensitivity with the careful use of technical language, attending to both mathematical integrity and usability by learners.
- Construct and evaluate multiple representations of a mathematical idea or process, and establish correspondences between representations.
- Use questions to effectively probe mathematical understanding and make productive use of responses.
- Develop learners' skills at clear and coherent public mathematical communication in a classroom setting, using a variety of resources and media.
- Model and cultivate mathematical practices – questioning, representing, communicating, problem solving, inquiring and conjecturing, making connections, reasoning and proving, self-assessing.
- Use various instructional applications of technology and be able to make judicious, mathematically and pedagogically grounded use of them.
- Construct and evaluate mathematical contexts or story problems for various mathematical situations and processes.
- Analyze and evaluate student work, and design appropriate responses.
- Develop skillful and flexible use of grouping strategies – whole group, small group, partner, and individual – in support of learning goals.

- Know the resources for managing the diversities – cultural, linguistic, gender, socioeconomic, developmental – of the classroom, and ways to skillfully deploy them in support of the learning of all students.
- Understand and address diversity in planning and implementing instruction and in helping other teachers foster mathematics classroom environments that support the learning of all students.
- Understand and use various cultural approaches to learning mathematics.

Knowledge of mathematics curriculum and assessment:

- Know common developmental paths (learning trajectories) related to foundational mathematical topics (e.g., place value, rational numbers and equivalence); understand these developmental paths, and use them to sequence activities to build learning environments that are developmentally appropriate and effective.
- Select, use, adapt and determine the suitability of mathematics curricula and teaching materials (e.g., textbooks, technology, manipulatives) for particular learning goals.
- Be cognizant of a formative assessment cycle (administering a formative assessment task, analyzing student work from the task, using that analysis to enhance teacher knowledge, and designing and teaching reengagement lessons) and resources.
- Provide appropriate interpretations of assessment results, and communicate results (in context) to specific individuals and groups (e.g., students, parents, caregivers, colleagues, administrators, policymakers, community members).

(Standards for Elementary Mathematics Specialists, Association of Mathematics Teacher Educators, 2009)

Possible Assignments:

- Mathematical problem solving tasks: Strategically selected mathematical tasks focused on developing teacher understanding of numbers and operations in a problem solving arena.
- Clinical interview: Design an interview to determine student understanding of numbers and operations, including interview questions with extensions to press for student understanding. Videotape and conduct the interview. Analyze the video using a rubric that assesses both student understanding and the interview process.
- Differentiated lesson: Select one of the course objectives. Develop a lesson to meet the objective for the whole group. Then develop differentiation activities for at least three populations (e.g., gifted students, English Language Learners, Student with Disabilities).
- Action research project: Action research is a particular approach to research that aims to improve practice by addressing real-world situations and needs through focused, reflective practice and the application of interventions or procedures that are designed for improving performance. Develop a classroom research project about teaching concepts of fractions for your own classroom. Consult the

mathematics education literature, citing at least three sources from peer-reviewed journals. Develop a research question which could be a new teaching, assessment, or curriculum strategy for rational numbers in your class. Collect data, and analyze the results.

- Case study: Write a case presenting the mathematical thinking of a student or group of students. Be sure to include details in your narrative, such as student dialogue, your questions, and what you were thinking as you listened to the students. Analyze the student thinking and discuss the questions that are raised for you in the students' mathematical thinking.
- Lesson study cycle: Work with a group to develop a task-based lesson including anticipated student responses, questions for classroom discourse, and formative assessment. One person in the group will teach the lesson while other members of the group observe student thinking. Group members refine the lesson in response to the observation data and then teach and analyze the lesson in their own classes.
- Reflections on your own mathematical understanding: Keep a reflections log that discusses: a mathematical idea from the session that is new or important for you, a question you have, and an application of the mathematical idea for your classroom.
- Content exams, performance tasks, and homework for determining participants' mathematical knowledge for teaching on the topic of numbers and operations.
- Reading research – choose from possible articles and an assignment based on the article; discussions, agreement/disagreement, impact in classrooms.
- Lesson study via videotape

Suggested Texts:

Beckmann, Sybilla (2008). *Mathematics for Elementary Teachers (2nd ed.)*. Boston: Pearson.

Parker, Thomas H. and Baldridge, Scott J. (2005). *Elementary Mathematics for Teachers*. Portland, OR: Sefton-Ash Publishers.

Van De Walle, John A. et. al., (2009). *Elementary and Middle School Mathematics, Teaching Developmentally (7th Edition).* Boston, MA: Allyn & Bacon.

Carpenter, Thomas P. et. al., (2003). *Thinking Mathematically: Integrating Arithmetic and Algebra in Elementary School*. Chicago, IL: Heinemann.

Schifter, Deborah, et. al., (2007). *Reasoning Algebraically About Operations Casebook: A Collaborative Project by the Staff and Participants of Teaching to the Big Ideas (Developing Mathematical Ideas).* Palo Alto, CA: Dale Seymour Publications.

Schifter, Deborah, et. al. (2007). *Building A System Of Tens: Casebook: Numbers and Operations (Developing Mathematical Ideas)* Palo Alto, CA: Dale Seymour Publications.

Supplementary Texts/Readings:

Fennema, Elizabeth (1992). *Cognitively Guided Instruction.* Washington, DC: Office of Educational Research and Improvement (ED).

Ashlock, R. A. (2009). *Error patterns in computation: Using Error Patterns to Improve Iinstruction. (10th ed).* Boston: Allyn & Bacon.

National Council of Teachers of Mathematics (2000). *Principles and Standards for School Mathematics.* Reston, VA: NCTM.

Van de Walle, John A. (2009). *Elementary and Middle School Mathematics: Teaching Developmentally (7th edition).* Boston: Allyn & Bacon.

Journal of Research in Mathematics Education, Teaching Children Mathematics; various as appropriate. Reston, VA: NCTM

Ministry of Education, Singapore. *Singapore Mathematics Singapore Primary Mathematics Textbooks 3A, 4A, 5A, and 6A.* Oregon City, OR: Marshall Cavendish Education.

Ministry of Education, Singapore. *Singapore Primary Mathematics Workbook 5A.* Oregon City, OR: Marshall Cavendish Education.