MATHEMATICS
Middle/Junior High (6–8)

Utah STATE OFFICE of Education

UTAH CORE STATE STANDARDS
for
MATHEMATICS
MIDDLE/JUNIOR HIGH SCHOOL GRADES (6–8)

Adopted August 2010

by the

Utah State Board of Education

Revised September 2015–April 2016

UTAH STATE BOARD OF EDUCATION

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2/2016

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INTRODUCTION

Organization of the Standards

The Utah Core Standards are organized into **strands**, which represent significant areas of learning within content areas. Depending on the core area, these strands may be designated by time periods, thematic principles, modes of practice, or other organizing principles.

Within each strand are **standards**. A standard is an articulation of the demonstrated proficiency to be obtained. A standard represents an essential element of the learning that is expected. While some standards within a strand may be more comprehensive than others, all standards are essential for mastery.

Understanding Mathematics

These standards define what students should understand and be able to do in their study of mathematics. Asking a student to understand something means asking a teacher to assess whether the student has understood it. But what does mathematical understanding look like? One hallmark of mathematical understanding is the ability to justify, in a way appropriate to the student's mathematical maturity, why a particular mathematical statement is true or where a mathematical rule comes from. Mathematical understanding and procedural skill are equally important, and both are assessable using mathematical tasks of sufficient richness.

The standards set grade-specific standards but do not dictate curriculum or teaching methods, nor do they define the intervention methods or materials necessary to support students who are well below or well above grade-level expectations. It is also beyond the scope of the Standards to define the full range of supports appropriate for English language learners and for students with special needs. At the same time, all students must have the opportunity to learn and meet the same high standards if they are to access the knowledge and skills necessary in their post-school lives. The standards should be read as allowing for the widest possible range of students to participate fully from the outset, along with appropriate accommodations to ensure maximum participation of students with special education needs. No set of grade-specific standards can fully reflect the great variety in abilities, needs, learning rates, and achievement levels of students in any given classroom. However, the standards do provide clear signposts along the way to the goal of college and career readiness for all students.

What students can learn at any particular grade level depends upon what they have learned before. Ideally then, each standard in this document might have been phrased in the form, "Students who already know… should next come to learn ..." Grade placements for specific topics have been made on the basis of state and international comparisons and the collective experience and collective professional judgment of educators, researchers and mathematicians. Learning opportunities will continue to vary across schools and school systems, and educators should make every effort to meet the needs of individual students based on their current understanding.

Utah STATE OFFICE of Education

UTAH CORE STATE STANDARDS
for
MATHEMATICS

6–8

Mathematics | Grade 6

In Grade 6, instructional time should focus on four critical areas: (1) connecting ratio and rate to whole number multiplication and division and using concepts of ratio and rate to solve problems; (2) completing understanding of division of fractions and extending the notion of number to the system of rational numbers, which includes negative numbers; (3) writing, interpreting, and using expressions and equations; and (4) developing understanding of statistical thinking.

(1) Students will use reasoning about multiplication and division to solve ratio and rate problems about quantities. By viewing equivalent ratios and rates as deriving from, and extending, pairs of rows (or columns) in the multiplication table, and by analyzing simple drawings that indicate the relative size of quantities, students will connect their understanding of multiplication and division with ratios and rates. Thus students will expand the scope of problems for which they can use multiplication and division to solve problems, and they connect ratios and fractions. Students will solve a wide variety of problems involving ratios and rates.

(2) Students will use the meaning of fractions, the meanings of multiplication and division, and the relationship between multiplication and division to understand and explain why the procedures for dividing fractions make sense. Students will use these operations to solve problems. Students will extend their previous understandings of number and the ordering of numbers to the full system of rational numbers, which includes negative rational numbers, and in particular negative integers. They will reason about the order and absolute value of rational numbers and about the location of points in all four quadrants of the coordinate plane.

(3) Students will understand the use of variables in mathematical expressions. They will write expressions and equations that correspond to given situations, evaluate expressions, and use expressions and formulas to solve problems. Students will understand that expressions in different forms can be equivalent, and they will use the properties of operations to rewrite expressions in equivalent forms. Students will know that the solutions of an equation are the values of the variables that make the equation true. Students will use properties of operations and the idea of maintaining the equality of both sides of an equation to solve simple one-step equations. Students will construct and analyze tables, such as tables of quantities that are in equivalent ratios, and they will use equations (such as $3x = y$) to describe relationships between quantities.

(4) Building on and reinforcing their understanding of number, students will begin to develop their ability to think statistically. Students will recognize that a data distribution may not have a definite center and that different ways to measure center yield different values. The median measures center in the sense that it is roughly the middle value. The mean measures center in the sense that it is the value that each data point would take on if the total of the data values were redistributed equally, and also in the sense that it is a balance point. Students will recognize that a measure of variability (interquartile range or mean absolute deviation) can also be useful for summarizing data because two very different

sets of data can have the same mean and median yet be distinguished by their variability. Students will learn to describe and summarize numerical data sets, identifying clusters, peaks, gaps, and symmetry, considering the context in which the data were collected.

(5) Students in Grade 6 will also build on their work with area in elementary school by reasoning about relationships among shapes to determine area, surface area, and volume. They will find areas of right triangles, other triangles, and special quadrilaterals by decomposing these shapes, rearranging or removing pieces, and relating the shapes to rectangles. Using these methods, students will discuss, develop, and justify formulas for areas of triangles and parallelograms. Students will find areas of polygons and surface areas of prisms and pyramids by decomposing them into pieces whose area they can determine. They will reason about right rectangular prisms with fractional side lengths to extend formulas for the volume of a right rectangular prism to fractional side lengths. They will prepare for work on scale drawings and constructions in Grade 7 by drawing polygons in the coordinate plane.

Strand: MATHEMATICAL PRACTICES (6.MP)

The Standards for Mathematical Practice in Sixth Grade describe mathematical habits of mind that teachers should seek to develop in their students. Students become mathematically proficient in engaging with mathematical content and concepts as they learn, experience, and apply these skills and attitudes **(Standards 6.MP.1–8).**

- **Standard 6.MP.1 Make sense of problems and persevere in solving them.** Explain the meaning of a problem and look for entry points to its solution. Analyze givens, constraints, relationships, and goals. Make conjectures about the form and meaning of the solution, plan a solution pathway, and continually monitor progress asking, "Does this make sense?" Consider analogous problems, make connections between multiple representations, identify the correspondence between different approaches, look for trends, and transform algebraic expressions to highlight meaningful mathematics. Check answers to problems using a different method.

- **Standard 6.MP.2 Reason abstractly and quantitatively.** Make sense of the quantities and their relationships in problem situations. Translate between context and algebraic representations by contextualizing and decontextualizing quantitative relationships. This includes the ability to decontextualize a given situation, representing it algebraically and manipulating symbols fluently as well as the ability to contextualize algebraic representations to make sense of the problem.

- **Standard 6.MP.3 Construct viable arguments and critique the reasoning of others.** Understand and use stated assumptions, definitions, and previously established results in constructing arguments. Make conjectures and build a logical progression of statements to explore the truth of their conjectures. Justify conclusions and communicate them to others. Respond to the arguments of others by listening, asking clarifying questions, and critiquing the reasoning of others.

- **Standard 6.MP.4 Model with mathematics.** Apply mathematics to solve problems arising in everyday life, society, and the workplace. Make assumptions and approximations, identifying important quantities to construct a mathematical model. Routinely interpret mathematical results in the context of the situation and reflect on whether the results make sense, possibly improving the model if it has not served its purpose.

- **Standard 6.MP.5 Use appropriate tools strategically.** Consider the available tools and be sufficiently familiar with them to make sound decisions about when each tool might be helpful, recognizing both the insight to be gained as well as the limitations. Identify relevant external mathematical resources and use them to pose or solve problems. Use tools to explore and deepen their understanding of concepts.

- **Standard 6.MP.6 Attend to precision.** Communicate precisely to others. Use explicit definitions in discussion with others and in their own reasoning. They state the meaning of the symbols they choose. Specify units of measure and label axes to clarify the correspondence with quantities in a problem. Calculate accurately and efficiently, and express numerical answers with a degree of precision appropriate for the problem context.

■ **Standard 6.MP.7 Look for and make use of structure.** Look closely at mathematical relationships to identify the underlying structure by recognizing a simple structure within a more complicated structure. See complicated things, such as some algebraic expressions, as single objects or as being composed of several objects. For example, see 5 – 3(x – y)2 as 5 minus a positive number times a square and use that to realize that its value cannot be more than 5 for any real numbers x and y.

■ **Standard 6.MP.8 Look for and express regularity in repeated reasoning.** Notice if reasoning is repeated, and look for both generalizations and shortcuts. Evaluate the reasonableness of intermediate results by maintaining oversight of the process while attending to the details.

Strand: RATIOS AND PROPORTIONAL RELATIONSHIPS (6.RP)

Understand ratio concepts and use ratio reasoning to solve problems **(Standards 6.RP.1–3).**

■ **Standard 6.RP.1** Understand the concept of a ratio and use ratio language to describe a ratio relationship between two quantities. The following are examples of ratio language: *"The ratio of wings to beaks in the bird house at the zoo was 2:1, because for every two wings there was one beak." "For every vote candidate A received, candidate C received nearly three votes."*

■ **Standard 6.RP.2** Understand the concept of a unit rate a/b associated with a ratio $a:b$ with $b \neq 0$, and use rate language in the context of a ratio relationship. The following are examples of rate language: *"This recipe has a ratio of four cups of flour to two cups of sugar, so the rate is two cups of flour for each cup of sugar." "We paid $75 for 15 hamburgers, which is a rate of $5 per hamburger."* (In sixth grade, unit rates are limited to non-complex fractions.)

■ **Standard 6.RP.3** Use ratio and rate reasoning to solve real-world (with a context) and mathematical (void of context) problems, using strategies such as reasoning about tables of equivalent ratios, tape diagrams, double number line diagrams, or equations involving unit rate problems.

   a. Make tables of equivalent ratios relating quantities with whole-number measurements, find missing values in the tables, and plot the pairs of values on the coordinate plane. Use tables to compare ratios.

   b. Solve unit rate problems including those involving unit pricing and constant speed. *For example, if it took four hours to mow eight lawns, how many lawns could be mowed in 32 hours? What is the hourly rate at which lawns were being mowed?*

   c. Find a percent of a quantity as a rate per 100. Solve problems involving finding the whole, given a part and the percent. *(For example, 30% of a quantity means 30/100 times the quantity.)*

   d. Use ratio reasoning to convert measurement units; manipulate and transform units appropriately when multiplying or dividing quantities.

Strand: THE NUMBER SYSTEM (6.NS)

Apply and extend previous understandings of multiplication and division of whole numbers to divide fractions by fractions **(Standard 6.NS.1).** Compute (add, subtract, multiply and divide) fluently with multi-digit numbers and decimals and find common factors and multiples **(Standards 6.NS.2–4).** Apply and extend previous understandings of numbers to the system of rational numbers **(Standards 6.NS.5–8).**

■ **Standard 6.NS.1**   Interpret and compute quotients of fractions.

  a. Compute quotients of fractions by fractions, *for example, by applying strategies such as visual fraction models, equations, and the relationship between multiplication and division, to represent problems.*

  b. Solve real-world problems involving division of fractions by fractions. *For example, how much chocolate will each person get if three people share 1/2 pound of chocolate equally? How many 3/4-cup servings are in 2/3 of a cup of yogurt? How wide is a rectangular strip of land with length 3/4 mile and area 1/2 square mile?*

  c. Explain the meaning of quotients in fraction division problems. *For example, create a story context for (2/3) ÷ (3/4) and use a visual fraction model to show the quotient. Use the relationship between multiplication and division to explain that (2/3) ÷ (3/4) = 8/9 because 3/4 of 8/9 is 2/3. (In general, (a/b) ÷ (c/d) = ad/bc.)*

■ **Standard 6.NS.2**   Fluently divide multi-digit numbers using the standard algorithm.

■ **Standard 6.NS.3**   Fluently add, subtract, multiply, and divide multi-digit decimals using the standard algorithm for each operation.

  a. Fluently divide multi-digit decimals using the standard algorithm, limited to a whole number dividend with a decimal divisor or a decimal dividend with a whole number divisor.

  b. Solve division problems in which both the dividend and the divisor are multi-digit decimals; develop the standard algorithm by using models, the meaning of division, and place value understanding.

■ **Standard 6.NS.4**   Find the greatest common factor of two whole numbers less than or equal to 100 and the least common multiple of two whole numbers less than or equal to 12. Use the distributive property to express a sum of two whole numbers 1–100 with a common factor as a multiple of a sum of two whole numbers with no common factor. *For example, express 36 + 8 as 4(9 + 2).*

■ **Standard 6.NS.5**   Understand that positive and negative numbers are used together to describe quantities having opposite directions or values *(for example, temperature above/below zero, elevation above/below sea level, credits/debits, positive/negative electric charge)*; use positive and negative numbers to represent quantities in real-world contexts, explaining the meaning of zero in each situation.

■ **Standard 6.NS.6** Understand a rational number as a point on the number line. Extend number line diagrams and coordinate axes familiar from previous grades to represent points on the line and in the plane with negative number coordinates.

   a. Recognize opposite signs of numbers as indicating locations on opposite sides of zero on the number line; recognize that the opposite of the opposite of a number is the number itself. *For example, -(-3) = 3, and zero is its own opposite.*

   b. Understand that the signs of numbers in ordered pairs indicate their location in quadrants of the coordinate plane; recognize that when two ordered pairs differ only by signs, the locations of the points are related by reflections across one or both axes.

   c. Find and position integers and other rational numbers on a horizontal or vertical number line diagram; find and position pairs of integers and other rational numbers on a coordinate plane.

■ **Standard 6.NS.7** Understand ordering and absolute value of rational numbers.

   a. Interpret statements of inequality as statements about the relative position of two numbers on a number line diagram. *For example, interpret –3 > –7 as a statement that –3 is located to the right of –7 on a number line oriented from left to right.*

   b. Write, interpret, and explain statements of order for rational numbers in real-world contexts. *For example, write –3° C > –7° C to express the fact that –3° C is warmer than –7° C.*

   c. Understand the absolute value of a rational number as its distance from zero on the number line; interpret absolute value as magnitude for a positive or negative quantity in a real-world context. *For example, for an account balance of –30 dollars, write |–30|= 30 to describe the size of the debt in dollars.*

   d. Distinguish comparisons of absolute value from statements about order. *For example: Recognize that an account balance less than –30 dollars represents a debt greater than 30 dollars.*

■ **Standard 6.NS.8** Solve real-world and mathematical problems by graphing points in all four quadrants of the coordinate plane. Include use of coordinates and absolute value to find distances between points with the same *x*-coordinate or the same *y*-coordinate.

Strand: EXPRESSIONS AND EQUATIONS (6.EE)

Apply and extend previous understandings of arithmetic to algebraic expressions involving exponents and variables **(Standards 6.EE.1–4).** They reason about and solve one-variable equations and inequalities **(Standards 6.EE.5–8).** Represent and analyze quantitative relationships between dependent and independent variables in a real-world context **(Standard 6.EE.9).**

■ **Standard 6.EE.1** Write and evaluate numerical expressions involving whole-number exponents.

■ **Standard 6.EE.2** Write, read, and evaluate expressions in which letters represent numbers.

   a. Write expressions that record operations with numbers and with letters representing numbers. *For example, express the calculation "Subtract* y *from 5" as 5 –* y *and express "Jane had $105.00 in her bank account. One year later, she had* x *dollars more. Write an expression that shows her new balance" as $105.00 +* x.

   b. Identify parts of an expression using mathematical terms (for example, sum, term, product, factor, quotient, coefficient); view one or more parts of an expression as a single entity and a sum of two terms. *For example, describe the expression 2(8 + 7) as a product of two factors; view (8 + 7) as both a single entity and a sum of two terms.*

   c. Evaluate expressions at specific values of their variables. Include expressions that arise from formulas used in real-world problems. Perform arithmetic operations, including those involving whole-number exponents, applying the Order of Operations when there are no parentheses to specify a particular order. *For example, use the formulas* $V = s^3$ *and* $A = 6s^2$ *to find the volume and surface area of a cube with sides of length* s = 1/2.

■ **Standard 6.EE.3** Apply the properties of operations to generate equivalent expressions. *For example, apply the distributive property to the expression 3(2 +* x*) to produce the equivalent expression 6 + 3x; apply the distributive property to the expression 24x + 18y to produce the equivalent expression 6(4x + 3y); apply properties of operations to* y + y + y *to produce the equivalent expression 3*y.

■ **Standard 6.EE.4** Identify when two expressions are equivalent. *For example, the expressions* y + y + y *and 3*y *are equivalent because they name the same number, regardless of which number* y *represents.*

■ **Standard 6.EE.5** Understand solving an equation or inequality as a process of answering the question: Which values from a specified set, if any, make the equation or inequality true? Use substitution to determine whether a given number in a specified set makes an equation or inequality true.

■ **Standard 6.EE.6** Use variables to represent numbers and write expressions when solving a real-world or mathematical problem; understand that a variable can represent an unknown number, or, depending on the purpose at hand, any number in a specified set.

■ **Standard 6.EE.7** Solve real-world and mathematical problems by writing and solving equations of the form $x + a = b$ and $ax = b$ for cases in which a, b and x are all non-negative rational numbers.

■ **Standard 6.EE.8** Write an inequality of the form $x > c$ or $x < c$ to represent a constraint or condition in a real-world or mathematical problem. Recognize that inequalities of the form $x > c$ or x < c have infinitely many solutions; represent solutions of such inequalities on number line diagrams.

■ **Standard 6.EE.9** Use variables to represent two quantities in a real-world problem that change in relationship to one another; write an equation to express one quantity,

thought of as the dependent variable, in terms of the other quantity, thought of as the independent variable. Analyze the relationship between the dependent and independent variables using graphs and tables, and relate these to the equation. *For example, in a problem involving motion at constant speed, list and graph ordered pairs of distances and times, and write the equation* d = 65t *to represent the relationship between distance and time.*

Strand: GEOMETRY (6.G)

Solve real-world and mathematical problems involving area, surface area, and volume **(Standards 6.G.1–4).**

■ **Standard 6.G.1**   Find the area of right triangles, other triangles, special quadrilaterals, and polygons by composing and decomposing into rectangles, triangles and/or other shapes; apply these techniques in the context of solving real-world and mathematical problems.

■ **Standard 6.G.2**   Find the volume of a right rectangular prism with appropriate unit fraction edge lengths by packing it with cubes of the appropriate unit fraction edge lengths *(for example, 3½ x 2 x 6)*, and show that the volume is the same as would be found by multiplying the edge lengths of the prism. Apply the formulas $V = lwh$ and $V = bh$ to find volumes of right rectangular prisms with fractional edge lengths in the context of solving real-world and mathematical problems. (Note: Model the packing using drawings and diagrams.)

■ **Standard 6.G.3**   Draw polygons in the coordinate plane given coordinates for the vertices; use coordinates to find the length of a side joining points with the same *x* coordinate or the same *y* coordinate. Apply these techniques in the context of solving real-world and mathematical problems.

■ **Standard 6.G.4**   Represent three-dimensional figures using nets made up of rectangles and triangles, and use the nets to find the surface area of these figures. Apply these techniques in the context of solving real-world and mathematical problems.

Strand: STATISTICS AND PROBABILITY (6.SP)

Develop understanding of statistical variability of data **(Standards 6.SP.1–3).** Summarize and describe distributions **(Standards 6.SP.4–5).**

■ **Standard 6.SP.1** Recognize a statistical question as one that anticipates variability in the data related to the question and accounts for it in the answers. *For example, "How old am I?" is not a statistical question, but "How old are the students in my school?" is a statistical question because one anticipates variability in students' ages.*

■ **Standard 6.SP.2** Understand that a set of data collected to answer a statistical question has a distribution that can be described by its center, spread/range and overall shape.

■ **Standard 6.SP.3** Recognize that a measure of center for a numerical data set summarizes all of its values with a single number, while a measure of variation describes how its values vary with a single number.

■ **Standard 6.SP.4** Display numerical data in plots on a number line, including dot plots, histograms and box plots. Choose the most appropriate graph/plot for the data collected.

■ **Standard 6.SP.5** Summarize numerical data sets in relation to their context, such as by:

    a. Reporting the number of observations.

    b. Describing the nature of the attribute under investigation, including how it was measured and its units of measurement.

    c. Giving quantitative measures of center (median and/or mean) and variability (interquartile range and/or mean absolute deviation), as well as describing any overall pattern and any striking deviations (for example, outliers) from the overall pattern with reference to the context in which the data were gathered.

    d. Relating the choice of measures of center and variability to the shape of the data distribution and the context in which the data were gathered.

Mathematics | Grade 7

In Grade 7, instructional time should focus on four critical areas: (1) developing understanding of and applying proportional relationships; (2) developing understanding of operations with rational numbers and working with expressions and linear equations; (3) solving problems involving scale drawings and informal geometric constructions, and working with two- and three-dimensional shapes to solve problems involving area, surface area, and volume; and (4) drawing inferences about populations based on samples.

(1) Students extend their understanding of ratios and develop understanding of proportionality to solve single- and multi-step problems. Students use their understanding of ratios and proportionality to solve a wide variety of percent problems, including those involving discounts, interest, taxes, tips, and percent increase or decrease. Students solve problems about scale drawings by relating corresponding lengths between the objects or by using the fact that relationships of lengths within an object are preserved in similar objects. Students graph proportional relationships and understand the unit rate informally as a measure of the steepness of the related line, called the slope. They distinguish proportional relationships from other relationships.

(2) Students develop a unified understanding of number, recognizing fractions, decimals (that have a finite or a repeating decimal representation), and percents as different representations of rational numbers. Students extend addition, subtraction, multiplication, and division to all rational numbers, maintaining the properties of operations and the relationships between addition and subtraction, and between multiplication and division. By applying these properties, and by viewing negative numbers in terms of everyday contexts (e.g., amounts owed or temperatures below zero), students explain and interpret the rules for adding, subtracting, multiplying, and dividing with negative numbers. They use the arithmetic of rational numbers as they formulate expressions and equations in one variable and use these equations to solve problems.

(3) Students continue their work with area from Grade 6, solving problems involving the area and circumference of a circle and surface area of three-dimensional objects. In preparation for work on congruence and similarity in Grade 8, they reason about relationships among two-dimensional figures using scale drawings and informal geometric constructions, and they gain familiarity with the relationships between angles formed by intersecting lines. Students work with three-dimensional figures, relating them to two-dimensional figures by examining cross-sections. They solve real-world and mathematical problems involving area, surface area, and volume of two- and three-dimensional objects composed of triangles, quadrilaterals, polygons, cubes and right prisms.

(4) Students build on their previous work with single-data distributions to compare two-data distributions and address questions about differences between populations. They begin informal work with random sampling to generate data sets and learn about the importance of representative samples for drawing inferences.

Strand: MATHEMATICAL PRACTICES (7.MP)

The Standards for Mathematical Practice in Seventh Grade describe mathematical habits of mind that teachers should seek to develop in their students. Students become mathematically proficient in engaging with mathematical content and concepts as they learn, experience, and apply these skills and attitudes **(Standards 7.MP.1–8).**

- **Standard 7.MP.1 Make sense of problems and persevere in solving them.** Explain the meaning of a problem and look for entry points to its solution. Analyze givens, constraints, relationships, and goals. Make conjectures about the form and meaning of the solution, plan a solution pathway, and continually monitor progress asking, "Does this make sense?" Consider analogous problems, make connections between multiple representations, identify the correspondence between different approaches, look for trends, and transform algebraic expressions to highlight meaningful mathematics. Check answers to problems using a different method.

- **Standard 7.MP.2 Reason abstractly and quantitatively.** Make sense of the quantities and their relationships in problem situations. Translate between context and algebraic representations by contextualizing and decontextualizing quantitative relationships. This includes the ability to decontextualize a given situation, representing it algebraically and manipulating symbols fluently as well as the ability to contextualize algebraic representations to make sense of the problem.

- **Standard 7.MP.3 Construct viable arguments and critique the reasoning of others.** Understand and use stated assumptions, definitions, and previously established results in constructing arguments. Make conjectures and build a logical progression of statements to explore the truth of their conjectures. Justify conclusions and communicate them to others. Respond to the arguments of others by listening, asking clarifying questions, and critiquing the reasoning of others.

- **Standard 7.MP.4 Model with mathematics.** Apply mathematics to solve problems arising in everyday life, society, and the workplace. Make assumptions and approximations, identifying important quantities to construct a mathematical model. Routinely interpret mathematical results in the context of the situation and reflect on whether the results make sense, possibly improving the model if it has not served its purpose.

- **Standard 7.MP.5 Use appropriate tools strategically.** Consider the available tools and be sufficiently familiar with them to make sound decisions about when each tool might be helpful, recognizing both the insight to be gained as well as the limitations. Identify relevant external mathematical resources and use them to pose or solve problems. Use tools to explore and deepen their understanding of concepts.

- **Standard 7.MP.6 Attend to precision.** Communicate precisely to others. Use explicit definitions in discussion with others and in their own reasoning. They state the meaning of the symbols they choose. Specify units of measure and label axes to clarify the correspondence with quantities in a problem. Calculate accurately and efficiently, express numerical answers with a degree of precision appropriate for the problem context.

■ **Standard 7.MP.7 Look for and make use of structure.** Look closely at mathematical relationships to identify the underlying structure by recognizing a simple structure within a more complicated structure. See complicated things, such as some algebraic expressions, as single objects or as being composed of several objects. *For example, see 5 – 3(x – y)² as 5 minus a positive number times a square and use that to realize that its value cannot be more than 5 for any real numbers x and y.*

■ **Standard 7.MP.8 Look for and express regularity in repeated reasoning.** Notice if reasoning is repeated, and look for both generalizations and shortcuts. Evaluate the reasonableness of intermediate results by maintaining oversight of the process while attending to the details.

Strand: RATIOS AND PROPORTIONAL RELATIONSHIPS (7.RP)

Analyze proportional relationships and use them to solve real-world and mathematical problems **(Standards 7.RP.1–3).**

■ **Standard 7.RP.1** Compute unit rates associated with ratios of fractions, including ratios of lengths, areas and other quantities measured in like or different units. *For example, if a person walks ½ mile in each ¼ hour, compute the unit rate as the complex fraction ½/¼ miles per hour, equivalently 2 miles per hour.*

■ **Standard 7.RP.2** Recognize and represent proportional relationships between quantities.

   a. Decide whether two quantities are in a proportional relationship, e.g., by testing for equivalent ratios in a table or graphing on a coordinate plane and observing whether the graph is a straight line through the origin.

   b. Identify the constant of proportionality (unit rate) in tables, graphs, equations, diagrams, and verbal descriptions of proportional relationships.

   c. Represent proportional relationships by equations. *For example, if total cost t is proportional to the number n of items purchased at a constant price p, the relationship between the total cost and the number of items can be expressed as* t = pn.

   d. Explain what a point *(x, y)* on the graph of a proportional relationship means in terms of the situation, with special attention to the points (0, 0) and (1, *r*) where *r* is the unit rate.

■ **Standard 7.RP.3** Use proportional relationships to solve multi-step ratio and percent problems. *Examples: simple interest, tax, markups and markdowns, gratuities and commissions, fees, percent increase and decrease, percent error.*

Strand: THE NUMBER SYSTEM (7.NS)

Apply and extend previous understandings of operations with fractions to add, subtract, multiply, and divide rational numbers **(Standards 7.NS.1–3).**

■ **Standard 7.NS.1** Apply and extend previous understandings of addition and subtraction to add and subtract rational numbers; represent addition and subtraction on a horizontal or vertical number line diagram.

  a. Describe situations in which opposite quantities combine to make 0. *For example, a hydrogen atom has 0 charge because its two constituents are oppositely charged.*

  b. Understand $p + q$ as the number located a distance $|q|$ from p in the positive or negative direction, depending on whether q is positive or negative. Show that a number and its opposite have a sum of 0 (are additive inverses). Interpret sums of rational numbers by describing real-world contexts.

  c. Understand subtraction of rational numbers as adding the additive inverse, $p - q = p + (-q)$. Show that the distance between two rational numbers on the number line is the absolute value of their difference, and apply this principle in real-world contexts.

  d. Apply properties of operations as strategies to add and subtract rational numbers.

■ **Standard 7.NS.2** Apply and extend previous understandings of multiplication and division and of fractions to multiply and divide rational numbers.

  a. Understand that multiplication is extended from fractions to rational numbers by requiring that operations continue to satisfy the properties of operations, particularly the distributive property, leading to products such as $(-1)(-1) = 1$ and the rules for multiplying signed numbers. Interpret products of rational numbers by describing real-world contexts.

  b. Understand that integers can be divided, provided the divisor is not zero, and that every quotient of integers (with non-zero divisor) is a rational number. If p and q are integers, then $-(p/q) = (-p)/q = p/(-q)$. Interpret quotients of rational numbers by describing real-world contexts.

  c. Apply properties of operations as strategies to multiply and divide rational numbers.

  d. Convert a rational number to a decimal using long division; know that the decimal form of a rational number terminates in 0s or eventually repeats.

■ **Standard 7.NS.3** Solve real-world and mathematical problems involving the four operations with rational numbers. Computations with rational numbers extend the rules for manipulating fractions to complex fractions.

Strand: EXPRESSIONS AND EQUATIONS (7.EE)

Use properties of operations to generate equivalent expressions **(Standards 7.EE.1–2).** Solve real-life and mathematical problems using numerical and algebraic expressions and equations **(Standards 7.EE.3–4).**

■ **Standard 7.EE.1** Apply properties of operations as strategies to add, subtract, factor, and expand linear expressions with rational coefficients.

■ **Standard 7.EE.2** Understand that rewriting an expression in different forms in a problem context can shed light on the problem, and how the quantities in it are related. *For example,* a + *0.05*a = *1.05*a *means that "increase by 5%" is the same as "multiply by 1.05."*

■ **Standard 7.EE.3** Solve multi-step real-life and mathematical problems posed with positive and negative rational numbers in any form (whole numbers, fractions, and decimals), using tools strategically. Apply properties of operations to calculate with numbers in any form, convert between forms as appropriate, and assess the reasonableness of answers using mental computation and estimation strategies. *For example: If a woman making $25 an hour gets a 10% raise, she will make an additional 1/10 of her salary an hour, or $2.50, for a new salary of $27.50. If you want to place a towel bar 9 3/4 inches long in the center of a door that is 27 1/2 inches wide, you will need to place the bar about 9 inches from each edge; this estimate can be used as a check on the exact computation.*

■ **Standard 7.EE.4** Use variables to represent quantities in a real-world or mathematical problem, and construct simple equations and inequalities to solve problems by reasoning about the quantities.

   a. Solve word problems leading to equations of the form $px + q = r$ and $p(x + q) = r$, where p, q, and r are specific rational numbers. Solve equations of these forms fluently. Compare an algebraic solution to an arithmetic solution, identifying the sequence of the operations used in each approach. *For example, the perimeter of a rectangle is 54 cm. Its length is 6 cm. What is its width?*

   b. Solve word problems leading to inequalities of the form $px + q > r$ or $px + q < r$, where p, q, and r are specific rational numbers. Graph the solution set of the inequality and interpret it in the context of the problem. *For example: As a salesperson, you are paid $50 per week plus $3 per sale. This week you want your pay to be at least $100. Write an inequality for the number of sales you need to make, and describe the solutions.*

Strand: GEOMETRY (7.G)

Draw, construct, and describe geometrical figures, and describe the relationships between them **(Standards 7.G.1–3).** Solve real-life and mathematical problems involving angle measure, area, surface area, and volume **(Standards 7.G.4–6).**

■ **Standard 7.G.1**   Solve problems involving scale drawings of geometric figures, such as computing actual lengths and areas from a scale drawing and reproducing a scale drawing at a different scale.

■ **Standard 7.G.2**   Draw (freehand, with ruler and protractor, and with technology) geometric shapes with given conditions. Focus on constructing triangles from three measures of angles or sides, noticing when the conditions determine a unique triangle, more than one triangle, or no triangle.

■ **Standard 7.G.3**   Describe the two-dimensional figures that result from slicing three-dimensional figures, as in plane sections of right rectangular prisms and right rectangular pyramids.

- **Standard 7.G.4**   Know the formulas for the area and circumference of a circle, and solve problems; give an informal derivation of the relationship between the circumference and area of a circle.

- **Standard 7.G.5**   Use facts about supplementary, complementary, vertical, and adjacent angles in a multi-step problem to write, and use them to solve simple equations for an unknown angle in a figure.

- **Standard 7.G.6**   Solve real-world and mathematical problems involving area, volume and surface area of two- and three-dimensional objects composed of triangles, quadrilaterals, polygons, cubes, and right prisms.

Strand: STATISTICS AND PROBABILITY (7.SP)

Use random sampling to draw inferences about a population **(Standards 7.SP.1–2).** Draw informal comparative inferences about two populations **(Standards 7.SP.3–4).** Investigate chance processes and develop, use, and evaluate probability models **(Standards 7.SP.5–8).**

- **Standard 7.SP.1** Understand that statistics can be used to gain information about a population by examining a sample of the population, and that generalizations about a population from a sample are valid only if the sample is representative of that population. Understand that random sampling is more likely to produce representative samples and support valid inferences.

- **Standard 7.SP.2** Use data from a random sample to draw inferences about a population with an unknown characteristic of interest. Generate multiple samples (or simulated samples) of the same size to gauge the variation in estimates or predictions. *For example, estimate the mean word length in a book by randomly sampling words from the book; predict the winner of a school election based on randomly sampled survey data. Gauge how far off the estimate or prediction might be.*

- **Standard 7.SP.3** Informally assess the degree of visual overlap of two numerical data distributions with similar variabilities, estimating the difference between the centers by expressing it as a multiple of a measure of variability. *For example, the mean height of players on the basketball team is 10 cm greater than the mean height of players on the soccer team, approximately twice the variability (mean absolute deviation) on either team; on a dot plot, the separation between the two distributions of heights is noticeable.*

- **Standard 7.SP.4** Use measures of center and measures of variability for numerical data from random samples to draw informal comparative inferences about two populations. *For example, decide whether the words in a chapter of a seventh grade science book are generally longer than the words in a chapter of a fourth grade science book.*

- **Standard 7.SP.5** Understand that the probability of a chance event is a number between 0 and 1 that expresses the likelihood of the event occurring. Larger numbers indicate greater likelihood. A probability near 0 indicates an unlikely event, a probability around ½ indicates an event that is neither unlikely nor likely, and a probability near 1 indicates a likely event.

■ **Standard 7.SP.6** Approximate the probability of a chance event by collecting data on the chance process that produces it and observing its long-run relative frequency, and predict the approximate relative frequency given the probability. *For example, when rolling a number cube 600 times, predict that a 3 or 6 would be rolled roughly 200 times, but probably not exactly 200 times.*

■ **Standard 7.SP.7** Develop a probability model and use it to find probabilities of events. Compare probabilities from a model to observed frequencies; if the agreement is not good, explain possible sources of the discrepancy.

> **a.** Develop a uniform probability model by assigning equal probability to all outcomes, and use the model to determine probabilities of events. *For example, if a student is selected at random from a class, find the probability that Jane will be selected and the probability that a girl will be selected.*

> **b.** Develop a probability model (which may not be uniform) by observing frequencies in data generated from a chance process. *For example, find the approximate probability that a spinning penny will land heads up or that a tossed paper cup will land open-end down. Do the outcomes for the spinning penny appear to be equally likely based on the observed frequencies?*

■ **Standard 7.SP.8** Find probabilities of compound events using organized lists, tables, tree diagrams, and simulation.

> **a.** Understand that, just as with simple events, the probability of a compound event is the fraction of outcomes in the sample space for which the compound event occurs.

> **b.** Represent sample spaces for compound events using methods such as organized lists, tables and tree diagrams. For an event described in everyday language (e.g., "rolling double sixes"), identify the outcomes in the sample space which compose the event.

> **c.** Design and use a simulation to generate frequencies for compound events. *For example, use random digits as a simulation tool to approximate the answer to the question: If 40% of donors have type A blood, what is the probability that it will take at least 4 donors to find one with type A blood?*

Mathematics | Grade 8

In Grade 8, instructional time should focus on three critical areas: (1) formulating and reasoning about expressions and equations, including modeling an association in bivariate data with a linear equation, and solving linear equations and systems of linear equations; (2) grasping the concept of a function and using functions to describe quantitative relationships; (3) analyzing two- and three-dimensional space and figures using distance, angle, similarity, and congruence, and understanding and applying the Pythagorean Theorem.

(1) Students use linear equations, linear inequalities, and systems of linear equations to represent, analyze, and solve a variety of problems. Students recognize equations for proportions *(y/x = m or y = mx)* as special linear equations *(y = mx + b)*, understanding that the constant of proportionality *(m)* is the slope, and the graphs are lines through the origin. They understand that the slope *(m)* of a line is a constant rate of change, so that if the input or *x*-coordinate changes by an amount A, the output or *y*-coordinate changes by the amount $m{\times}A$. Students also use a linear equation to describe the association between two quantities in bivariate data (such as arm span vs. height for students in a classroom). At this grade, fitting the model and assessing its fit to the data are done informally. Interpreting the model in the context of the data requires students to express a relationship between the two quantities in question and to interpret components of the relationship (such as slope and *y*-intercept) in terms of the situation.

Students strategically choose and efficiently implement procedures to solve linear equations in one variable, understanding that when they use the properties of equality and the concept of logical equivalence, they maintain the solutions of the original equation. Students solve systems of two linear equations in two variables and relate the systems to pairs of lines in the plane; these intersect, are parallel, or are the same line. Students use linear equations, linear inequalities, systems of linear equations, linear functions, and their understanding of slope of a line to analyze situations and solve problems.

(2) Students grasp the concept of a function as a rule that assigns to each input exactly one output. They understand that functions describe situations where one quantity determines another. They can translate among representations and partial representations of functions (noting that tabular and graphical representations may be partial representations), and they describe how aspects of the function are reflected in the different representations.

(3) Students use ideas about distance and angles; how they behave under translations, rotations, reflections, and dilations; and ideas about congruence and similarity to describe and analyze two-dimensional figures and to solve problems. Students show that the sum of the angles in a triangle is the angle formed by a straight line, and that various configurations of lines give rise to similar triangles because of the angles created when a transversal cuts parallel lines. Students understand the statement of the Pythagorean Theorem and its converse, and can explain why the Pythagorean Theorem holds, for example, by decomposing a square in two different ways. They apply the Pythagorean Theorem to find distances between points on the coordinate plane, to find lengths, and to analyze polygons. Students complete their work on volume by solving problems involving cones, cylinders, and spheres.

Strand: MATHEMATICAL PRACTICES (8.MP)

The Standards for Mathematical Practice in Eighth Grade describe mathematical habits of mind that teachers should seek to develop in their students. Students become mathematically proficient in engaging with mathematical content and concepts as they learn, experience, and apply these skills and attitudes **(Standards 8.MP.1–8).**

- **Standard 8.MP.1 Make sense of problems and persevere in solving them.** Explain the meaning of a problem and look for entry points to its solution. Analyze givens, constraints, relationships, and goals. Make conjectures about the form and meaning of the solution, plan a solution pathway, and continually monitor progress asking, "Does this make sense?" Consider analogous problems, make connections between multiple representations, identify the correspondence between different approaches, look for trends, and transform algebraic expressions to highlight meaningful mathematics. Check answers to problems using a different method.

- **Standard 8.MP.2 Reason abstractly and quantitatively.** Make sense of the quantities and their relationships in problem situations. Translate between context and algebraic representations by contextualizing and decontextualizing quantitative relationships. This includes the ability to decontextualize a given situation, representing it algebraically and manipulating symbols fluently as well as the ability to contextualize algebraic representations to make sense of the problem.

- **Standard 8.MP.3 Construct viable arguments and critique the reasoning of others.** Understand and use stated assumptions, definitions, and previously established results in constructing arguments. Make conjectures and build a logical progression of statements to explore the truth of their conjectures. Justify conclusions and communicate them to others. Respond to the arguments of others by listening, asking clarifying questions, and critiquing the reasoning of others.

- **Standard 8.MP.4 Model with mathematics.** Apply mathematics to solve problems arising in everyday life, society, and the workplace. Make assumptions and approximations, identifying important quantities to construct a mathematical model. Routinely interpret mathematical results in the context of the situation and reflect on whether the results make sense, possibly improving the model if it has not served its purpose.

- **Standard 8.MP.5 Use appropriate tools strategically.** Consider the available tools and be sufficiently familiar with them to make sound decisions about when each tool might be helpful, recognizing both the insight to be gained as well as the limitations. Identify relevant external mathematical resources and use them to pose or solve problems. Use tools to explore and deepen their understanding of concepts.

- **Standard 8.MP.6 Attend to precision.** Communicate precisely to others. Use explicit definitions in discussion with others and in their own reasoning. They state the meaning of the symbols they choose. Specify units of measure and label axes to clarify the correspondence with quantities in a problem. Calculate accurately and efficiently, express numerical answers with a degree of precision appropriate for the problem context.

- **Standard 8.MP.7 Look for and make use of structure.** Look closely at mathematical relationships to identify the underlying structure by recognizing a simple structure within a more complicated structure. See complicated things, such as some algebraic expressions, as single objects or as being composed of several objects. *For example, see 5 – 3(x – y)² as 5 minus a positive number times a square and use that to realize that its value cannot be more than 5 for any real numbers* x *and* y.

- **Standard 8.MP.8 Look for and express regularity in repeated reasoning.** Notice if reasoning is repeated, and look for both generalizations and shortcuts. Evaluate the reasonableness of intermediate results by maintaining oversight of the process while attending to the details.

Strand: NUMBER SYSTEM (8.NS)

Know that there are numbers that are not rational, and approximate them by rational numbers **(Standards 8.NS.1–3).**

- **Standard 8.NS.1** Know that numbers that are not rational are called irrational. Understand informally that every number has a decimal expansion; for rational numbers, show that the decimal expansion repeats eventually, and convert a decimal expansion which repeats eventually into a rational number.

- **Standard 8.NS.2** Use rational approximations of irrational numbers to compare the size of irrational numbers, locate them approximately on a number line diagram, and estimate the value of expressions (e.g., π^2). *For example, by truncating the decimal expansion of $\sqrt{2}$, show that $\sqrt{2}$ is between 1 and 2, then between 1.4 and 1.5, and explain how to continue on to get better approximations.*

- **Standard 8.NS.3** Understand how to perform operations and simplify radicals with emphasis on square roots.

Strand: EXPRESSIONS AND EQUATIONS (8.EE)

Work with radical and integer exponents **(Standards 8.EE.1–4).** Understand the connections between proportional relationships, lines, and linear relationships **(Standards 8.EE.5–6).** Analyze and solve linear equations and inequalities and pairs of simultaneous linear equations **(Standards 8.EE.7–8).**

- **Standard 8.EE.1** Know and apply the properties of integer exponents to generate equivalent numerical expressions. *For example, $3^2 \times 3^{-5} = 3^{-3} = 1/3^3 = 1/27$.*

- **Standard 8.EE.2** Use square root and cube root symbols to represent solutions to equations of the form $x^2 = p$ and $x^3 = p$, where p is a positive rational number. Evaluate square roots of small perfect squares and cube roots of small perfect cubes. Know that $\sqrt{2}$ is irrational.

- **Standard 8.EE.3** Use numbers expressed in the form of a single digit times an integer

power of 10 to estimate very large or very small quantities, and to express how many times as much one is than the other. *For example, estimate the population of the United States as* 3×10^8 *and the population of the world as* 7×10^9*, and determine that the world population is more than 20 times larger.*

■ **Standard 8.EE.4** Perform operations with numbers expressed in scientific notation, including problems where both decimal and scientific notation are used. Use scientific notation and choose units of appropriate size for measurements of very large or very small quantities (e.g., use millimeters per year for seafloor spreading). Interpret scientific notation that has been generated by technology.

■ **Standard 8.EE.5** Graph proportional relationships, interpreting the unit rate as the slope of the graph. Compare two different proportional relationships represented in different ways. *For example, compare a distance-time graph to a distance-time equation to determine which of two moving objects has greater speed.*

■ **Standard 8.EE.6** Use similar triangles to explain why the slope m is the same between any two distinct points on a non-vertical line in the coordinate plane; derive the equation $y = mx$ for a line through the origin and the equation $y = mx + b$ for a line intercepting the vertical axis at b.

■ **Standard 8.EE.7** Solve linear equations and inequalities in one variable.

    a. Give examples of linear equations in one variable with one solution, infinitely many solutions, or no solutions. Show which of these possibilities is the case by successively transforming the given equation into simpler forms, until an equivalent equation of the form $x = a$, $a = a$, or $a = b$ results (where a and b are different numbers).

    b. Solve single-variable linear equations and inequalities with rational number coefficients, including equations and inequalities whose solutions require expanding expressions using the distributive property and collecting like terms.

    c. Solve single-variable absolute value equations.

■ **Standard 8.EE.8** Analyze and solve pairs of simultaneous linear equations.

    a. Understand that solutions to a system of two linear equations in two variables correspond to points of intersection of their graphs, because points of intersection satisfy both equations simultaneously.

    b. Solve systems of two linear equations in two variables graphically, approximating when solutions are not integers and estimate solutions by graphing the equations. Solve simple cases by inspection. *For example,* $3x + 2y = 5$ *and* $3x + 2y = 6$ *have no solution because* $3x + 2y$ *cannot simultaneously be* 5 *and* 6.

    c. Solve real-world and mathematical problems leading to two linear equations in two variables graphically. *For example, given coordinates for two pairs of points, determine whether the line through the first pair of points intersects the line through the second pair.*

Strand: FUNCTIONS (8.F)

Define, evaluate, and compare functions **(Standards 8.F.1–3).** Use functions to model relationships between quantities **(Standards 8.F.4–5).**

■ **Standard 8.F.1** Understand that a function is a rule that assigns to each input exactly one output. The graph of a function is the set of ordered pairs consisting of an input and the corresponding output. (Function notation is not required in grade 8.)

■ **Standard 8.F.2** Compare properties of two functions, each represented in a different way (algebraically, graphically, numerically in tables, or by verbal descriptions). *For example, given a linear function represented by a table of values and a linear function represented by an algebraic expression, determine which function has the greater rate of change.*

■ **Standard 8.F.3** Interpret the equation $y = mx + b$ as defining a linear function whose graph is a straight line; give examples of functions that are not linear. *For example, the function* A = s², *giving the area of a square as a function of its side length, is not linear because its graph contains the points* (1,1), (2,4) *and* (3,9)*, which are not on a straight line.*

■ **Standard 8.F.4** Construct a function to model a linear relationship between two quantities. Determine the rate of change and initial value of the function from a description of a relationship or from two *(x, y)* values, including reading these from a table or from a graph. Interpret the rate of change and initial value of a linear function in terms of the situation it models, and in terms of its graph or a table of values.

■ **Standard 8.F.5** Describe qualitatively the functional relationship between two quantities by analyzing a graph (e.g., where the function is increasing or decreasing, linear or nonlinear). Sketch a graph that exhibits the qualitative features of a function that has been described verbally.

Strand: GEOMETRY (8.G)

Understand congruence and similarity using physical models, transparencies, or geometry software **(Standards 8.G.1–5).** Understand and apply the Pythagorean Theorem and its converse **(Standards 8.G.6–8).** Solve real-world and mathematical problems involving volume of cylinders, cones, and spheres **(Standard 8.G.9).**

■ **Standard 8.G.1** Verify experimentally the properties of rotations, reflections, and translations:

    a. Lines are taken to lines, and line segments to line segments of the same length.

    b. Angles are taken to angles of the same measure.

    c. Parallel lines are taken to parallel lines.

■ **Standard 8.G.2** Understand that a two-dimensional figure is congruent to another if the second can be obtained from the first by a sequence of rotations, reflections, and translations; given two congruent figures, describe a sequence that exhibits the congruence between them.

■ **Standard 8.G.3**   Observe that orientation of the plane is preserved in rotations and translations, but not with reflections. Describe the effect of dilations, translations, rotations, and reflections on two-dimensional figures using coordinates.

■ **Standard 8.G.4**   Understand that a two-dimensional figure is similar to another if the second can be obtained from the first by a sequence of rotations, reflections, translations, and dilations; given two similar two-dimensional figures, describe a sequence that exhibits the similarity between them.

■ **Standard 8.G.5**   Use informal arguments to establish facts about the angle sum and exterior angle of triangles, about the angles created when parallel lines are cut by a transversal, and the angle-angle criterion for similarity of triangles. *For example, arrange three copies of the same triangle so that the sum of the three angles appears to form a line, and give an argument in terms of transversals why this is so.*

■ **Standard 8.G.6**   Explore and explain proofs of the Pythagorean Theorem and its converse.

■ **Standard 8.G.7**   Apply the Pythagorean Theorem to determine unknown side lengths in right triangles in real-world and mathematical problems in two and three dimensions.

■ **Standard 8.G.8**   Apply the Pythagorean Theorem to find the distance between two points in a coordinate system.

■ **Standard 8.G.9**   Know the formulas for the volumes of cones, cylinders, and spheres, and use them to solve real-world and mathematical problems.

Strand: STATISTICS AND PROBABILITY (8.SP)

Investigate patterns of association in bivariate data **(Standards 8.SP.1–4).**

■ **Standard 8.SP.1** Construct and interpret scatter plots for bivariate measurement data to investigate patterns of association between two quantities. Describe patterns such as clustering, outliers, positive or negative association, linear association, and nonlinear association.

■ **Standard 8.SP.2** Know that straight lines are widely used to model relationships between two quantitative variables. For scatter plots that suggest a linear association, informally fit a straight line, and informally assess the model fit by judging the closeness of the data points to the line.

■ **Standard 8.SP.3** Use the equation of a linear model to solve problems in the context of bivariate measurement data, interpreting the slope and intercept. *For example, in a linear model for a biology experiment, interpret a slope of 1.5 cm/hr as meaning that an additional hour of sunlight each day is associated with an additional 1.5 cm in mature plant height.* (Calculating equations for a linear model is not expected in grade 8.)

■ **Standard 8.SP.4** Understand that patterns of association can also be seen in bivariate categorical data by displaying frequencies and relative frequencies in a two-way table.

Construct and interpret a two-way table summarizing data on two categorical variables collected from the same subjects. Use relative frequencies calculated for rows or columns to describe possible association between the two variables. *For example, collect data from students in your class on whether or not they have a curfew on school nights and whether or not they have assigned chores at home. Is there evidence that those who have a curfew also tend to have chores?*

inside of back cover

MATHEMATICS
High School (9–12)

UTAH CORE STATE STANDARDS
for
MATHEMATICS
HIGH SCHOOL (9–12)

Adopted August 2010

by the

Utah State Board of Education

Revised January 2016

The Utah State Board of Education, in January of 1984, established policy requiring the identification of specific core standards to be met by all K–12 students in order to graduate from Utah's secondary schools. The Utah State Board of Education regularly updates the Utah Core Standards, while parents, teachers, and local school boards continue to control the curriculum choices that reflect local values.

The Utah Core Standards are aligned to scientifically based content standards. They drive high quality instruction through statewide comprehensive expectations for all students. The standards outline essential knowledge, concepts, and skills to be mastered at each grade level or within a critical content area. The standards provide a foundation for ensuring learning within the classroom.
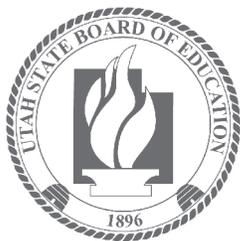
UTAH STATE BOARD OF EDUCATION

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3/2016

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INTRODUCTION

Organization of the Standards

The Utah Core Standards are organized into **strands**, which represent significant areas of learning within content areas. Depending on the core area, these strands may be designated by time periods, thematic principles, modes of practice, or other organizing principles.

Within each strand are **standards**. A standard is an articulation of the demonstrated proficiency to be obtained. A standard represents an essential element of the learning that is expected. While some standards within a strand may be more comprehensive than others, all standards are essential for mastery.

Understanding Mathematics

These standards define what students should understand and be able to do in their study of mathematics. Asking a student to understand something means asking a teacher to assess whether the student has understood it. But what does mathematical understanding look like? One hallmark of mathematical understanding is the ability to justify, in a way appropriate to the student's mathematical maturity, why a particular mathematical statement is true or where a mathematical rule comes from. Mathematical understanding and procedural skill are equally important, and both are assessable using mathematical tasks of sufficient richness.

The standards set grade-specific standards but do not dictate curriculum or teaching methods, nor do they define the intervention methods or materials necessary to support students who are well below or well above grade-level expectations. It is also beyond the scope of the Standards to define the full range of supports appropriate for English language learners and for students with special needs. At the same time, all students must have the opportunity to learn and meet the same high standards if they are to access the knowledge and skills necessary in their post-school lives. The standards should be read as allowing for the widest possible range of students to participate fully from the outset, along with appropriate accommodations to ensure maximum participation of students with special education needs. No set of grade-specific standards can fully reflect the great variety in abilities, needs, learning rates, and achievement levels of students in any given classroom. However, the standards do provide clear signposts along the way to the goal of college and career readiness for all students.

What students can learn at any particular grade level depends upon what they have learned before. Ideally then, each standard in this document might have been phrased in the form, "Students who already know… should next come to learn ..." Grade placements for specific topics have been made on the basis of state and international comparisons and the collective experience and collective professional judgment of educators, researchers and mathematicians. Learning opportunities will continue to vary across schools and school systems, and educators should make every effort to meet the needs of individual students based on their current understanding.

UTAH CORE STATE STANDARDS

for

MATHEMATICS HIGH SCHOOL COURSES

SECONDARY MATHEMATICS I

THE FUNDAMENTAL PURPOSE OF SECONDARY MATHEMATICS I is to formalize and extend the mathematics that students learned in the middle grades. The critical areas, organized into units, deepen and extend understanding of linear relationships, in part by contrasting them with exponential phenomena, and in part by applying linear models to data that exhibit a linear trend. Secondary Mathematics I uses properties and theorems involving congruent figures to deepen and extend understanding of geometric knowledge from prior grades. The final unit in the course ties together the algebraic and geometric ideas studied. The Mathematical Practice Standards apply throughout each course and, together with the content standards, prescribe that students experience mathematics as a coherent, useful, and logical subject that makes use of their ability to make sense of problem situations.

CRITICAL AREA 1: By the end of eighth grade, students have had a variety of experiences working with expressions and creating equations. Students continue this work by using quantities to model and analyze situations, to interpret expressions, and by creating equations to describe situations.

CRITICAL AREA 2: In earlier grades, students define, evaluate, and compare functions, and use them to model relationships between quantities. Students will learn function notation and develop the concepts of domain and range. They move beyond viewing functions as processes that take inputs and yield outputs, and start viewing functions as objects in their own right. They explore many examples of functions, including sequences; they interpret functions given graphically, numerically, symbolically, and verbally, translate between representations, and understand the limitations of various representations. They work with functions given by graphs and tables, keeping in mind that, depending upon the context, these representations are likely to be approximate and incomplete. Their work includes functions that can be described or approximated by formulas, as well as those that cannot. When functions describe relationships between quantities arising from a context, students reason with the units in which those quantities are measured. Students build on and informally extend their understanding of integer exponents to consider exponential functions. They compare and contrast linear and exponential functions, distinguishing between additive and multiplicative change. They interpret arithmetic sequences as linear functions and geometric sequences as exponential functions.

CRITICAL AREA 3: By the end of eighth grade, students have learned to solve linear equations in one variable and have applied graphical and algebraic methods to analyze and solve systems of linear equations in two variables. This area builds on these earlier experiences by asking students to analyze and explain the process of solving an equation and to justify the process used in solving a system of equations. Students develop fluency writing, interpreting, and translating between various forms of linear equations and inequalities, and using them to solve problems. They master the solution of linear equations and apply related solution techniques and the laws of exponents to the creation and solution of simple exponential equations. Students explore

systems of equations and inequalities, and they find and interpret their solutions. All of this work is grounded on understanding quantities and on relationships between them.

CRITICAL AREA 4: This area builds upon students' prior experiences with data, providing students with more formal means of assessing how a model fits data. Students use regression techniques to describe approximately linear relationships between quantities. They use graphical representations and knowledge of the context to make judgments about the appropriateness of linear models. With linear models, they look at residuals to analyze the goodness of fit.

CRITICAL AREA 5: In previous grades, students were asked to draw triangles based on given measurements. They also have prior experience with rigid motions (translations, reflections, and rotations) and have used these to develop notions about what it means for two objects to be congruent. In this unit, students establish triangle congruence criteria, based on analyses of rigid motions and formal constructions. They solve problems about triangles, quadrilaterals, and other polygons. They apply reasoning to complete geometric constructions and explain why they work.

CRITICAL AREA 6: Building on their work with the Pythagorean Theorem in eighth grade to find distances, students use a rectangular coordinate system to verify geometric relationships, including properties of special triangles and quadrilaterals and slopes of parallel and perpendicular lines.

Strand: MATHEMATICAL PRACTICES (MP)

The Standards for Mathematical Practice in Secondary Mathematics I describe mathematical habits of mind that teachers should seek to develop in their students. Students become mathematically proficient in engaging with mathematical content and concepts as they learn, experience, and apply these skills and attitudes **(Standards MP.1–8).**

■ **Standard SI.MP.1   Make sense of problems and persevere in solving them.** Explain the meaning of a problem and look for entry points to its solution. Analyze givens, constraints, relationships, and goals. Make conjectures about the form and meaning of the solution, plan a solution pathway, and continually monitor progress asking, "Does this make sense?" Consider analogous problems, make connections between multiple representations, identify the correspondence between different approaches, look for trends, and transform algebraic expressions to highlight meaningful mathematics. Check answers to problems using a different method.

■ **Standard SI.MP.2   Reason abstractly and quantitatively.** Make sense of the quantities and their relationships in problem situations. Translate between context and algebraic representations by contextualizing and decontextualizing quantitative relationships. This includes the ability to decontextualize a given situation, representing it algebraically and manipulating symbols fluently as well as the ability to contextualize algebraic representations to make sense of the problem.

■ **Standard SI.MP.3   Construct viable arguments and critique the reasoning of others.** Understand and use stated assumptions, definitions, and previously established results in constructing arguments. Make conjectures and build a logical progression of statements to explore the truth of their conjectures. Justify conclusions and communicate them to others. Respond to the arguments of others by listening, asking clarifying questions, and critiquing the reasoning of others.

■ **Standard SI.MP.4   Model with mathematics.** Apply mathematics to solve problems arising in everyday life, society, and the workplace. Make assumptions and approximations, identifying important quantities to construct a mathematical model. Routinely interpret mathematical results in the context of the situation and reflect on whether the results make sense, possibly improving the model if it has not served its purpose.

■ **Standard SI.MP.5   Use appropriate tools strategically.** Consider the available tools and be sufficiently familiar with them to make sound decisions about when each tool might be helpful, recognizing both the insight to be gained as well as the limitations. Identify relevant external mathematical resources and use them to pose or solve problems. Use tools to explore and deepen their understanding of concepts.

■ **Standard SI.MP.6   Attend to precision.** Communicate precisely to others. Use explicit definitions in discussion with others and in their own reasoning. They state the meaning of the symbols they choose. Specify units of measure and label axes to clarify the correspondence with quantities in a problem. Calculate accurately and efficiently, express numerical answers with a degree of precision appropriate for the problem context.

- **Standard SI.MP.7   Look for and make use of structure.** Look closely at mathematical relationships to identify the underlying structure by recognizing a simple structure within a more complicated structure. See complicated things, such as some algebraic expressions, as single objects or as being composed of several objects. *For example, see $5 - 3(x - y)^2$ as 5 minus a positive number times a square and use that to realize that its value cannot be more than 5 for any real numbers* x *and* y.

- **Standard SI.MP.8   Look for and express regularity in repeated reasoning.** Notice if reasoning is repeated, and look for both generalizations and shortcuts. Evaluate the reasonableness of intermediate results by maintaining oversight of the process while attending to the details.

Strand: NUMBER AND QUANTITY—Quantities (N.Q)

Reason quantitatively and use units to solve problems. Working with quantities and the relationships between them provides grounding for work with expressions, equations, and functions **(Standards N.Q.1–3).**

- **Standard N.Q.1** Use units as a way to understand problems and to guide the solution of multi-step problems; choose and interpret units consistently in formulas; choose and interpret the scale and the origin in graphs and data displays.

- **Standard N.Q.2** Define appropriate quantities for the purpose of descriptive modeling.

- **Standard N.Q.3** Choose a level of accuracy appropriate to limitations on measurement when reporting quantities.

Strand: ALGEBRA—Seeing Structure in Expressions (A.SSE)

Interpret the structure of expressions **(Standard A.SSE.1).**

- **Standard A.SSE.1**   Interpret linear expressions and exponential expressions with integer exponents that represent a quantity in terms of its context. ★

    a. Interpret parts of an expression, such as terms, factors, and coefficients.

    b. Interpret complicated expressions by viewing one or more of their parts as a single entity. *For example, interpret* $P(1+r)^n$ *as the product of* P *and a factor not depending on* P.

Strand: ALGEBRA—Creating Equations (A.CED)

Create equations that describe numbers or relationships. Limit these to linear equations and inequalities, and exponential equations. In the case of exponential equations, limit to situations requiring evaluation of exponential functions at integer inputs **(Standards A.CED.1–4).**

- **Standard A.CED.1**   Create equations and inequalities in one variable and use them to solve problems. Include equations arising from linear and simple exponential functions.

■ **Standard A.CED.2** Create equations in two or more variables to represent relationships between quantities; graph equations on coordinate axes with labels and scales.

■ **Standard A.CED.3** Represent constraints by equations or inequalities and by systems of equations and/or inequalities, and interpret solutions as viable or non-viable options in a modeling context. *For example, represent inequalities describing nutritional and cost constraints on combinations of different foods.*

■ **Standard A.CED.4**   Rearrange formulas to highlight a quantity of interest, using the same reasoning as in solving equations. *For example, rearrange Ohm's Law* V = IR *to highlight resistance* R.

Strand: ALGEBRA—Reasoning With Equations and Inequalities (A.REI)

Understand solving equations as a process of reasoning and explain the reasoning **(Standard A.REI.1).** Solve equations and inequalities in one variable **(Standard A.REI.3).** Solve systems of equations. Build on student experiences graphing and solving systems of linear equations from middle school. Include cases where the two equations describe the same line—yielding infinitely many solutions—and cases where two equations describe parallel lines—yielding no solution; connect to GPE.5, which requires students to prove the slope criteria for parallel lines **(Standards A.REI.5–6).** Represent and solve equations and inequalities graphically **(Standards A.REI.10–12).**

■ **Standard A.REI.1**   Explain each step in solving a linear equation as following from the equality of numbers asserted at the previous step, starting from the assumption that the original equation has a solution. Construct a viable argument to justify a solution method. Students will solve exponential equations with logarithms in Secondary Mathematics III.

■ **Standard A.REI.3**   Solve equations and inequalities in one variable.

  a. Solve one-variable equations and literal equations to highlight a variable of interest.

  b. Solve compound inequalities in one variable, including absolute value inequalities.

  c. Solve simple exponential equations that rely only on application of the laws of exponents (limit solving exponential equations to those that can be solved without logarithms). *For example, $5^x = 125$ or $2^x = 1/16$.*

■ **Standard A.REI.5**   Prove that, given a system of two equations in two variables, replacing one equation by the sum of that equation and a multiple of the other produces a system with the same solutions.

■ **Standard A.REI.6**   Solve systems of linear equations exactly and approximately (numerically, algebraically, graphically), focusing on pairs of linear equations in two variables.

■ **Standard A.REI.10**   Understand that the graph of an equation in two variables is the set of all its solutions plotted in the coordinate plane, often forming a curve (which could be a line).

■ **Standard A.REI.11** Explain why the *x*-coordinates of the points where the graphs of the equations *y = f(x)* and *y = g(x)* intersect are the solutions of the equation *f(x) = g(x)*; find the solutions approximately; e.g., using technology to graph the functions, make tables of values, or find successive approximations. Include cases where *f(x)* and/or *g(x)* are linear and exponential functions. ★

■ **Standard A.REI.12** Graph the solutions to a linear inequality in two variables as a half-plane (excluding the boundary in the case of a strict inequality), and graph the solution set to a system of linear inequalities in two variables as the intersection of the corresponding half-planes.

Strand: FUNCTIONS—Interpreting Linear and Exponential Functions (F.IF)

Understand the concept of a linear or exponential function and use function notation. Recognize arithmetic and geometric sequences as examples of linear and exponential functions **(Standards F.IF.1–3).** Interpret linear or exponential functions that arise in applications in terms of a context **(Standards F.IF.4–6).** Analyze linear or exponential functions using different representations **(Standards F.IF.7,9).**

■ **Standard F.IF.1** Understand that a function from one set (called the domain) to another set (called the range) assigns to each element of the domain exactly one element of the range. If *f* is a function and *x* is an element of its domain, then *f(x)* denotes the output of *f* corresponding to the input *x*. The graph of *f* is the graph of the equation *y = f(x)*.

■ **Standard F.IF.2** Use function notation, evaluate functions for inputs in their domains, and interpret statements that use function notation in terms of a context.

■ **Standard F.IF.3** Recognize that sequences are functions, sometimes defined recursively, whose domain is a subset of the integers. Emphasize arithmetic and geometric sequences as examples of linear and exponential functions. *For example, the Fibonacci sequence is defined recursively by* f(0) = f(1) = 1, f(n+1) = f(n) + f(n-1) *for* n ≥ 1.

■ **Standard F.IF.4** For a function that models a relationship between two quantities, interpret key features of graphs and tables in terms of the quantities, and sketch graphs showing key features given a verbal description of the relationship. *Key features include intercepts; intervals where the function is increasing, decreasing, positive, or negative; relative maximums and minimums; symmetries; and end behavior.* ★

■ **Standard F.IF.5** Relate the domain of a function to its graph and, where applicable, to the quantitative relationship it describes. *For example, if the function* h(n) *gives the number of person-hours it takes to assemble n engines in a factory, then the positive integers would be an appropriate domain for the function.* ★

■ **Standard F.IF.6** Calculate and interpret the average rate of change of a function (presented symbolically or as a table) over a specified interval. Estimate the rate of change from a graph. ★

■ **Standard F.IF.7** Graph functions expressed symbolically and show key features of the

graph, by hand in simple cases and using technology for more complicated cases. ★

    a. Graph linear functions and show intercepts.

    e. Graph exponential functions, showing intercepts and end behavior.

■ **Standard F.IF.9** Compare properties of two functions, each represented in a different way (algebraically, graphically, numerically in tables, or by verbal descriptions). *For example, compare the growth of two linear functions, or two exponential functions such as* $y=3^n$ *and* $y=100 \cdot 2^n$.

Strand: FUNCTIONS—Building Linear or Exponential Functions (F.BF)

Build a linear or exponential function that models a relationship between two quantities **(Standards F.BF.1–2).** Build new functions from existing functions **(Standard F.BF.3).**

■ **Standard F.BF.1** Write a function that describes a relationship between two quantities. ★

    a. Determine an explicit expression, a recursive process, or steps for calculation from a context.

    b. Combine standard function types using arithmetic operations. *For example, build a function that models the temperature of a cooling body by adding a constant function to a decaying exponential, and relate these functions to the model.*

■ **Standard F.BF.2** Write arithmetic and geometric sequences both recursively and with an explicit formula, use them to model situations, and translate between the two forms. Limit to linear and exponential functions. Connect arithmetic sequences to linear functions and geometric sequences to exponential functions. ★

■ **Standard F.BF.3** Identify the effect on the graph of replacing $f(x)$ by $f(x) + k$, for specific values of k (both positive and negative); find the value of k given the graphs. Relate the vertical translation of a linear function to its y-intercept. Experiment with cases and illustrate an explanation of the effects on the graph using technology.

Strand: FUNCTIONS—Linear and Exponential (F.LE)

Construct and compare linear and exponential models and solve problems **(Standards F.LE.1–3).** Interpret expressions for functions in terms of the situation they model. **(Standard F.LE.5).**

■ **Standard F.LE.1** Distinguish between situations that can be modeled with linear functions and with exponential functions.

    a. Prove that linear functions grow by equal differences over equal intervals; exponential functions grow by equal factors over equal intervals.

    b. Recognize situations in which one quantity changes at a constant rate per unit interval relative to another.

c. Recognize situations in which a quantity grows or decays by a constant percent rate per unit interval relative to another.

■ **Standard F.LE.2** Construct linear and exponential functions, including arithmetic and geometric sequences, given a graph, a description of a relationship, or two input-output pairs (include reading these from a table).

■ **Standard F.LE.3** Observe using graphs and tables that a quantity increasing exponentially eventually exceeds a quantity increasing linearly.

■ **Standard F.LE.5** Interpret the parameters in a linear or exponential function in terms of a context. Limit exponential functions to those of the form $f(x) = b^x + k$.

Strand: GEOMETRY—Congruence (G.CO)

Experiment with transformations in the plane. Build on student experience with rigid motions from earlier grades **(Standards G.CO.1–5).** Understand congruence in terms of rigid motions. Rigid motions are at the foundation of the definition of congruence. Reason from the basic properties of rigid motions (that they preserve distance and angle), which are assumed without proof. Rigid motions and their assumed properties can be used to establish the usual triangle congruence criteria, which can then be used to prove other theorems **(Standards G.CO.6–8).** Make geometric constructions **(Standards G.CO.12–13).**

■ **Standard G.CO.1** Know precise definitions of angle, circle, perpendicular line, parallel line, and line segment, based on the undefined notions of point, line, distance along a line, and distance around a circular arc.

■ **Standard G.CO.2** Represent transformations in the plane using, for example, transparencies and geometry software; describe transformations as functions that take points in the plane as inputs and give other points as outputs. Compare transformations that preserve distance and angle to those that do not (e.g., translation versus horizontal stretch).

■ **Standard G.CO.3** Given a rectangle, parallelogram, trapezoid, or regular polygon, describe the rotations and reflections that carry it onto itself.

■ **Standard G.CO.4** Develop definitions of rotations, reflections, and translations in terms of angles, circles, perpendicular lines, parallel lines, and line segments.

■ **Standard G.CO.5** Given a geometric figure and a rotation, reflection, or translation, draw the transformed figure using, for example, graph paper, tracing paper, or geometry software. Specify a sequence of transformations that will carry a given figure onto another. Point out the basis of rigid motions in geometric concepts, for example, translations move points a specified distance along a line parallel to a specified line; rotations move objects along a circular arc with a specified center through a specified angle.

■ **Standard G.CO.6** Use geometric descriptions of rigid motions to transform figures and to predict the effect of a given rigid motion on a given figure; given two figures, use

the definition of congruence in terms of rigid motions to decide whether they are congruent.

■ **Standard G.CO.7** Use the definition of congruence in terms of rigid motions to show that two triangles are congruent if and only if corresponding pairs of sides and corresponding pairs of angles are congruent.

■ **Standard G.CO.8** Explain how the criteria for triangle congruence (ASA, SAS, and SSS) follow from the definition of congruence in terms of rigid motions.

■ **Standard G.CO.12** Make formal geometric constructions with a variety of tools and methods (compass and straightedge, string, reflective devices, paper folding, dynamic geometric software, etc.). Emphasize the ability to formalize and defend how these constructions result in the desired objects. *For example, copying a segment; copying an angle; bisecting a segment; bisecting an angle; constructing perpendicular lines, including the perpendicular bisector of a line segment; and constructing a line parallel to a given line through a point not on the line.*

■ **Standard G.CO.13** Construct an equilateral triangle, a square, and a regular hexagon inscribed in a circle. Emphasize the ability to formalize and defend how these constructions result in the desired objects.

Strand: GEOMETRY—Expressing Geometric Properties With Equations (G.GPE)

Use coordinates to prove simple geometric theorems algebraically **(Standards G.GPE.4–5, 7).**

■ **Standard G.GPE.4** Use coordinates to prove simple geometric theorems algebraically. *For example, prove or disprove that a figure defined by four given points in the coordinate plane is a rectangle; prove or disprove that the point $(1, \sqrt{3})$ lies on the circle centered at the origin and containing the point (0, 2).*

■ **Standard G.GPE.5** Prove the slope criteria for parallel and perpendicular lines; use them to solve geometric problems (e.g., find the equation of a line parallel or perpendicular to a given line that passes through a given point).

■ **Standard G.GPE.7** Use coordinates to compute perimeters of polygons and areas of triangles and rectangles; connect with The Pythagorean Theorem and the distance formula.★

Strand: STATISTICS AND PROBABILITY— Interpreting Categorical and Quantitative Data (S.ID)

Summarize, represent, and interpret data on a single count or measurement variable **(Standards S.ID.1–3).** Summarize, represent, and interpret data on two categorical and quantitative variables **(Standard S.ID.6).** Interpret linear models building on students' work with linear relationships, and introduce the correlation coefficient **(Standards S.ID.7–9).**

■ **Standard S.ID.1** Represent data with plots on the real number line (dot plots, histograms, and box plots).

■ **Standard S.ID.2** Use statistics appropriate to the shape of the data distribution to compare center (median, mean) and spread (interquartile range, standard deviation) of two or more different data sets.

■ **Standard S.ID.3** Interpret differences in shape, center, and spread in the context of the data sets, accounting for possible effects of extreme data points (outliers). Calculate the weighted average of a distribution and interpret it as a measure of center.

■ **Standard S.ID.6** Represent data on two quantitative variables on a scatter plot, and describe how the variables are related.

    a. Fit a linear function to the data; use functions fitted to data to solve problems in the context of the data. Use given functions, or choose a function suggested by the context. Emphasize linear and exponential models.

    b. Informally assess the fit of a function by plotting and analyzing residuals. Focus on situations for which linear models are appropriate.

    c. Fit a linear function for scatter plots that suggest a linear association.

■ **Standard S.ID.7** Interpret the slope (rate of change) and the intercept (constant term) of a linear model in the context of the data.

■ **Standard S.ID.8** Compute (using technology) and interpret the correlation coefficient of a linear fit.

■ **Standard S.ID.9** Distinguish between correlation and causation.

SECONDARY MATHEMATICS I—HONORS STANDARDS

Strand: NUMBER AND QUANTITY: VECTOR AND MATRIX QUANTITIES (N.VM)

Represent and model with vector quantities **(Standards N.VM.1–3).** Perform operations on vectors **(Standards N.VM.4–5).** Perform operations on matrices and use matrices in applications **(Standards N.VM.6–13).**

■ **Standard N.VM.1** Recognize vector quantities as having both magnitude and direction. Represent vector quantities by directed line segments, and use appropriate symbols for vectors and their magnitudes (e.g., v, $|v|$, $\|v\|$, v).

■ **Standard N.VM.2** Find the components of a vector by subtracting the coordinates of an initial point from the coordinates of a terminal point.

■ **Standard N.VM.3** Solve problems involving velocity and other quantities that can be represented by vectors.

■ **Standard N.VM.4** Add and subtract vectors.

    a. Add vectors end-to-end, component-wise, and by the parallelogram rule. Understand that the magnitude of a sum of two vectors is typically not the sum of the magnitudes.

    b. Given two vectors in magnitude and direction form, determine the magnitude and direction of their sum.

    c. Understand vector subtraction $v - w$ as $v + (-w)$, where $-w$ is the additive inverse of w, with the same magnitude as w and pointing in the opposite direction. Represent vector subtraction graphically by connecting the tips in the appropriate order, and perform vector subtraction component-wise.

■ **Standard N.VM.5** Multiply a vector by a scalar.

    a. Represent scalar multiplication graphically by scaling vectors and possibly reversing their direction; perform scalar multiplication component-wise, e.g., as $c(v_x, v_y) = (cv_x, cv_y)$.

    b. Compute the magnitude of a scalar multiple cv using $\|cv\| = |c|v$. Compute the direction of cv knowing that when $|c|v \neq 0$, the direction of cv is either along v (for $c > 0$) or against vs (for $c < 0$).

■ **Standard N.VM.6** Use matrices to represent and manipulate data, e.g., to represent pay-offs or incidence relationships in a network.

■ **Standard N.VM.7** Multiply matrices by scalars to produce new matrices, e.g., as when all of the pay-offs in a game are doubled.

■ **Standard N.VM.8** Add, subtract, and multiply matrices of appropriate dimensions.

■ **Standard N.VM.9** Understand that, unlike multiplication of numbers, matrix multiplication for square matrices is not a commutative operation, but still satisfies the associative and distributive properties.

■ **Standard N.VM.10** Understand that the zero and identity matrices play a role in matrix addition and multiplication similar to the role of 0 and 1 in the real numbers. The determinant of a square matrix is nonzero if and only if the matrix has a multiplicative inverse.

■ **Standard N.VM.11** Multiply a vector (regarded as a matrix with one column) by a matrix of suitable dimensions to produce another vector. Work with matrices as transformations of vectors.

■ **Standard N.VM.12** Work with 2×2 matrices as transformations of the plane, and interpret the absolute value of the determinant in terms of area.

■ **Standard N.VM.13** Solve systems of linear equations up to three variables using matrix row reduction.

SECONDARY MATHEMATICS II

THE FOCUS OF SECONDARY MATHEMATICS II is on quadratic expressions, equations, and functions and on comparing their characteristics and behavior to those of linear and exponential relationships from Secondary Mathematics I as organized into six critical areas, or units. The need for extending the set of rational numbers arises, and real and complex numbers are introduced so that all quadratic equations can be solved. The link between probability and data is explored through conditional probability and counting methods, including their use in making and evaluating decisions. The study of similarity leads to an understanding of right triangle trigonometry and connects to quadratics through Pythagorean relationships. Circles, with their quadratic algebraic representations, round out the course. The Mathematical Practice Standards apply throughout each course and, together with the content standards, prescribe that students experience mathematics as a coherent, useful, and logical subject that makes use of their ability to make sense of problem situations.

> **CRITICAL AREA 1:** Students extend the laws of exponents to rational exponents and explore distinctions between rational and irrational numbers by considering their decimal representations. Students learn that when quadratic equations do not have real solutions the number system must be extended so that solutions exist, analogous to the way in which extending the whole numbers to the negative numbers allows $x+1 = 0$ to have a solution. Students explore relationships between number systems: whole numbers, integers, rational numbers, real numbers, and complex numbers. The guiding principle is that equations with no solutions in one number system may have solutions in a larger number system.

> **CRITICAL AREA 2:** Students consider quadratic functions, comparing the key characteristics of quadratic functions to those of linear and exponential functions. They select from among these functions to model phenomena. Students learn to anticipate the graph of a quadratic function by interpreting various forms of quadratic expressions. In particular, they identify the real solutions of a quadratic equation as the zeros of a related quadratic function. When quadratic equations do not have real solutions, students learn that the graph of the related quadratic function does not cross the horizontal axis. They expand their experience with functions to include more specialized functions—absolute value, step, and those that are piecewise-defined.

> **CRITICAL AREA 3:** Students begin this unit by focusing on the structure of expressions, rewriting expressions to clarify and reveal aspects of the relationship they represent. They create and solve equations, inequalities, and systems of equations involving exponential and quadratic expressions.

> **CRITICAL AREA 4:** Building on probability concepts that began in the middle grades, students use the languages of set theory to expand their ability to compute and interpret theoretical and experimental probabilities for compound events, attending to mutually exclusive events, independent events, and conditional probability. Students should make use of geometric probability models wherever possible. They use probability to make informed decisions.

CRITICAL AREA 5: Students apply their earlier experience with dilations and proportional reasoning to build a formal understanding of similarity. They identify criteria for similarity of triangles, use similarity to solve problems, and apply similarity in right triangles to understand right triangle trigonometry, with particular attention to special right triangles and the Pythagorean Theorem. It is in this unit that students develop facility with geometric proof. They use what they know about congruence and similarity to prove theorems involving lines, angles, triangles, and other polygons. They explore a variety of formats for writing proofs.

CRITICAL AREA 6: Students prove basic theorems about circles, such as a tangent line is perpendicular to a radius, inscribed angle theorem, and theorems about chords, secants, and tangents dealing with segment lengths and angle measures. In the Cartesian coordinate system, students use the distance formula to write the equation of a circle when given the radius and the coordinates of its center, and the equation of a parabola with vertical axis when given an equation of its directrix and the coordinates of its focus. Given an equation of a circle, they draw the graph in the coordinate plane, and apply techniques for solving quadratic equations to determine intersections between lines and circles or a parabola and between two circles. Students develop informal arguments justifying common formulas for circumference, area, and volume of geometric objects, especially those related to circles.

Strand: MATHEMATICAL PRACTICES (MP)

The Standards for Mathematical Practice in Secondary Mathematics II describe mathematical habits of mind that teachers should seek to develop in their students. Students become mathematically proficient in engaging with mathematical content and concepts as they learn, experience, and apply these skills and attitudes **(Standards MP.1–8).**

■ **Standard SII.MP.1 Make sense of problems and persevere in solving them.** Explain the meaning of a problem and look for entry points to its solution. Analyze givens, constraints, relationships, and goals. Make conjectures about the form and meaning of the solution, plan a solution pathway, and continually monitor progress asking, "Does this make sense?" Consider analogous problems, make connections between multiple representations, identify the correspondence between different approaches, look for trends, and transform algebraic expressions to highlight meaningful mathematics. Check answers to problems using a different method.

■ **Standard SII.MP.2 Reason abstractly and quantitatively.** Make sense of the quantities and their relationships in problem situations. Translate between context and algebraic representations by contextualizing and decontextualizing quantitative relationships. This includes the ability to decontextualize a given situation, representing it algebraically and manipulating symbols fluently as well as the ability to contextualize algebraic representations to make sense of the problem.

■ **Standard SII.MP.3 Construct viable arguments and critique the reasoning of others.** Understand and use stated assumptions, definitions, and previously established results in constructing arguments. Make conjectures and build a logical progression of statements to explore the truth of their conjectures. Justify conclusions and communicate them to others. Respond to the arguments of others by listening, asking clarifying questions, and critiquing the reasoning of others.

■ **Standard SII.MP.4 Model with mathematics.** Apply mathematics to solve problems arising in everyday life, society, and the workplace. Make assumptions and approximations, identifying important quantities to construct a mathematical model. Routinely interpret mathematical results in the context of the situation and reflect on whether the results make sense, possibly improving the model if it has not served its purpose.

■ **Standard SII.MP.5 Use appropriate tools strategically.** Consider the available tools and be sufficiently familiar with them to make sound decisions about when each tool might be helpful, recognizing both the insight to be gained as well as the limitations. Identify relevant external mathematical resources and use them to pose or solve problems. Use tools to explore and deepen their understanding of concepts.

■ **Standard SII.MP.6 Attend to precision.** Communicate precisely to others. Use explicit definitions in discussion with others and in their own reasoning. They state the meaning of the symbols they choose. Specify units of measure and label axes to clarify the correspondence with quantities in a problem. Calculate accurately and efficiently, express numerical answers with a degree of precision appropriate for the problem context.

■ **Standard SII.MP.7   Look for and make use of structure.** Look closely at mathematical relationships to identify the underlying structure by recognizing a simple structure within a more complicated structure. See complicated things, such as some algebraic expressions, as single objects or as being composed of several objects. *For example, see $5 - 3(x - y)^2$ as 5 minus a positive number times a square and use that to realize that its value cannot be more than 5 for any real numbers* x *and* y.

■ **Standard SII.MP.8   Look for and express regularity in repeated reasoning.** Notice if reasoning is repeated, and look for both generalizations and shortcuts. Evaluate the reasonableness of intermediate results by maintaining oversight of the process while attending to the details.

Strand: NUMBER AND QUANTITY—The Real Number System (**N.RN**)

Extend the properties of exponents to rational exponents **(Standards N.RN.1–2).** Use properties of rational and irrational numbers **(Standard N.RN. 3).**

■ **Standard N.RN.1   **Explain how the definition of the meaning of rational exponents follows from extending the properties of integer exponents to those values, allowing for a notation for radicals in terms of rational exponents. *For example, we define $5^{1/3}$ to be the cube root of 5 because we want $(5^{1/3})^3 = 5^{(1/3)3}$ to hold, so $(5^{1/3})^3$ must equal 5.*

■ **Standard N.RN.2   **Rewrite expressions involving radicals and rational exponents using the properties of exponents.

■ **Standard N.RN.3   **Explain why sums and products of rational numbers are rational, that the sum of a rational number and an irrational number is irrational, and that the product of a nonzero rational number and an irrational number is irrational. Connect to physical situations (e.g., finding the perimeter of a square of area 2).

Strand: NUMBER AND QUANTITY—The Complex Number System (N.CN)

Perform arithmetic operations with complex numbers **(Standards N.CN.1–2).** Use complex numbers in polynomial identities and equations **(Standards N.CN.7–9).**

■ **Standard N.CN.1   **Know there is a complex number i such that $i^2 = -1$, and every complex number has the form $a + bi$ with a and b real.

■ **Standard N.CN.2   **Use the relation $i^2 = -1$ and the commutative, associative, and distributive properties to add, subtract, and multiply complex numbers. Limit to multiplications that involve i^2 as the highest power of i.

■ **Standard N.CN.7   **Solve quadratic equations with real coefficients that have complex solutions.

■ **Standard N.CN.8   **Extend polynomial identities to the complex numbers. Limit to quadratics with real coefficients. *For example, rewrite* $x^2 + 4$ *as (x + 2i)(x – 2i)*.

■ **Standard N.CN.9** Know the Fundamental Theorem of Algebra; show that it is true for quadratic polynomials.

Strand: ALGEBRA—Seeing Structure in Expression (A.SSE)

Interpret the structure of expressions **(Standards A.SSE.1–2).** Write expressions in equivalent forms to solve problems, balancing conceptual understanding and procedural fluency in work with equivalent expressions **(Standard A.SSE.3).**

■ **Standard A.SSE.1** Interpret quadratic and exponential expressions that represent a quantity in terms of its context. ★

   a. Interpret parts of an expression, such as terms, factors, and coefficients.

   b. Interpret increasingly more complex expressions by viewing one or more of their parts as a single entity. Exponents are extended from the integer exponents to rational exponents focusing on those that represent square or cube roots.

■ **Standard A.SSE.2** Use the structure of an expression to identify ways to rewrite it. For example, see $x^4 - y^4$ as $(x^2)^2 - (y^2)^2$, thus recognizing it as a difference of squares that can be factored as $(x^2 - y^2)(x^2 + y^2)$.

■ **Standard A.SSE.3** Choose and produce an equivalent form of an expression to reveal and explain properties of the quantity represented by the expression. *For example, development of skill in factoring and completing the square goes hand in hand with understanding what different forms of a quadratic expression reveal.* ★

   a. Factor a quadratic expression to reveal the zeros of the function it defines.

   b. Complete the square in a quadratic expression to reveal the maximum or minimum value of the function it defines.

   c. Use the properties of exponents to transform expressions for exponential functions. *For example, the expression 1.15^t can be rewritten as $(1.15^{1/12})^{12t} \approx 1.012^{12t}$ to reveal the approximate equivalent monthly interest rate if the annual rate is 15%.*

Strand: ALGEBRA—Arithmetic With Polynomials and Rational Expressions (A.APR)

Perform arithmetic operations on polynomials. Focus on polynomial expressions that simplify to forms that are linear or quadratic in a positive integer power of *x* **(Standard A.APR.1).**

■ **Standard A.APR.1** Understand that polynomials form a system analogous to the integers, namely, they are closed under the operations of addition, subtraction, and multiplication; add, subtract, and multiply polynomials.

Strand: ALGEBRA—Creating Equations (A.CED)

Create equations that describe numbers or relationships. Extend work on linear and exponential equations to quadratic equations **(Standards A.CED.1–2, 4).**

■ **Standard A.CED.1**   Create equations and inequalities in one variable and use them to solve problems. Include equations arising from linear and quadratic functions, and simple rational and exponential functions.

■ **Standard A.CED.2**   Create equations in two or more variables to represent relationships between quantities; graph equations on coordinate axes with labels and scales.

■ **Standard A.CED.4**   Rearrange formulas to highlight a quantity of interest, using the same reasoning as in solving equations; extend to formulas involving squared variables. *For example, rearrange the formula for the volume of a cylinder* $V = \pi r^2 h$.

Strand: ALGEBRA—Reasoning With Equations and Inequalities (A.REI)

Solve equations and inequalities in one variable **(Standard A.REI.4).** Solve systems of equations. Extend the work of systems to include solving systems consisting of one linear and one nonlinear equation **(Standard A.REI.7).**

■ **Standard A.REI.4**   Solve quadratic equations in one variable.

    a.   Use the method of completing the square to transform any quadratic equation in x into an equation of the form $(x – p)^2 = q$ that has the same solutions. Derive the quadratic formula from this form.

    b.   Solve quadratic equations by inspection (e.g., for $x^2 = 49$), taking square roots, completing the square, the quadratic formula and factoring, as appropriate to the initial form of the equation. Recognize when the quadratic formula gives complex solutions and write them as $a \pm bi$ for real numbers a and b.

■ **Standard A.REI.7**   Solve a simple system consisting of a linear equation and a quadratic equation in two variables algebraically and graphically. *For example, find the points of intersection between the line* $y = –3x$ *and the circle* $x^2 + y^2 = 3$.

Strand: FUNCTIONS—Interpret Functions (F.IF)

Interpret quadratic functions that arise in applications in terms of a context **(Standards F.IF.4–6).** Analyze functions using different representations **(Standards F.IF.7–9).**

■ **Standard F.IF.4**   For a function that models a relationship between two quantities, interpret key features of graphs and tables in terms of the quantities, and sketch graphs showing key features given a verbal description of the relationship. *Key features include intercepts; intervals where the function is increasing, decreasing, positive, or negative; relative maximums and minimums; symmetries; and end behavior.* ★

■ **Standard F.IF.5**   Relate the domain of a function to its graph and, where applicable, to the quantitative relationship it describes. Focus on quadratic functions; compare with linear and exponential functions. *For example, if the function* h(n) *gives the number of person-hours it takes to assemble n engines in a factory, then the positive integers would be an appropriate domain for the function.* ★

■ **Standard F.IF.6**   Calculate and interpret the average rate of change of a function (presented symbolically or as a table) over a specified interval. Estimate the rate of change from a graph. ★

■ **Standard F.IF.7**   Graph functions expressed symbolically and show key features of the graph, by hand in simple cases and using technology for more complicated cases. ★

  a. Graph linear and quadratic functions and show intercepts, maxima, and minima.

  b. Graph piecewise-defined functions and absolute value functions. Compare and contrast absolute value and piecewise-defined functions with linear, quadratic, and exponential functions. Highlight issues of domain, range, and usefulness when examining piecewise-defined functions.

■ **Standard F.IF.8**   Write a function defined by an expression in different but equivalent forms to reveal and explain different properties of the function.

  a. Use the process of factoring and completing the square in a quadratic function to show zeros, extreme values, and symmetry of the graph, and interpret these in terms of a context.

  b. Use the properties of exponents to interpret expressions for exponential functions. *For example, identify percent rate of change in functions such as* $y = (1.02)^t$, $y = (0.97)^t$, $y = (1.01)^{12t}$, $y = (1.2)^{t/10}$, *and classify them as representing exponential growth or decay.*

■ **Standard F.IF.9**   Compare properties of two functions, each represented in a different way (algebraically, graphically, numerically in tables, or by verbal descriptions). Extend work with quadratics to include the relationship between coefficients and roots, and that once roots are known, a quadratic equation can be factored. *For example, given a graph of one quadratic function and an algebraic expression for another, say which has the larger maximum.*

Strand: FUNCTIONS—Building Functions (F.BF)

Build a function that models a relationship between two quantities **(Standard F.BF.1).** Build new functions from existing functions **(Standard F.BF.3).**

■ **Standard F.BF.1**   Write a quadratic or exponential function that describes a relationship between two quantities. ★

  a. Determine an explicit expression, a recursive process, or steps for calculation from a context.

b. Combine standard function types using arithmetic operations. *For example, build a function that models the temperature of a cooling body by adding a constant function to a decaying exponential, and relate these functions to the model.*

■ **Standard F.BF.3** Identify the effect on the graph of replacing *f(x) by f(x) + k, k f(x), f(kx), and f(x + k)* for specific values of *k* (both positive and negative); find the value of *k* given the graphs. Focus on quadratic functions and consider including absolute value functions. Experiment with cases and illustrate an explanation of the effects on the graph using technology. Include recognizing even and odd functions from their graphs and algebraic expressions for them.

Strand: FUNCTIONS—Linear, Quadratic, and Exponential Models (F.LE)

Construct and compare linear, quadratic, and exponential models and solve problems **(Standard F.LE.3).**

■ **Standard F.LE.3** Observe using graphs and tables that a quantity increasing exponentially eventually exceeds a quantity increasing linearly, quadratically, or (more generally) as a polynomial function. Compare linear and exponential growth to quadratic growth.

Strand: FUNCTIONS—Trigonometric Functions (F.TF)

Prove and apply trigonometric identities. Limit θ to angles between 0 and 90 degrees. Connect with the Pythagorean Theorem and the distance formula **(Standard F.TF.8).**

■ **Standard F.TF.8** Prove the Pythagorean identity $\sin^2(\theta) + \cos^2(\theta) = 1$ and use it to find sin (θ), cos (θ), or tan (θ), given sin (θ), cos (θ), or tan (θ), and the quadrant of the angle.

Strand: GEOMETRY—Congruence (G.CO)

Prove geometric theorems. Encourage multiple ways of writing proofs, such as narrative paragraphs, flow diagrams, two-column format, and diagrams without words. Focus on the validity of the underlying reasoning while exploring a variety of formats for expressing that reasoning **(Standards G.CO.9–11).**

■ **Standard G.CO.9** Prove theorems about lines and angles. *Theorems include: vertical angles are congruent; when a transversal crosses parallel lines, alternate interior angles are congruent and corresponding angles are congruent; points on a perpendicular bisector of a line segment are exactly those equidistant from the segment's endpoints.*

■ **Standard G.CO.10**   Prove theorems about triangles. *Theorems include: measures of interior angles of a triangle sum to 180°; base angles of isosceles triangles are congruent; the segment joining midpoints of two sides of a triangle is parallel to the third side and half the length; the medians of a triangle meet at a point.*

■ **Standard G.CO.11**   Prove theorems about parallelograms. *Theorems include: opposite sides are congruent, opposite angles are congruent, the diagonals of a parallelogram bisect each other, and conversely, rectangles are parallelograms with congruent diagonals.*

Strand: GEOMETRY—Similarity, Right Triangles, and Trigonometry (G.SRT)

Understand similarity in terms of similarity transformations **(Standards G.SRT.1–3).** Prove theorems involving similarity **(Standards G.SRT.4–5).** Define trigonometric ratios and solve problems involving right triangles **(Standards G.SRT.6–8).**

■ **Standard G.SRT.1** Verify experimentally the properties of dilations given by a center and a scale factor.

   a. A dilation takes a line not passing through the center of the dilation to a parallel line, and leaves a line passing through the center unchanged.

   b. The dilation of a line segment is longer or shorter in the ratio given by the scale factor.

■ **Standard G.SRT.2** Given two figures, use the definition of similarity in terms of similarity transformations to decide whether they are similar; explain using similarity transformations the meaning of similarity for triangles as the equality of all corresponding pairs of angles and the proportionality of all corresponding pairs of sides.

■ **Standard G.SRT.3** Use the properties of similarity transformations to establish the AA criterion for two triangles to be similar.

■ **Standard G.SRT.4** Prove theorems about triangles. *Theorems include: a line parallel to one side of a triangle divides the other two proportionally and conversely; the Pythagorean Theorem (proved using triangle similarity).*

■ **Standard G.SRT.5** Use congruence and similarity criteria for triangles to solve problems and to prove relationships in geometric figures.

■ **Standard G.SRT.6** Understand that by similarity, side ratios in right triangles are properties of the angles in the triangle, leading to definitions of trigonometric ratios for acute angles.

■ **Standard G.SRT.7** Explain and use the relationship between the sine and cosine of complementary angles.

■ **Standard G.SRT.8** Use trigonometric ratios and the Pythagorean Theorem to solve right triangles in applied problems.

Strand: GEOMETRY—Circles (G.C)

Understand and apply theorems about circles **(Standard G.C.1–4).** Find arc lengths and areas of sectors of circles. Use this as a basis for introducing the radian as a unit of measure. It is not intended that it be applied to the development of circular trigonometry in this course **(Standard G.C.5).**

■ **Standard G.C.1** Prove that all circles are similar.

- **Standard G.C.2** Identify and describe relationships among inscribed angles, radii, and chords. *Relationships include the relationship between central, inscribed, and circumscribed angles; inscribed angles on a diameter are right angles; the radius of a circle is perpendicular to the tangent where the radius intersects the circle.*

- **Standard G.C.3** Construct the inscribed and circumscribed circles of a triangle, and prove properties of angles for a quadrilateral inscribed in a circle.

- **Standard G.C.4** Construct a tangent line from a point outside a given circle to the circle.

- **Standard G.C.5** Derive, using similarity, the fact that the length of the arc intercepted by an angle is proportional to the radius, and define the radian measure of the angle as the constant of proportionality; derive the formula for the area of a sector.

Strand: GEOMETRY—Expressing Geometric Properties With Equations (G.GPE)

Translate between the geometric description and the equation for a conic section **(Standard G.GPE.1).** Use coordinates to prove simple geometric theorems algebraically. Include simple proofs involving circles **(Standard G.GPE.4).** Use coordinates to prove simple geometric theorems algebraically **(Standard G.GPE.6).**

- **Standard G.GPE.1**   Derive the equation of a circle of given center and radius using the Pythagorean Theorem; complete the square to find the center and radius of a circle given by an equation.

- **Standard G.GPE.4**   Use coordinates to prove simple geometric theorems algebraically. *For example, prove or disprove that a figure defined by four given points in the coordinate plane is a rectangle; prove or disprove that the point (1, √3) lies on the circle centered at the origin and containing the point (0, 2).*

- **Standard G.GPE.6**   Find the point on a directed line segment between two given points that partitions the segment in a given ratio.

Strand: GEOMETRY—Geometric Measurement and Dimension (G.GMD)

Explain volume formulas and use them to solve problems **(Standards G.GMD.1, 3).**

- **Standard G.GMD.1** Give an informal argument for the formulas for the circumference of a circle, area of a circle, volume of a cylinder, pyramid, and cone. Informal arguments for area formulas can make use of the way in which area scale under similarity transformations: when one figure in the plane results from another by applying a similarity transformation with scale factor k, its area is k^2 times the area of the first. *Use dissection arguments, Cavalieri's principle, and informal limit arguments.*

- **Standard G.GMD.3** Use volume formulas for cylinders, pyramids, cones, and spheres to solve problems. Informal arguments for volume formulas can make use of the way in which volume scale under similarity transformations: when one figure results from

another by applying a similarity transformation, volumes of solid figures scale by k^3 under a similarity transformation with scale factor k. ★

Strand: STATISTICS—Interpreting Categorical and Quantitative Data (S.ID)

Summarize, represent, and interpret data on two categorical or quantitative variables **(Standard S.ID.5).**

■ **Standard S.ID.5** Summarize categorical data for two categories in two-way frequency tables. Interpret relative frequencies in the context of the data (including joint, marginal, and condition reltive frequencies). Recognize possible associations and trends in the date.

Strand: STATISTICS—Conditional Probability and the Rules of Probability (S.CP)

Understand independence and conditional probability and use them to interpret data **(Standards S.CP.1, 4–5).** Use the rules of probability to compute probabilities of compound events in a uniform probability model **(Standard S.CP.6).**

■ **Standard S.CP.1** Describe events as subsets of a sample space (the set of outcomes) using characteristics (or categories) of the outcomes, or as unions, intersections, or complements of other events ("or," "and," "not").

■ **Standard S.CP.4** Construct and interpret two-way frequency tables of data when two categories are associated with each object being classified. Use the two-way table as a sample space to decide if events are independent and to approximate conditional probabilities. *For example, collect data from a random sample of students in your school on their favorite subject among math, science, and English. Estimate the probability that a randomly selected student from your school will favor science given that the student is in tenth grade. Do the same for other subjects and compare the results.*

■ **Standard S.CP.5** Recognize and explain the concepts of conditional probability and independence in everyday language and everyday situations. *For example, compare the chance of having lung cancer if you are are a smoker with the chance of being a smoker if you have lung cancer.*

■ **Standard S.CP.6** Find the conditional probability of *A* given *B* as the fraction of *B*'s outcomes that also belong to *A*, and interpret the answer in terms of the model.

SECONDARY MATHEMATICS II—HONORS STANDARDS

Strand: NUMBER AND QUANTITY—Complex Number System (N.CN)

Perform arithmetic operations with complex numbers **(Standard N.CN.3).** Represent complex numbers and their operations on the complex plane **(Standards N.CN.4–5).**

■ **Standard N.CN.3** Find the conjugate of a complex number; use conjugates to find moduli and quotients of complex numbers.

■ **Standard N.CN.4** Represent complex numbers on the complex plane in rectangular form, and explain why the rectangular form of a given complex number represents the same number.

■ **Standard N.CN.5** Represent addition, subtraction, and multiplication geometrically on the complex plane; use properties of this representation for computation. *For example, (-1 + √3 i)3 = 8 because (-1 + √3 i) has modulus 2 and argument 120°.*

Strand: ALGEBRA—Reasoning With Equations and Inequalities (A.REI)

Solve systems of equations **(Standards A.REI.8–9).**

■ **Standard A.REI.8** Represent a system of linear equations as a single-matrix equation in a vector variable.

■ **Standard A.REI.9** Find the inverse of a matrix if it exists, and use it to solve systems of linear equations (using technology for matrices of dimension 3 x 3 or greater).

Strand: FUNCTIONS—Interpreting Functions (F.IF)

Analyze functions using different representations **(Standards F.IF.10–11).**

■ **Standard F.IF.10** Use sigma notation to represent the sum of a finite arithmetic or geometric series.

■ **Standard F.IF.11** Represent series algebraically, graphically, and numerically.

Strand: GEOMETRY—Expressing Geometric Properties With Equations (G-GPE)

Translate between the geometric description and the equation for a conic section **(Standards G.GPE.2–3).**

■ **Standard G.GPE.2** Derive the equation of a parabola given a focus and directrix.

■ **Standard G.GPE.3** Derive the equations of ellipses and hyperbolas given the foci, using the fact that the sum or difference of distances from the foci is constant.

Strand: STATISTICS AND PROBABILITY—Conditional Probability and the Rules of Probability (S.CP)

Understand independence and conditional probability and use them to interpret data **(Standards S.CP.2–3).** Use the rules of probability to compute probabilities of compound events in a uniform probability model **(Standards S.CP.7–8).**

- **Standard S.CP.2** Understand that two events A and B are independent if the probability of A and B occurring together is the product of their probabilities, and use this characterization to determine if they are independent.

- **Standard S.CP.3** Understand the conditional probability of *A* given *B* as *P(A and B)/P(B)*, and interpret independence of A and *B* as saying that the conditional probability of *B* given *A* is the same as the probability of *B*.

- **Standard S.CP.7** Apply the Addition Rule, *P(A or B) = P(A) + P(B) – P(A and B)*, and interpret the answer in terms of the model.

- **Standard S.CP.8** Apply the general Multiplication Rule in a uniform probability model, *P(A and B) = P(A)P(B|A) = P(B)P(A|B)*, and interpret the answer in terms of the model.

SECONDARY MATHEMATICS III

IN SECONDARY MATHEMATICS III students pull together and apply the accumulation of learning that they have from their previous courses, with content grouped into four critical areas, organized into units. They apply methods from probability and statistics to draw inferences and conclusions from data. Students expand their repertoire of functions to include polynomial, rational, and radical functions. They expand their study of right triangle trigonometry to include general triangles. And, finally, students bring together all of their experience with functions and geometry to create models and solve contextual problems. The Mathematical Practice Standards apply throughout each course and, together with the content standards, prescribe that students experience mathematics as a coherent, useful, and logical subject that makes use of their ability to make sense of problem situations.

> **CRITICAL AREA 1:** Students see how the visual displays and summary statistics they learned in earlier grades relate to different types of data and to probability distributions. They identify different ways of collecting data—including sample surveys, experiments, and simulations—and the role that randomness and careful design play in the conclusions that can be drawn.

> **CRITICAL AREA 2:** This area develops the structural similarities between the system of polynomials and the system of integers. Students draw on analogies between polynomial arithmetic and base-ten computation, focusing on properties of operations, particularly the distributive property. Students connect multiplication of polynomials with multiplication of multi-digit integers, and division of polynomials with long division of integers. Students identify zeros of polynomials and make connections between zeros of polynomials and solutions of polynomial equations. The unit culminates with the fundamental theorem of algebra. Rational numbers extend the arithmetic of integers by allowing division by all numbers except 0. Similarly, rational expressions extend the arithmetic of polynomials by allowing division by all polynomials except the zero polynomial. A central theme of this unit is that the arithmetic of rational expressions is governed by the same rules as the arithmetic of rational numbers.

> **CRITICAL AREA 3:** Students develop the Laws of Sines and Cosines in order to find missing measures of general (not necessarily right) triangles. They are able to distinguish whether three given measures (angles or sides) define 0, 1, 2, or infinitely many triangles. This discussion of general triangles opens up the idea of trigonometry applied beyond the right triangle—that is, at least to obtuse angles. Students build on this idea to develop the notion of radian measure for angles and extend the domain of the trigonometric functions to all real numbers. They apply this knowledge to model simple periodic phenomena.

> **CRITICAL AREA 4:** Students synthesize and generalize what they have learned about a variety of function families. They extend their work with exponential functions to include solving exponential equations with logarithms. They explore the effects of transformations on graphs of diverse functions, including functions arising in an application, in order to abstract the general principle that transformations on a graph

always have the same effect regardless of the type of the underlying functions. They identify appropriate types of functions to model a situation, adjust parameters to improve the model, and compare models by analyzing the appropriateness of the fit and making judgments about the domain over which a model is a good fit. The description of modeling as "the process of choosing and using mathematics and statistics to analyze empirical situations, to understand them better, and to make decisions" is at the heart of this area. The narrative discussion and diagram of the modeling cycle should be considered when knowledge of functions, statistics, and geometry is applied in a modeling context.

Strand: MATHEMATICAL PRACTICES (MP)

The Standards for Mathematical Practice in Secondary Mathematics III describe mathematical habits of mind that teachers should seek to develop in their students. Students become mathematically proficient in engaging with mathematical content and concepts as they learn, experience, and apply these skills and attitudes **(Standards MP.1–8).**

■ **Standard SIII.MP.1 Make sense of problems and persevere in solving them.** Explain the meaning of a problem and look for entry points to its solution. Analyze givens, constraints, relationships, and goals. Make conjectures about the form and meaning of the solution, plan a solution pathway, and continually monitor progress asking, "Does this make sense?" Consider analogous problems, make connections between multiple representations, identify the correspondence between different approaches, look for trends, and transform algebraic expressions to highlight meaningful mathematics. Check answers to problems using a different method.

■ **Standard SIII.MP.2 Reason abstractly and quantitatively.** Make sense of the quantities and their relationships in problem situations. Translate between context and algebraic representations by contextualizing and decontextualizing quantitative relationships. This includes the ability to decontextualize a given situation, representing it algebraically and manipulating symbols fluently as well as the ability to contextualize algebraic representations to make sense of the problem.

■ **Standard SIII.MP.3 Construct viable arguments and critique the reasoning of others.** Understand and use stated assumptions, definitions, and previously established results in constructing arguments. Make conjectures and build a logical progression of statements to explore the truth of their conjectures. Justify conclusions and communicate them to others. Respond to the arguments of others by listening, asking clarifying questions, and critiquing the reasoning of others.

■ **Standard SIII.MP.4 Model with mathematics.** Apply mathematics to solve problems arising in everyday life, society, and the workplace. Make assumptions and approximations, identifying important quantities to construct a mathematical model. Routinely interpret mathematical results in the context of the situation and reflect on whether the results make sense, possibly improving the model if it has not served its purpose.

■ **Standard SIII.MP.5 Use appropriate tools strategically.** Consider the available tools and be sufficiently familiar with them to make sound decisions about when each tool might be helpful, recognizing both the insight to be gained as well as the limitations. Identify relevant external mathematical resources and use them to pose or solve problems. Use tools to explore and deepen their understanding of concepts.

■ **Standard SIII.MP.6 Attend to precision.** Communicate precisely to others. Use explicit definitions in discussion with others and in their own reasoning. They state the meaning of the symbols they choose. Specify units of measure and label axes to clarify the correspondence with quantities in a problem. Calculate accurately and efficiently, express numerical answers with a degree of precision appropriate for the problem context.

■ **Standard SIII.MP.7 Look for and make use of structure.** Look closely at mathematical relationships to identify the underlying structure by recognizing a simple structure within a more complicated structure. See complicated things, such as some algebraic expressions, as single objects or as being composed of several objects. *For example, see $5 - 3(x - y)^2$ as 5 minus a positive number times a square and use that to realize that its value cannot be more than 5 for any real numbers x and y.*

■ **Standard SIII.MP.8 Look for and express regularity in repeated reasoning.** Notice if reasoning is repeated, and look for both generalizations and shortcuts. Evaluate the reasonableness of intermediate results by maintaining oversight of the process while attending to the details.

Strand: NUMBER AND QUANTITY—The Complex Number System (**N.CN**)

Use complex numbers in polynomial identities and equations. Build on work with quadratic equations in Secondary Mathematics II **(Standards N.CN.8–9).**

■ **Standard N.CN.8** Extend polynomial identities to the complex numbers. *For example, rewrite $x^2 + 4$ as $(x + 2i)(x - 2i)$.*

■ **Standard N.CN.9** Know the Fundamental Theorem of Algebra; show that it is true for quadratic polynomials. Limit to polynomials with real coefficients.

Strand: ALGEBRA—Seeing Structures in Expressions (A.SSE)

Interpret the structure of expressions. Extend to polynomial and rational expressions **(Standards A.SSE.1–2).** Write expressions in equivalent forms to solve problems **(Standard A.SSE.4).**

■ **Standard A.SSE.1**   Interpret polynomial and rational expressions that represent a quantity in terms of its context.★

   a. Interpret parts of an expression, such as terms, factors, and coefficients.

   b. Interpret complex expressions by viewing one or more of their parts as a single entity. *For example, examine the behavior of $P(1+r/n)^{nt}$ as n becomes large.*

■ **Standard A.SSE.2**   Use the structure of an expression to identify ways to rewrite it. *For example, see $x^4 - y^4$ as $(x^2)^2 - (y^2)^2$, thus recognizing it as a difference of squares that can be factored as $(x^2 - y^2)(x^2 + y^2)$.*

■ **Standard A.SSE.4**   Understand the formula for the sum of a series and use the formula to solve problems.

   a. Derive the formula for the sum of an arithmetic series.

   b. Derive the formula for the sum of a geometric series, and use the formula to solve problems. Extend to infinite geometric series. *For example, calculate mortgage payments.*★

Strand: ALGEBRA—Arithmetic With Polynomials and Rational Expressions (A.APR)

Perform arithmetic operations on polynomials, extending beyond the quadratic polynomials **(Standard A.APR.1).** Understand the relationship between zeros and factors of polynomials **(Standards A.APR.2–3).** Use polynomial identities to solve problems **(Standards A.APR.4–5).** Rewrite rational expressions **(Standards A.APR.6–7).**

■ **Standard A.APR.1** Understand that all polynomials form a system analogous to the integers, namely, they are closed under the operations of addition, subtraction, and multiplication; add, subtract, and multiply polynomials.

■ **Standard A.APR.2** Know and apply the Remainder Theorem: For a polynomial $p(x)$ and a number a, the remainder on division by $x - a$ is $p(a)$, so $p(a) = 0$ if and only if $(x - a)$ is a factor of $p(x)$.

■ **Standard A.APR.3** Identify zeros of polynomials when suitable factorizations are available, and use the zeros to construct a rough graph of the function defined by the polynomial.

■ **Standard A.APR.4** Prove polynomial identities and use them to describe numerical relationships. *For example, the polynomial identity $(x^2 + y^2)^2 = (x^2 - y^2)^2 + (2xy)^2$ can be used to generate Pythagorean triples.*

■ **Standard A.APR.5** Know and apply the Binomial Theorem for the expansion of $(x + y)^n$ in powers of x and y for a positive integer n, where x and y are any numbers. *For example, with coefficients determined by Pascal's Triangle.*

■ **Standard A.APR.6** Rewrite simple rational expressions in different forms; write $a(x)/b(x)$ in the form $q(x) + r(x)/b(x)$, where $a(x), b(x), q(x),$ and $r(x)$ are polynomials with the degree of $r(x)$ less than the degree of $b(x)$, using inspection, long division or, for the more complicated examples, a computer algebra system.

■ **Standard A.APR.7** Understand that rational expressions form a system analogous to the rational numbers, closed under addition, subtraction, multiplication, and division by a nonzero rational expression; add, subtract, multiply, and divide rational expressions.

Strand: ALGEBRA: CREATING EQUATIONS (A.CED)

Create equations that describe numbers or relationships, using all available types of functions to create such equations **(Standards A.CED.1–4).**

■ **Standard A.CED.1** Create equations and inequalities in one variable and use them to solve problems. Include equations arising from linear and quadratic functions, and simple rational and exponential functions.

■ **Standard A.CED.2** Create equations in two or more variables to represent relationships between quantities; graph equations on coordinate axes with labels and scales.

■ **Standard A.CED.3** Represent constraints by equations or inequalities, and by systems of equations and/or inequalities, and interpret solutions as viable or non-viable options in

a modeling context. *For example, maximizing the volume of a box for a given surface area while drawing attention to the practical domain.*

■ **Standard A.CED.4**   Rearrange formulas to highlight a quantity of interest, using the same reasoning as in solving equations. *For example, rearrange the compound interest formula to solve for* t: $A = P(1+r/n)^{nt}$

Strand: ALGEBRA: REASONING WITH EQUATIONS AND INEQUALITIES (A.REI)

Understand solving equations as a process of reasoning and explain the reasoning **(Standard A.REI.2).** Represent and solve equations and inequalities graphically **(Standard A.REI.11).**

■ **Standard A.REI.2**   Solve simple rational and radical equations in one variable, and give examples showing how extraneous solutions may arise.

■ **Standard A.REI.11**   Explain why the *x*-coordinates of the points where the graphs of the equations *y = f(x) and y = g(x)* intersect are the solutions of the equation *f(x) = g(x)*; find the solutions approximately, for example, using technology to graph the functions, make tables of values, or find successive approximations. Include cases where *f(x)* and/ or *g(x)* are linear, polynomial, rational, absolute value, exponential, and logarithmic functions.★

Strand: FUNCTIONS—Interpreting Functions (F.IF)

Interpret functions that arise in applications in terms of a context **(Standards F.IF.4–6).** Analyze functions using different representations **(Standards F.IF.7–9).**

■ **Standard F.IF.4**   For a function that models a relationship between two quantities, interpret key features of graphs and tables in terms of the quantities, and sketch graphs showing key features given a verbal description of the relationship. *Key features include intercepts; intervals where the function is increasing, decreasing, positive, or negative; relative maximums and minimums; symmetries; end behavior; and periodicity.*★

■ **Standard F.IF.5**   Relate the domain of a function to its graph and, where applicable, to the quantitative relationship it describes. *For example, if the function* h(n) *gives the number of person-hours it takes to assemble* n *engines in a factory, then the positive integers would be an appropriate domain for the function.*★

■ **Standard F.IF.6**   Calculate and interpret the average rate of change of a function (presented symbolically or as a table) over a specified interval. Estimate the rate of change from a graph.★

■ **Standard F.IF.7**   Graph functions expressed symbolically and show key features of the graph, by hand in simple cases and using technology for more complicated cases.★

  b.   Graph square root, cube root, and piecewise-defined functions, including step functions and absolute value functions. Compare and contrast square root, cubed root, and step functions with all other functions.

   c. Graph polynomial functions, identifying zeros when suitable factorizations are available, and showing end behavior.

   d. Graph rational functions, identifying zeros and asymptotes when suitable factorizations are available, and showing end behavior.

   e. Graph exponential and logarithmic functions, showing intercepts and end behavior; and trigonometric functions, showing period, midline, and amplitude.

■ **Standard F.IF.8**   Write a function defined by an expression in different but equivalent forms to reveal and explain different properties of the function.

■ **Standard F.IF.9**   Compare properties of two functions each represented in a different way (algebraically, graphically, numerically in tables, or by verbal descriptions). *For example, given a graph of one quadratic function and an algebraic expression for another, say which has the larger maximum.*

Strand: FUNCTIONS—Building Functions (F.BF)

Build a function that models a relationship between two quantities. Develop models for more complex or sophisticated situations **(Standards F.BF.1).** Build new functions from existing functions **(Standards F.BF.3–4).**

■ **Standard F.BF.1**   Write a function that describes a relationship between two quantities. ★

   b. Combine standard function types using arithmetic operations. *For example, build a function that models the temperature of a cooling body by adding a constant function to a decaying exponential, and relate these functions to the model.*

■ **Standard F.BF.3**   Identify the effect on the graph of replacing $f(x)$ by $f(x) + k$, $k\,f(x)$, $f(kx)$, and $f(x + k)$ for specific values of k (both positive and negative); find the value of k given the graphs. Note the effect of multiple transformations on a single function and the common effect of each transformation across function types. Include functions defined only by a graph. Experiment with cases and illustrate an explanation of the effects on the graph using technology. Include recognizing even and odd functions from their graphs and algebraic expressions for them.

■ **Standard F.BF.4**   Find inverse functions.

   a. Solve an equation of the form $f(x) = c$ for a simple function f that has an inverse and write an expression for the inverse. Include linear, quadratic, exponential, logarithmic, rational, square root, and cube root functions. *For example,* $f(x) = 2x^3$ *or* $f(x) = $ *(x+1)/(x-1) for* $x \neq 1$.

Strand: FUNCTIONS—Linear, Quadratic, and Exponential Models (F.LE)

Construct and compare linear, quadratic, and exponential models and solve problems **(Standards F.LE.3–4).** Interpret expressions for functions in terms of the situation it models.

Introduce $f(x) = e^x$ as a model for continuous growth **(Standard F.LE.5).**

■ **Standard F.LE.3** Observe using graphs and tables that a quantity increasing exponentially eventually exceeds a quanitity increasing linearly, quadratically, or (more generally) as a polynomial function.

■ **Standard F.LE.4** For exponential models, express as a logarithm the solution to $ab^{ct} = d$ where *a, c,* and *d* are numbers and the base *b* is 2, 10, or *e*; evaluate the logarithm using technology. Include the relationship between properties of logarithms and properties of exponents, such as the connection between the properties of exponents and the basic logarithm property that log *xy = log x + log y.*

■ **Standard F.LE.5** Interpret the parameters in a linear, quadratic, or exponential function in terms of a context.

Strand: FUNCTIONS—Trigonometric Functions (F.TF)

Extend the domain of trigonometric functions using the unit circle **(Standards F.TF.1–3).** Model periodic phenomena with trigonometric functions **(Standards F.TF.5–7).**

■ **Standard F.TF.1** Understand radian measure of an angle as the length of the arc on the unit circle subtended by the angle.

■ **Standard F.TF.2** Explain how the unit circle in the coordinate plane enables the extension of trigonometric functions to all real numbers, interpreted as radian measures of angles traversed counterclockwise around the unit circle.

■ **Standard F.TF.3** Use special triangles to determine geometrically the values of sine, co-sine, tangent for $\pi/3$, $\pi/4$ and $\pi6$, and use the unit circle to express the values of sine, cosine, and tangent for $\pi - x$, $\pi + x$, and $2\pi - x$ in terms of their values for *x,* where *x* is any real number.

■ **Standard F.TF.5** Choose trigonometric functions to model periodic phenomena with specified amplitude, frequency, and midline.★

■ **Standard F.TF.7** Use inverse functions to solve trignometric equations that arise in mod-eling context; evaluate the solutions using technology and interpret them in terms of context. Limit solutions to a given interval. ★

Strand: GEOMETRY—Similarity, Right Triangles, and Trigonometry (G.SRT)

Apply trigonometry to general triangles. With respect to the general case of the Laws of Sines and Cosines, the definitions of sine and cosine must be extended to obtuse angles **(Standards G.SRT.9–11).**

■ **Standard G.SRT.9** Derive the formula $A = \frac{1}{2} ab \sin(C)$ for the area of a triangle by drawing an auxiliary line from a vertex perpendicular to the opposite side.

■ **Standard G.SRT.10** Prove the Laws of Sines and Cosines and use them to solve problems.

■ **Standard G.SRT.11** Understand and apply the Law of Sines and the Law of Cosines to find unknown measurements in right and non-right triangles (e.g., surveying problems, resultant forces).

Strand: GEOMETRY—Geometric Measurement and Dimension (G.GMD)

Visualize relationships between two-dimensional and three-dimensional objects **(Standards G.MD.4).**

■ **Standard G.GMD.4** Identify the shapes of two-dimensional cross-sections of three-dimensional objects, and identify three-dimensional objects generated by rotations of two-dimensional objects.

Strand: GEOMETRY—Modeling With Geometry (G.MG)

Apply geometric concepts in modeling situations **(Standards G.MG.1–3).**

■ **Standard G.MG.1** Use geometric shapes, their measures, and their properties to describe objects (e.g., modeling a tree trunk or a human torso as a cylinder).★

■ **Standard G.MG.2** Apply concepts of density based on area and volume in modeling situations (e.g., persons per square mile, BTUs per cubic foot).★

■ **Standard G.MG.3** Apply geometric methods to solve design problems (e.g., designing an object or structure to satisfy physical constraints or minimize cost; working with typographic grid systems based on ratios).★

Strand: STATISTICS—Interpreting Categorical and Quantitative Data (S.ID)

Summarize, represent, and interpret data on a single count or measurement variable. While students may have heard of the normal distribution, it is unlikely that they will have prior experience using it to make specific estimates. Build on students' understanding of data distributions to help them see how the normal distribution uses area to make estimates of frequencies (which can be expressed as probabilities). Emphasize that only some data are well described by a normal distribution **(Standard S.ID.4).**

■ **Standard S.ID.4** Use the mean and standard deviation of a data set to fit it to a normal distribution and to estimate population percentages. Recognize that there are data sets for which such a procedure is not appropriate. Use calculators, spreadsheets, and tables to estimate areas under the normal curve.

Strand: STATISTICS—Making Inferences and Justifying Conclusions (S.IC)

Understand and evaluate random processes underlying statistical experiments **(Standard S.IC.1).** Draw and justify conclusions from sample surveys, experiments, and observational studies. In earlier grades, students are introduced to different ways of collecting data and use

graphical displays and summary statistics to make comparisons. These ideas are revisited with a focus on how the way in which data is collected determines the scope and nature of the conclusions that can be drawn from that data. The concept of statistical significance is developed informally through simulation as meaning a result that is unlikely to have occurred solely as a result of random selection in sampling or random assignment in an experiment. For S.IC.4, focus on the variability of results from experiments—that is, focus on statistics as a way of dealing with, not eliminating, inherent randomness **(Standards S.IC.3–4, 6).**

■ **Standard S.IC.1**   Understand that statistics allow inferences to be made about population parameters based on a random sample from that population.

■ **Standard S.IC.3**   Recognize the purposes of and differences among sample surveys, experiments, and observational studies; explain how randomization relates to each.

■ **Standard S.IC.4**   Use data from a sample survey to estimate a population mean or proportion; develop a margin of error through the use of simulation models for random sampling.

■ **Standard S.IC.6**   Evaluate reports based on data.

SECONDARY MATHEMATICS III—HONORS STANDARDS

Strand: NUMBER AND QUANTITY—Complex Number System (N.CN)

Perform arithmetic operations with complex numbers **(Standard N.CN.3)**. Represent complex numbers and their operations on the complex plane **(Standard N.CN.4–6)**. Use complex numbers in polynomial identities and equations **(Standard N.CN.10)**.

- **Standard N.CN.3** Find the conjugate of a complex number; use conjugates to find moduli and quotients of complex numbers.

- **Standard N.CN.4** Represent complex numbers on the complex plane in rectangular form and polar form (including real and imaginary numbers), and explain why the rectangular form of a given complex number represents the same number.

- **Standard N.CN.5** Represent addition, subtraction, multiplication, and conjugation of complex numbers geometrically on the complex plane; use properties of this representation for computation. *For example, $(-1 + \sqrt{3}\,i)3 = 8$ because $(-1 + \sqrt{3}\,i)$ has modulus 2 and argument 120°.*

- **Standard N.CN.6** Calculate the distance between numbers in the complex plane as the modulus of the difference, and the midpoint of a segment as the average of the numbers at its endpoints.

- **Standard N.CN.10** Multiply complex numbers in polar form and use DeMoivre's Theorem to find roots of complex numbers.

Strand: FUNCTIONS—Interpreting Functions (F.IF)

Analyze functions using different representations **(Standard F.IF.7, d and f)**.

- **Standard F.IF.7** Graph functions expressed symbolically, and show key features of the graph, by hand in simple cases and using technology for more complicated cases.

    d. Graph rational functions, identifying zeros, asymptotes, and point discontinuities when suitable factorizations are available, and showing end behavior.

    f. Define a curve parametrically and draw its graph.

Strand: FUNCTIONS—Building Functions (F.BF).

Build a function that models a relationship between two quantities **(Standard F.BF.1.c)**. Build new functions from existing functions **(Standards F.BF.4, b,c,d–5)**.

- **Standard F.BF.1** Write a function that describes a relationship between two quantities.

    c. Compose functions. *For example, if T(y) is the temperature in the atmosphere as a function of height, and h(t) is the height of a weather balloon as a function of time, then T(h(t)) is the temperature at the location of the weather balloon as a function of time.*

■ **Standard F.BF.4** Find inverse functions.

   b. Verify by composition that one function is the inverse of another.

   c. Read values of an inverse function from a graph or a table, given that the function has an inverse.

   d. Produce an invertible function from a non-invertible function by restricting the domain.

■ **Standard F.BF.5** Understand the inverse relationship between exponents and logarithms, and use this relationship to solve problems involving logarithms and exponents.

Strand: FUNCTIONS—Trigonometric Functions (F.TF)

Extend the domain of trigonometric functions using the unit circle **(Standard T.FT.4).** Model periodic phenomena with trigonometric functions **(Standards T.FT.6–7).** Prove and apply trigonometric identities **(Standard T.FT.9).**

■ **Standard F.TF.4** Use the unit circle to explain symmetry (odd and even) and periodicity of trigonometric functions.

■ **Standard F.TF.6** Understand that restricting a trigonometric function to a domain on which it is always increasing or always decreasing allows its inverse to be constructed.

■ **Standard F.TF.7** Use the inverse functions to solve trigonometric equations that arise in the modeling contexts; evaluate the solutions using technology, and interpret them in terms of the context.

■ **Standard F.TF.9** Prove the addition and subtraction formulas for sine, cosine, and tangent, and use them to solve problems.

Strand: GEOMETRY—Geometric Measurement and Dimension (G.GMD)

Explain volume formulas and use them to solve problems **(Standard G.GMD.2).**

■ **Standard G.GMD.2** Give an informal argument using Cavalieri's principle for the formulas for the volume of a sphere and other solid figures.

Strand: STATISTICS AND PROBABILITY—Conditional Probability and the Rules of Probability (S.CP)

Use the rules of probability to compute probabilities of compound events in a uniform probability model **(Standard S.CP.9).**

■ **Standard S.CP.9** Use permutations and combinations to compute probabilities of compound events and solve problems.

PRECALCULUS

Number and Quantity

- **Vector and Matrix Quantities**
- **Complex Number System**

Algebra

- **Reasoning With Equations and Inequalities**

Functions

- **Interpreting Functions**
- **Building Functions**
- **Trigonometric Functions**

Geometry

- **Geometric Measurement and Dimension**
- **Expressing Geometric Properties with Equations**

Statistics

- **Conditional Probability and the Rules of Probability**

MATHEMATICAL PRACTICES

1. Make sense of problems and persevere in solving them.

2. Reason abstractly and quantitatively.

3. Construct viable arguments and critique the reasoning of others.

4. Model with mathematics.

5. Use appropriate tools strategically.

6. Attend to precision.

7. Look for and make use of structure.

8. Look for and express regularity in repeated reasoning.

Strand: MATHEMATICAL PRACTICES (MP)

The Standards for Mathematical Practice in PreCalculus describe mathematical habits of mind that teachers should seek to develop in their students. Students become mathematically proficient in engaging with mathematical content and concepts as they learn, experience, and apply these skills and attitudes **(Standards MP.1–8).**

■ **Standard P.MP.1   Make sense of problems and persevere in solving them.** Explain the meaning of a problem and look for entry points to its solution. Analyze givens, constraints, relationships, and goals. Make conjectures about the form and meaning of the solution, plan a solution pathway, and continually monitor progress asking, "Does this make sense?" Consider analogous problems, make connections between multiple representations, identify the correspondence between different approaches, look for trends, and transform algebraic expressions to highlight meaningful mathematics. Check answers to problems using a different method.

■ **Standard P.MP.2   Reason abstractly and quantitatively.** Make sense of the quantities and their relationships in problem situations. Translate between context and algebraic representations by contextualizing and decontextualizing quantitative relationships. This includes the ability to decontextualize a given situation, representing it algebraically and manipulating symbols fluently as well as the ability to contextualize algebraic representations to make sense of the problem.

■ **Standard P.MP.3   Construct viable arguments and critique the reasoning of others.** Understand and use stated assumptions, definitions, and previously established results in constructing arguments. Make conjectures and build a logical progression of statements to explore the truth of their conjectures. Justify conclusions and communicate them to others. Respond to the arguments of others by listening, asking clarifying questions, and critiquing the reasoning of others.

■ **Standard P.MP.4   Model with mathematics.** Apply mathematics to solve problems arising in everyday life, society, and the workplace. Make assumptions and approximations, identifying important quantities to construct a mathematical model. Routinely interpret mathematical results in the context of the situation and reflect on whether the results make sense, possibly improving the model if it has not served its purpose.

■ **Standard P.MP.5   Use appropriate tools strategically.** Consider the available tools and be sufficiently familiar with them to make sound decisions about when each tool might be helpful, recognizing both the insight to be gained as well as the limitations. Identify relevant external mathematical resources and use them to pose or solve problems. Use tools to explore and deepen their understanding of concepts.

■ **Standard P.MP.6   Attend to precision.** Communicate precisely to others. Use explicit definitions in discussion with others and in their own reasoning. They state the meaning of the symbols they choose. Specify units of measure and label axes to clarify the correspondence with quantities in a problem. Calculate accurately and efficiently, express numerical answers with a degree of precision appropriate for the problem context.

■ **Standard P.MP.7   Look for and make use of structure.** Look closely at mathematical relationships to identify the underlying structure by recognizing a simple structure within a more complicated structure. See complicated things, such as some algebraic expressions, as single objects or as being composed of several objects. *For example, see 5 – 3(x – y)^2 as 5 minus a positive number times a square and use that to realize that its value cannot be more than 5 for any real numbers* x *and* y.

■ **Standard P.MP.8   Look for and express regularity in repeated reasoning.** Notice if reasoning is repeated, and look for both generalizations and shortcuts. Evaluate the reasonableness of intermediate results by maintaining oversight of the process while attending to the details.

Strand: NUMBER AND QUANTITY—Vector and Matrix Quantities (N.VM)

Represent and model with vector quantities **(Standards 1–3).** Perform operations on vectors **(Standards 4–5).** Perform operations on matrices and use matrices in applications **(Standards 6–13).**

■ **Standard N.VM.1** Recognize vector quantities as having both magnitude and direction. Represent vector quantities by directed line segments, and use appropriate symbols for vectors and their magnitudes (e.g., *v, |v|, ||v||, v).*

■ **Standard N.VM.2** Find the components of a vector by subtracting the coordinates of an initial point from the coordinates of a terminal point.

■ **Standard N.VM.3** Solve problems involving velocity and other quantities that can be represented by vectors.

■ **Standard N.VM. 4** Add and subtract vectors.

    a. Add vectors end to end, component-wise, and by the parallelogram rule. Understand that the magnitude of a sum of two vectors is typically not the sum of the magnitudes.

    b. Given two vectors in magnitude and direction form, determine the magnitude and direction of their sum.

    c. Understand vector subtraction *v – w as v + (–w),* where *–w* is the additive inverse of *w,* with the same magnitude as *w* and pointing in the opposite direction. Represent vector subtraction graphically by connecting the tips in the appropriate order, and perform vector subtraction component-wise.

■ **Standard N.VM.5** Multiply a vector by a scalar.

    a. Represent scalar multiplication graphically by scaling vectors and possibly reversing their direction; perform scalar multiplication component-wise, e.g., *as c(vx , vy) = (cvx, cvy).*

    b. Compute the magnitude of a scalar multiple cv using $||cv|| = |c|v$. Compute the direction of cv knowing that when $|c|v \neq 0$, the direction of cv is either along v (for $c > 0$) or against vs (for $c < 0$).

■ **Standard N.VM.6** Use matrices to represent and manipulate data, e.g., to represent payoffs or incidence relationships in a network.

■ **Standard N.VM.7** Multiply matrices by scalars to produce new matrices, e.g., as when all of the payoffs in a game are doubled.

■ **Standard N.VM.8** Add, subtract, and multiply matrices of appropriate dimensions.

■ **Standard N.VM.9** Understand that, unlike multiplication of numbers, matrix multiplication for square matrices is not a commutative operation, but still satisfies the associative and distributive properties.

■ **Standard N.VM.10** Understand that the zero and identity matrices play a role in matrix addition and multiplication similar to the role of 0 and 1 in the real numbers. The determinant of a square matrix is nonzero if and only if the matrix has a multiplicative inverse.

■ **Standard N.VM.11** Multiply a vector (regarded as a matrix with one column) by a matrix of suitable dimensions to produce another vector. Work with matrices as transformations of vectors.

■ **Standard N.VM.12** Work with 2×2 matrices as transformations of the plane, and interpret the absolute value of the determinant in terms of area.

■ **Standard N.VM.13** Solve systems of linear equations up to three variables using matrix row reduction.

Strand: NUMBER AND QUANTITY—Complex Number Systems (N.CN)

Perform arithmetic operations with complex numbers **(Standard 3).** Represent complex numbers and their operations on the complex plane **(Standards 4–6).** Use complex numbers in polynomial identities and equations **(Standard 10).**

■ **Standard N.CN.3** Find the conjugate of a complex number; use conjugates to find moduli and quotients of complex numbers.

■ **Standard N.CN.4** Represent complex numbers on the complex plane in rectangular and polar form (including real and imaginary numbers), and explain why the rectangular and polar forms of a given complex number represent the same number.

■ **Standard N.CN.5** Represent addition, subtraction, multiplication, and conjugation of complex numbers geometrically on the complex plane; use properties of this representation for computation. *For example, $(-1 + \sqrt{3}\,i)^3 = 8$, because $(-1 + \sqrt{3}\,i)$ has modulus 2 and argument 120°.*

■ **Standard N.CN.6** Calculate the distance between numbers in the complex plane as the modulus of the difference, and the midpoint of a segment as the average of the numbers at its endpoints.

■ **Standard N.CN.10** Multiply complex numbers in polar form and use DeMoivre's Theorem to find roots of complex numbers.

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Strand: ALGEBRA: REASONING WITH EQUATIONS AND INEQUALITIES (A.REI)

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Solve systems of equations **(Standards 8–9).**

8. Represent a system of linear equations as a single matrix equation in a vector variable.

9. Find the inverse of a matrix, if it exists, and use it to solve systems of linear equations (using technology for matrices of dimension 3 x 3 or greater).

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Strand: FUNCTIONS—Interpreting Functions (F.IF)

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Analyze functions using different representations **(Standard 7, 10–11).**

■ **Standard F.IF.7** Graph functions expressed symbolically, and show key features of the graph, by hand in simple cases and using technology for more complicated cases.

d. Graph rational functions, identifying zeros, asymptotes, and point discontinuities when suitable factorizations are available, and showing end behavior.

f. Define a curve parametrically and draw its graph.

■ **Standard F.IF.10** Use sigma notation to represent the sum of a finite arithmetic or geometric series.

■ **Standard F.IF.11** Represent series algebraically, graphically, and numerically.

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Strand: FUNCTIONS—Building Functions (F.BF)

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Build a function that models a relationship between two quantities **(Standard 1).** Build new functions from existing functions **(Standard 4–5).**

■ **Standard F.BF.1** Write a function that describes a relationship between two quantities.

c. Compose functions. *For example, if* T(y) *is the temperature in the atmosphere as a function of height, and* h(t) *is the height of a weather balloon as a function of time, then* T(h(t)) *is the temperature at the location of the weather balloon as a function of time.*

■ **Standard F.BF.4** Find inverse functions.

b. Verify by composition that one function is the inverse of another.

   c. Read values of an inverse function from a graph or a table, given that the function has an inverse.

   d. Produce an invertible function from a non-invertible function by restricting the domain.

■ **Standard F.BF.5** Understand the inverse relationship between exponents and logarithms and use this relationship to solve problems involving logarithms and exponents.

Strand: FUNCTIONS—Trigonometric Functions (F.TF)

Extend the domain of trigonometric functions using the unit circle **(Standard 4).** Model periodic phenomena with trigonometric functions **(Standard 6–7).** Prove and apply trigonometric identities **(Standard 9).**

■ **Standard F.TF.4** Use the unit circle to explain symmetry (odd and even) and periodicity of trigonometric functions.

■ **Standard F.TF.6** Understand that restricting a trigonometric function to a domain on which it is always increasing or always decreasing allows its inverse to be constructed.

■ **Standard F.TF.7** Use inverse functions to solve trigonometric equations that arise in modeling contexts; evaluate the solutions using technology, and interpret them in terms of the context

■ **Standard F.TF.9** Prove the addition and subtraction formulas for sine, cosine, and tangent, and use them to solve problems.

Strand: GEOMETRY—Geometric Measurement and Dimension (G.GMD)

Explain volume formulas and use them to solve problems **(Standard 2).**

■ **Standard G.GMD.2** Give an informal argument using Cavalieri's principle for the formulas for the volume of a sphere and other solid figures.

Strand: GEOMETRY—Expressing Geometric Properties With Equations (G.GPE)

Translate between the geometric description and the equation for a conic section **(Standards 2–3).**

■ **Standard G.GPE.2** Derive the equation of a parabola given a focus and a directrix.

■ **Standard G.GPE.3** Derive the equations of ellipses and hyperbolas given the foci, using the fact that the sum or difference of distances from the foci is constant.

Strand: STATISTICS—Conditional Probability and the Rules of Probability (S.CP)

Understand independence and conditional probability and use them to interpret data **(Standards 2–3).** Use the rules of probability to compute probabilities of compound events in a uniform probability model **(Standards 7–9).**

■ **Standard S.CP.2** Understand that two events *A* and *B* are independent if the probability of *A* and *B* occurring together is the product of their probabilities, and use this characterization to determine if they are independent.

■ **Standard S.CP.3** Understand the conditional probability of *A* given *B* as *P(A and B)/P(B)*, and interpret independence of *A* and *B* as saying that the conditional probability of *B* given *A* is the same as the probability of *B*.

■ **Standard S.CP.7** Apply the Addition Rule, *P(A or B) = P(A) + P(B) – P(A and B)*, and interpret the answer in terms of the model.

■ **Standard S.CP.8** Apply the general Multiplication Rule in a uniform probability model, *P(A andB) = P(A)P(B|A) = P(B)P(A|B)*, and interpret the answer in terms of the model.

■ **Standard S.CP.9** Use permutations and combinations to compute probabilities of compound events and solve problems.

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