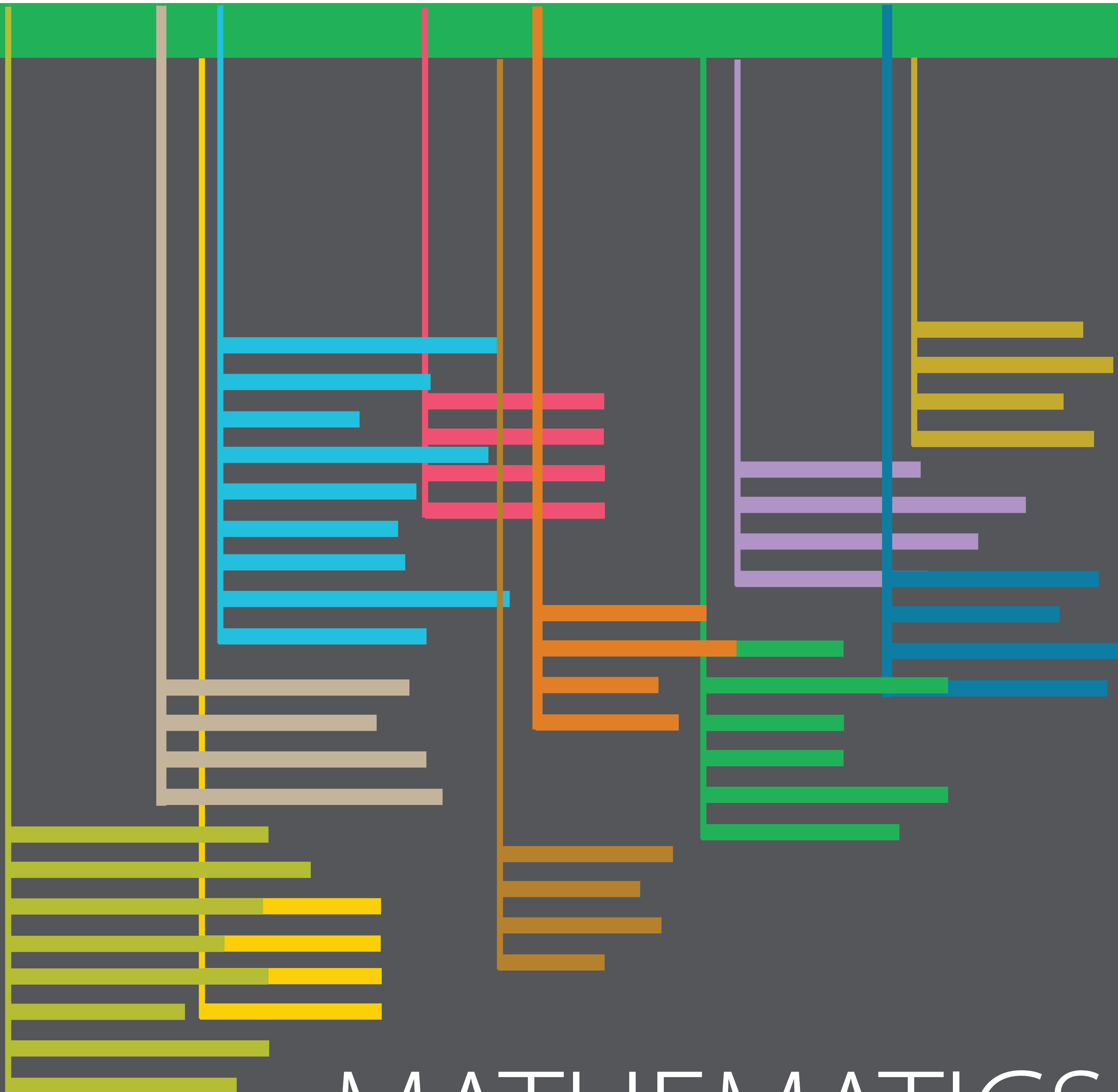

MATHEMATICS
High School (9–12)

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Utah State Office of Education

UTAH CORE STATE STANDARDS *for* MATHEMATICS

HIGH SCHOOL (9–12)

Adopted August 2010
by the
Utah State Board of Education

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The Utah State Board of Education, in January of 1984, established policy requiring the identification of specific core standards to be met by all K–12 students in order to graduate from Utah's secondary schools. The Utah State Board of Education regularly updates the Utah Core Standards, while parents, teachers, and local school boards continue to control the curriculum choices that reflect local values.

The Utah Core Standards are aligned to scientifically based content standards. They drive high quality instruction through statewide comprehensive expectations for all students. The standards outline essential knowledge, concepts, and skills to be mastered at each grade level or within a critical content area. The standards provide a foundation for ensuring learning within the classroom.

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INTRODUCTION

Organization of the Standards

The Utah Core Standards are organized into **strands**, which represent significant areas of learning within content areas. Depending on the core area, these strands may be designated by time periods, thematic principles, modes of practice, or other organizing principles.

Within each strand are **standards**. A standard is an articulation of the demonstrated proficiency to be obtained. A standard represents an essential element of the learning that is expected. While some standards within a strand may be more comprehensive than others, all standards are essential for mastery.

Understanding Mathematics

These standards define what students should understand and be able to do in their study of mathematics. Asking a student to understand something means asking a teacher to assess whether the student has understood it. But what does mathematical understanding look like? One hallmark of mathematical understanding is the ability to justify, in a way appropriate to the student's mathematical maturity, why a particular mathematical statement is true or where a mathematical rule comes from. Mathematical understanding and procedural skill are equally important, and both are assessable using mathematical tasks of sufficient richness.

The standards set grade-specific standards but do not dictate curriculum or teaching methods, nor do they define the intervention methods or materials necessary to support students who are well below or well above grade-level expectations. It is also beyond the scope of the Standards to define the full range of supports appropriate for English language learners and for students with special needs. At the same time, all students must have the opportunity to learn and meet the same high standards if they are to access the knowledge and skills necessary in their post-school lives. The standards should be read as allowing for the widest possible range of students to participate fully from the outset, along with appropriate accommodations to ensure maximum participation of students with special education needs. No set of grade-specific standards can fully reflect the great variety in abilities, needs, learning rates, and achievement levels of students in any given classroom. However, the standards do provide clear signposts along the way to the goal of college and career readiness for all students.

What students can learn at any particular grade level depends upon what they have learned before. Ideally then, each standard in this document might have been phrased in the form, "Students who already know… should next come to learn ..." Grade placements for specific topics have been made on the basis of state and international comparisons and the collective experience and collective professional judgment of educators, researchers and mathematicians. Learning opportunities will continue to vary across schools and school systems, and educators should make every effort to meet the needs of individual students based on their current understanding.

Utah State Office of Education

UTAH CORE STATE STANDARDS

for

MATHEMATICS HIGH SCHOOL COURSES

SECONDARY MATHEMATICS I

THE FUNDAMENTAL PURPOSE OF SECONDARY MATHEMATICS I is to formalize and extend the mathematics that students learned in the middle grades. The critical areas, organized into units, deepen and extend understanding of linear relationships, in part by contrasting them with exponential phenomena, and in part by applying linear models to data that exhibit a linear trend. Secondary Mathematics I uses properties and theorems involving congruent figures to deepen and extend understanding of geometric knowledge from prior grades. The final unit in the course ties together the algebraic and geometric ideas studied. The Mathematical Practice Standards apply throughout each course and, together with the content standards, prescribe that students experience mathematics as a coherent, useful, and logical subject that makes use of their ability to make sense of problem situations.

CRITICAL AREA 1: By the end of eighth grade, students have had a variety of experiences working with expressions and creating equations. Students continue this work by using quantities to model and analyze situations, to interpret expressions, and by creating equations to describe situations.

CRITICAL AREA 2: In earlier grades, students define, evaluate, and compare functions, and use them to model relationships between quantities. Students will learn function notation and develop the concepts of domain and range. They move beyond viewing functions as processes that take inputs and yield outputs, and start viewing functions as objects in their own right. They explore many examples of functions, including sequences; they interpret functions given graphically, numerically, symbolically, and verbally, translate between representations, and understand the limitations of various representations. They work with functions given by graphs and tables, keeping in mind that, depending upon the context, these representations are likely to be approximate and incomplete. Their work includes functions that can be described or approximated by formulas, as well as those that cannot. When functions describe relationships between quantities arising from a context, students reason with the units in which those quantities are measured. Students build on and informally extend their understanding of integer exponents to consider exponential functions. They compare and contrast linear and exponential functions, distinguishing between additive and multiplicative change. They interpret arithmetic sequences as linear functions and geometric sequences as exponential functions.

CRITICAL AREA 3: By the end of eighth grade, students have learned to solve linear equations in one variable and have applied graphical and algebraic methods to analyze and solve systems of linear equations in two variables. This area builds on these earlier experiences by asking students to analyze and explain the process of solving an equation and to justify the process used in solving a system of equations. Students develop fluency writing, interpreting, and translating between various forms of linear equations and inequalities, and using them to solve problems. They master the solution of linear equations and apply related solution techniques and the laws of exponents to the creation and solution of simple exponential equations. Students explore systems of equations and inequalities, and they find and interpret their solutions. All of this work is grounded on understanding quantities and on relationships between them.

CRITICAL AREA 4: This area builds upon students' prior experiences with data, providing students with more formal means of assessing how a model fits data. Students use regression techniques to describe approximately linear relationships between quantities. They use graphical representations and knowledge of the context to make judgments about the appropriateness of linear models. With linear models, they look at residuals to analyze the goodness of fit.

CRITICAL AREA 5: In previous grades, students were asked to draw triangles based on given measurements. They also have prior experience with rigid motions (translations, reflections, and rotations) and have used these to develop notions about what it means for two objects to be congruent. In this unit, students establish triangle congruence criteria, based on analyses of rigid motions and formal constructions. They solve problems about triangles, quadrilaterals, and other polygons. They apply reasoning to complete geometric constructions and explain why they work.

CRITICAL AREA 6: Building on their work with the Pythagorean Theorem in eighth grade to find distances, students use a rectangular coordinate system to verify geometric relationships, including properties of special triangles and quadrilaterals and slopes of parallel and perpendicular lines.

Strand: MATHEMATICAL PRACTICES (MP)

The Standards for Mathematical Practice in Secondary Mathematics I describe mathematical habits of mind that teachers should seek to develop in their students. Students become mathematically proficient in engaging with mathematical content and concepts as they learn, experience, and apply these skills and attitudes **(Standards MP.1–8).**

- **Standard SI.MP.1 Make sense of problems and persevere in solving them.** Explain the meaning of a problem and look for entry points to its solution. Analyze givens, constraints, relationships, and goals. Make conjectures about the form and meaning of the solution, plan a solution pathway, and continually monitor progress asking, "Does this make sense?" Consider analogous problems, make connections between multiple representations, identify the correspondence between different approaches, look for trends, and transform algebraic expressions to highlight meaningful mathematics. Check answers to problems using a different method.

- **Standard SI.MP.2 Reason abstractly and quantitatively.** Make sense of the quantities and their relationships in problem situations. Translate between context and algebraic representations by contextualizing and decontextualizing quantitative relationships. This includes the ability to decontextualize a given situation, representing it algebraically and manipulating symbols fluently as well as the ability to contextualize algebraic representations to make sense of the problem.

- **Standard SI.MP.3 Construct viable arguments and critique the reasoning of others.** Understand and use stated assumptions, definitions, and previously established results in constructing arguments. Make conjectures and build a logical progression of statements to explore the truth of their conjectures. Justify conclusions and communicate them to others. Respond to the arguments of others by listening, asking clarifying questions, and critiquing the reasoning of others.

- **Standard SI.MP.4 Model with mathematics.** Apply mathematics to solve problems arising in everyday life, society, and the workplace. Make assumptions and approximations, identifying important quantities to construct a mathematical model. Routinely interpret mathematical results in the context of the situation and reflect on whether the results make sense, possibly improving the model if it has not served its purpose.

- **Standard SI.MP.5 Use appropriate tools strategically.** Consider the available tools and be sufficiently familiar with them to make sound decisions about when each tool might be helpful, recognizing both the insight to be gained as well as the limitations. Identify relevant external mathematical resources and use them to pose or solve problems. Use tools to explore and deepen their understanding of concepts.

- **Standard SI.MP.6 Attend to precision.** Communicate precisely to others. Use explicit definitions in discussion with others and in their own reasoning. They state the meaning of the symbols they choose. Specify units of measure and label axes to clarify the correspondence with quantities in a problem. Calculate accurately and efficiently, express numerical answers with a degree of precision appropriate for the problem context.

- **Standard SI.MP.7 Look for and make use of structure.** Look closely at mathematical relationships to identify the underlying structure by recognizing a simple structure within a more complicated structure. See complicated things, such as some algebraic expressions, as single objects or as being composed of several objects. *For example, see* $5 - 3(x - y)^2$ *as 5 minus a positive number times a square and use that to realize that its value cannot be more than 5 for any real numbers* x *and* y.

- **Standard SI.MP.8 Look for and express regularity in repeated reasoning.** Notice if reasoning is repeated, and look for both generalizations and shortcuts. Evaluate the reasonableness of intermediate results by maintaining oversight of the process while attending to the details.

Strand: NUMBER AND QUANTITY—Quantities (N.Q)

Reason quantitatively and use units to solve problems. Working with quantities and the relationships between them provides grounding for work with expressions, equations, and functions **(Standards N.Q.1–3).**

- **Standard N.Q.1** Use units as a way to understand problems and to guide the solution of multi-step problems; choose and interpret units consistently in formulas; choose and interpret the scale and the origin in graphs and data displays.

- **Standard N.Q.2** Define appropriate quantities for the purpose of descriptive modeling.

- **Standard N.Q.3** Choose a level of accuracy appropriate to limitations on measurement when reporting quantities.

Strand: ALGEBRA—Seeing Structure in Expressions (A.SSE)

Interpret the structure of expressions **(Standard A.SSE.1).**

- **Standard A.SSE.1** Interpret linear expressions and exponential expressions with integer exponents that represent a quantity in terms of its context.★

  a. Interpret parts of an expression, such as terms, factors, and coefficients.

  b. Interpret complicated expressions by viewing one or more of their parts as a single entity. *For example, interpret* $P(1+r)^n$ *as the product of* P *and a factor not depending on* P.

Strand: ALGEBRA—Creating Equations (A.CED)

Create equations that describe numbers or relationships. Limit these to linear equations and inequalities, and exponential equations. In the case of exponential equations, limit to situations requiring evaluation of exponential functions at integer inputs **(Standards A.CED.1–4).**

- **Standard A.CED.1** Create equations and inequalities in one variable and use them to solve problems. Include equations arising from linear and simple exponential functions.

- **Standard A.CED.2** Create equations in two or more variables to represent relationships between quantities; graph equations on coordinate axes with labels and scales.

- **Standard A.CED.3** Represent constraints by equations or inequalities and by systems of equations and/or inequalities, and interpret solutions as viable or non-viable options in a modeling context. *For example, represent inequalities describing nutritional and cost constraints on combinations of different foods.*

- **Standard A.CED.4** Rearrange formulas to highlight a quantity of interest, using the same reasoning as in solving equations. *For example, rearrange Ohm's Law* V = IR *to highlight resistance* R.

Strand: ALGEBRA—Reasoning With Equations and Inequalities (A.REI)

Understand solving equations as a process of reasoning and explain the reasoning **(Standard A.REI.1).** Solve equations and inequalities in one variable **(Standard A.REI.3).** Solve systems of equations. Build on student experiences graphing and solving systems of linear equations from middle school. Include cases where the two equations describe the same line—yielding infinitely many solutions—and cases where two equations describe parallel lines—yielding no solution; connect to GPE.5, which requires students to prove the slope criteria for parallel lines **(Standards A.REI.5–6).** Represent and solve equations and inequalities graphically **(Standards A.REI.10–12).**

- **Standard A.REI.1** Explain each step in solving a linear equation as following from the equality of numbers asserted at the previous step, starting from the assumption that the original equation has a solution. Construct a viable argument to justify a solution method. Students will solve exponential equations with logarithms in Secondary Mathematics III.

- **Standard A.REI.3** Solve equations and inequalities in one variable.

  a. Solve one-variable equations and literal equations to highlight a variable of interest.

  b. Solve compound inequalities in one variable, including absolute value inequalities.

  c. Solve simple exponential equations that rely only on application of the laws of exponents (limit solving exponential equations to those that can be solved without logarithms). *For example,* $5^x = 125$ *or* $2^x = 1/16$.

- **Standard A.REI.5** Prove that, given a system of two equations in two variables, replacing one equation by the sum of that equation and a multiple of the other produces a system with the same solutions.

- **Standard A.REI.6** Solve systems of linear equations exactly and approximately (numerically, algebraically, graphically), focusing on pairs of linear equations in two variables.

- **Standard A.REI.10** Understand that the graph of an equation in two variables is the set of all its solutions plotted in the coordinate plane, often forming a curve (which could be a line).

- **Standard A.REI.11** Explain why the *x*-coordinates of the points where the graphs of the equations $y = f(x)$ and $y = g(x)$ intersect are the solutions of the equation $f(x) = g(x)$; find the solutions approximately; e.g., using technology to graph the functions, make tables of values, or find successive approximations. Include cases where $f(x)$ and/or $g(x)$ are linear and exponential functions.★

- **Standard A.REI.12** Graph the solutions to a linear inequality in two variables as a half-plane (excluding the boundary in the case of a strict inequality), and graph the solution set to a system of linear inequalities in two variables as the intersection of the corresponding half-planes.

Strand: FUNCTIONS—Interpreting Linear and Exponential Functions (F.IF)

Understand the concept of a linear or exponential function and use function notation. Recognize arithmetic and geometric sequences as examples of linear and exponential functions **(Standards F.IF.1–3).** Interpret linear or exponential functions that arise in applications in terms of a context **(Standards F.IF.4–6).** Analyze linear or exponential functions using different representations **(Standards F.IF.7,9).**

- **Standard F.IF.1** Understand that a function from one set (called the domain) to another set (called the range) assigns to each element of the domain exactly one element of the range. If *f* is a function and *x* is an element of its domain, then $f(x)$ denotes the output of *f* corresponding to the input *x*. The graph of *f* is the graph of the equation $y = f(x)$.

- **Standard F.IF.2** Use function notation, evaluate functions for inputs in their domains, and interpret statements that use function notation in terms of a context.

- **Standard F.IF.3** Recognize that sequences are functions, sometimes defined recursively, whose domain is a subset of the integers. Emphasize arithmetic and geometric sequences as examples of linear and exponential functions. *For example, the Fibonacci sequence is defined recursively by* $f(0) = f(1) = 1$, $f(n+1) = f(n) + f(n-1)$ *for* $n \geq 1$.

- **Standard F.IF.4** For a function that models a relationship between two quantities, interpret key features of graphs and tables in terms of the quantities, and sketch graphs showing key features given a verbal description of the relationship. *Key features include intercepts; intervals where the function is increasing, decreasing, positive, or negative; relative maximums and minimums; symmetries; and end behavior.*★

- **Standard F.IF.5** Relate the domain of a function to its graph and, where applicable, to the quantitative relationship it describes. *For example, if the function* h(n) *gives the number of person-hours it takes to assemble n engines in a factory, then the positive integers would be an appropriate domain for the function.*★

- **Standard F.IF.6** Calculate and interpret the average rate of change of a function (presented symbolically or as a table) over a specified interval. Estimate the rate of change from a graph.★

- **Standard F.IF.7** Graph functions expressed symbolically and show key features of the graph, by hand in simple cases and using technology for more complicated cases.★

a. Graph linear functions and show intercepts.

e. Graph exponential functions, showing intercepts and end behavior.

■ **Standard F.IF.9** Compare properties of two functions, each represented in a different way (algebraically, graphically, numerically in tables, or by verbal descriptions). *For example, compare the growth of two linear functions, or two exponential functions such as* $y=3^n$ *and* $y=100\cdot 2^n$.

Strand: FUNCTIONS—Building Linear or Exponential Functions (F.BF)

Build a linear or exponential function that models a relationship between two quantities **(Standards F.BF.1–2).** Build new functions from existing functions (**Standard F.BF.3).**

■ **Standard F.BF.1** Write a function that describes a relationship between two quantities.★

a. Determine an explicit expression, a recursive process, or steps for calculation from a context.

b. Combine standard function types using arithmetic operations. *For example, build a function that models the temperature of a cooling body by adding a constant function to a decaying exponential, and relate these functions to the model.*

■ **Standard F.BF.2** Write arithmetic and geometric sequences both recursively and with an explicit formula, use them to model situations, and translate between the two forms. Limit to linear and exponential functions. Connect arithmetic sequences to linear functions and geometric sequences to exponential functions. ★

■ **Standard F.BF.3** Identify the effect on the graph of replacing $f(x)$ by $f(x) + k$, for specific values of k (both positive and negative); find the value of k given the graphs. Relate the vertical translation of a linear function to its y-intercept. Experiment with cases and illustrate an explanation of the effects on the graph using technology.

Strand: FUNCTIONS—Linear and Exponential (F.LE)

Construct and compare linear and exponential models and solve problems **(Standards F.LE.1–3).** Interpret expressions for functions in terms of the situation they model. **(Standard F.LE.5).**

■ **Standard F.LE.1** Distinguish between situations that can be modeled with linear functions and with exponential functions.

a. Prove that linear functions grow by equal differences over equal intervals; exponential functions grow by equal factors over equal intervals.

b. Recognize situations in which one quantity changes at a constant rate per unit interval relative to another.

c. Recognize situations in which a quantity grows or decays by a constant percent rate per unit interval relative to another.

- **Standard F.LE.2** Construct linear and exponential functions, including arithmetic and geometric sequences, given a graph, a description of a relationship, or two input-output pairs (include reading these from a table).
- **Standard F.LE.3** Observe using graphs and tables that a quantity increasing exponentially eventually exceeds a quantity increasing linearly.
- **Standard F.LE.5** Interpret the parameters in a linear or exponential function in terms of a context. Limit exponential functions to those of the form $f(x) = b^x + k$.

Strand: GEOMETRY—Congruence (G.CO)

Experiment with transformations in the plane. Build on student experience with rigid motions from earlier grades **(Standards G.CO.1–5).** Understand congruence in terms of rigid motions. Rigid motions are at the foundation of the definition of congruence. Reason from the basic properties of rigid motions (that they preserve distance and angle), which are assumed without proof. Rigid motions and their assumed properties can be used to establish the usual triangle congruence criteria, which can then be used to prove other theorems **(Standards G.CO.6–8).** Make geometric constructions **(Standards G.CO.12–13).**

- **Standard G.CO.1** Know precise definitions of angle, circle, perpendicular line, parallel line, and line segment, based on the undefined notions of point, line, distance along a line, and distance around a circular arc.
- **Standard G.CO.2** Represent transformations in the plane using, for example, transparencies and geometry software; describe transformations as functions that take points in the plane as inputs and give other points as outputs. Compare transformations that preserve distance and angle to those that do not (e.g., translation versus horizontal stretch).
- **Standard G.CO.3** Given a rectangle, parallelogram, trapezoid, or regular polygon, describe the rotations and reflections that carry it onto itself.
- **Standard G.CO.4** Develop definitions of rotations, reflections, and translations in terms of angles, circles, perpendicular lines, parallel lines, and line segments.
- **Standard G.CO.5** Given a geometric figure and a rotation, reflection, or translation, draw the transformed figure using, for example, graph paper, tracing paper, or geometry software. Specify a sequence of transformations that will carry a given figure onto another. Point out the basis of rigid motions in geometric concepts, for example, translations move points a specified distance along a line parallel to a specified line; rotations move objects along a circular arc with a specified center through a specified angle.
- **Standard G.CO.6** Use geometric descriptions of rigid motions to transform figures and to predict the effect of a given rigid motion on a given figure; given two figures, use

the definition of congruence in terms of rigid motions to decide whether they are congruent.

- **Standard G.CO.7** Use the definition of congruence in terms of rigid motions to show that two triangles are congruent if and only if corresponding pairs of sides and corresponding pairs of angles are congruent.

- **Standard G.CO.8** Explain how the criteria for triangle congruence (ASA, SAS, and SSS) follow from the definition of congruence in terms of rigid motions.

- **Standard G.CO.12** Make formal geometric constructions with a variety of tools and methods (compass and straightedge, string, reflective devices, paper folding, dynamic geometric software, etc.). Emphasize the ability to formalize and defend how these constructions result in the desired objects. *For example, copying a segment; copying an angle; bisecting a segment; bisecting an angle; constructing perpendicular lines, including the perpendicular bisector of a line segment; and constructing a line parallel to a given line through a point not on the line.*

- **Standard G.CO.13** Construct an equilateral triangle, a square, and a regular hexagon inscribed in a circle. Emphasize the ability to formalize and defend how these constructions result in the desired objects.

Strand: GEOMETRY—Expressing Geometric Properties With Equations (G.GPE)

Use coordinates to prove simple geometric theorems algebraically **(Standards G.GPE.4–5, 7).**

- **Standard G.GPE.4** Use coordinates to prove simple geometric theorems algebraically. *For example, prove or disprove that a figure defined by four given points in the coordinate plane is a rectangle; prove or disprove that the point $(1, \sqrt{3})$ lies on the circle centered at the origin and containing the point $(0, 2)$.*

- **Standard G.GPE.5** Prove the slope criteria for parallel and perpendicular lines; use them to solve geometric problems (e.g., find the equation of a line parallel or perpendicular to a given line that passes through a given point).

- **Standard G.GPE.7** Use coordinates to compute perimeters of polygons and areas of triangles and rectangles; connect with The Pythagorean Theorem and the distance formula.★

**Strand: STATISTICS AND PROBABILITY—
Interpreting Categorical and Quantitative Data (S.ID)**

Summarize, represent, and interpret data on a single count or measurement variable **(Standards S.ID.1–3).** Summarize, represent, and interpret data on two categorical and quantitative variables **(Standard S.ID.6).** Interpret linear models building on students' work with linear relationships, and introduce the correlation coefficient **(Standards S.ID.7–9).**

- **Standard S.ID.1** Represent data with plots on the real number line (dot plots, histograms, and box plots).
- **Standard S.ID.2** Use statistics appropriate to the shape of the data distribution to compare center (median, mean) and spread (interquartile range, standard deviation) of two or more different data sets.
- **Standard S.ID.3** Interpret differences in shape, center, and spread in the context of the data sets, accounting for possible effects of extreme data points (outliers). Calculate the weighted average of a distribution and interpret it as a measure of center.
- **Standard S.ID.6** Represent data on two quantitative variables on a scatter plot, and describe how the variables are related.
  - **a.** Fit a linear function to the data; use functions fitted to data to solve problems in the context of the data. Use given functions, or choose a function suggested by the context. Emphasize linear and exponential models.
  - **b.** Informally assess the fit of a function by plotting and analyzing residuals. Focus on situations for which linear models are appropriate.
  - **c.** Fit a linear function for scatter plots that suggest a linear association.
- **Standard S.ID.7** Interpret the slope (rate of change) and the intercept (constant term) of a linear model in the context of the data.
- **Standard S.ID.8** Compute (using technology) and interpret the correlation coefficient of a linear fit.
- **Standard S.ID.9** Distinguish between correlation and causation.

SECONDARY MATHEMATICS I—HONORS STANDARDS

Strand: NUMBER AND QUANTITY: VECTOR AND MATRIX QUANTITIES (N.VM)

Represent and model with vector quantities **(Standards N.VM.1–3).** Perform operations on vectors **(Standards N.VM.4–5).** Perform operations on matrices and use matrices in applications **(Standards N.VM.6–13).**

- **Standard N.VM.1** Recognize vector quantities as having both magnitude and direction. Represent vector quantities by directed line segments, and use appropriate symbols for vectors and their magnitudes (e.g., v, $|v|$, $||v||$, v).

- **Standard N.VM.2** Find the components of a vector by subtracting the coordinates of an initial point from the coordinates of a terminal point.

- **Standard N.VM.3** Solve problems involving velocity and other quantities that can be represented by vectors.

- **Standard N.VM.4** Add and subtract vectors.

  a. Add vectors end-to-end, component-wise, and by the parallelogram rule. Understand that the magnitude of a sum of two vectors is typically not the sum of the magnitudes.

  b. Given two vectors in magnitude and direction form, determine the magnitude and direction of their sum.

  c. Understand vector subtraction $v – w$ as $v + (–w)$, where $–w$ is the additive inverse of w, with the same magnitude as w and pointing in the opposite direction. Represent vector subtraction graphically by connecting the tips in the appropriate order, and perform vector subtraction component-wise.

- **Standard N.VM.5** Multiply a vector by a scalar.

  a. Represent scalar multiplication graphically by scaling vectors and possibly reversing their direction; perform scalar multiplication component-wise, e.g., as $c(v_x, v_y) = (cv_x, cv_y)$.

  b. Compute the magnitude of a scalar multiple cv using $||cv|| = |c|v$. Compute the direction of cv knowing that when $|c|v \neq 0$, the direction of cv is either along v (for $c > 0$) or against vs (for $c < 0$).

- **Standard N.VM.6** Use matrices to represent and manipulate data, e.g., to represent payoffs or incidence relationships in a network.

- **Standard N.VM.7** Multiply matrices by scalars to produce new matrices, e.g., as when all of the pay-offs in a game are doubled.

- **Standard N.VM.8** Add, subtract, and multiply matrices of appropriate dimensions.
- **Standard N.VM.9** Understand that, unlike multiplication of numbers, matrix multiplication for square matrices is not a commutative operation, but still satisfies the associative and distributive properties.
- **Standard N.VM.10** Understand that the zero and identity matrices play a role in matrix addition and multiplication similar to the role of 0 and 1 in the real numbers. The determinant of a square matrix is nonzero if and only if the matrix has a multiplicative inverse.
- **Standard N.VM.11** Multiply a vector (regarded as a matrix with one column) by a matrix of suitable dimensions to produce another vector. Work with matrices as transformations of vectors.
- **Standard N.VM.12** Work with 2×2 matrices as transformations of the plane, and interpret the absolute value of the determinant in terms of area.
- **Standard N.VM.13** Solve systems of linear equations up to three variables using matrix row reduction.

SECONDARY MATHEMATICS II

THE FOCUS OF SECONDARY MATHEMATICS II is on quadratic expressions, equations, and functions and on comparing their characteristics and behavior to those of linear and exponential relationships from Secondary Mathematics I as organized into six critical areas, or units. The need for extending the set of rational numbers arises, and real and complex numbers are introduced so that all quadratic equations can be solved. The link between probability and data is explored through conditional probability and counting methods, including their use in making and evaluating decisions. The study of similarity leads to an understanding of right triangle trigonometry and connects to quadratics through Pythagorean relationships. Circles, with their quadratic algebraic representations, round out the course. The Mathematical Practice Standards apply throughout each course and, together with the content standards, prescribe that students experience mathematics as a coherent, useful, and logical subject that makes use of their ability to make sense of problem situations.

CRITICAL AREA 1: Students extend the laws of exponents to rational exponents and explore distinctions between rational and irrational numbers by considering their decimal representations. Students learn that when quadratic equations do not have real solutions the number system must be extended so that solutions exist, analogous to the way in which extending the whole numbers to the negative numbers allows $x+1 = 0$ to have a solution. Students explore relationships between number systems: whole numbers, integers, rational numbers, real numbers, and complex numbers. The guiding principle is that equations with no solutions in one number system may have solutions in a larger number system.

CRITICAL AREA 2: Students consider quadratic functions, comparing the key characteristics of quadratic functions to those of linear and exponential functions. They select from among these functions to model phenomena. Students learn to anticipate the graph of a quadratic function by interpreting various forms of quadratic expressions. In particular, they identify the real solutions of a quadratic equation as the zeros of a related quadratic function. When quadratic equations do not have real solutions, students learn that the graph of the related quadratic function does not cross the horizontal axis. They expand their experience with functions to include more specialized functions—absolute value, step, and those that are piecewise-defined.

CRITICAL AREA 3: Students begin this unit by focusing on the structure of expressions, rewriting expressions to clarify and reveal aspects of the relationship they represent. They create and solve equations, inequalities, and systems of equations involving exponential and quadratic expressions.

CRITICAL AREA 4: Building on probability concepts that began in the middle grades, students use the languages of set theory to expand their ability to compute and interpret theoretical and experimental probabilities for compound events, attending to mutually exclusive events, independent events, and conditional probability. Students should make use of geometric probability models wherever possible. They use probability to make informed decisions.

CRITICAL AREA 5: Students apply their earlier experience with dilations and proportional reasoning to build a formal understanding of similarity. They identify criteria for similarity of triangles, use similarity to solve problems, and apply similarity in right triangles to understand right triangle trigonometry, with particular attention to special right triangles and the Pythagorean Theorem. It is in this unit that students develop facility with geometric proof. They use what they know about congruence and similarity to prove theorems involving lines, angles, triangles, and other polygons. They explore a variety of formats for writing proofs.

CRITICAL AREA 6: Students prove basic theorems about circles, such as a tangent line is perpendicular to a radius, inscribed angle theorem, and theorems about chords, secants, and tangents dealing with segment lengths and angle measures. In the Cartesian coordinate system, students use the distance formula to write the equation of a circle when given the radius and the coordinates of its center, and the equation of a parabola with vertical axis when given an equation of its directrix and the coordinates of its focus. Given an equation of a circle, they draw the graph in the coordinate plane, and apply techniques for solving quadratic equations to determine intersections between lines and circles or a parabola and between two circles. Students develop informal arguments justifying common formulas for circumference, area, and volume of geometric objects, especially those related to circles.

Strand: MATHEMATICAL PRACTICES (MP)

The Standards for Mathematical Practice in Secondary Mathematics II describe mathematical habits of mind that teachers should seek to develop in their students. Students become mathematically proficient in engaging with mathematical content and concepts as they learn, experience, and apply these skills and attitudes **(Standards MP.1–8).**

- **Standard SII.MP.1 Make sense of problems and persevere in solving them.** Explain the meaning of a problem and look for entry points to its solution. Analyze givens, constraints, relationships, and goals. Make conjectures about the form and meaning of the solution, plan a solution pathway, and continually monitor progress asking, "Does this make sense?" Consider analogous problems, make connections between multiple representations, identify the correspondence between different approaches, look for trends, and transform algebraic expressions to highlight meaningful mathematics. Check answers to problems using a different method.
- **Standard SII.MP.2 Reason abstractly and quantitatively.** Make sense of the quantities and their relationships in problem situations. Translate between context and algebraic representations by contextualizing and decontextualizing quantitative relationships. This includes the ability to decontextualize a given situation, representing it algebraically and manipulating symbols fluently as well as the ability to contextualize algebraic representations to make sense of the problem.
- **Standard SII.MP.3 Construct viable arguments and critique the reasoning of others.** Understand and use stated assumptions, definitions, and previously established results in constructing arguments. Make conjectures and build a logical progression of statements to explore the truth of their conjectures. Justify conclusions and communicate them to others. Respond to the arguments of others by listening, asking clarifying questions, and critiquing the reasoning of others.
- **Standard SII.MP.4 Model with mathematics.** Apply mathematics to solve problems arising in everyday life, society, and the workplace. Make assumptions and approximations, identifying important quantities to construct a mathematical model. Routinely interpret mathematical results in the context of the situation and reflect on whether the results make sense, possibly improving the model if it has not served its purpose.
- **Standard SII.MP.5 Use appropriate tools strategically.** Consider the available tools and be sufficiently familiar with them to make sound decisions about when each tool might be helpful, recognizing both the insight to be gained as well as the limitations. Identify relevant external mathematical resources and use them to pose or solve problems. Use tools to explore and deepen their understanding of concepts.
- **Standard SII.MP.6 Attend to precision.** Communicate precisely to others. Use explicit definitions in discussion with others and in their own reasoning. They state the meaning of the symbols they choose. Specify units of measure and label axes to clarify the correspondence with quantities in a problem. Calculate accurately and efficiently, express numerical answers with a degree of precision appropriate for the problem context.

- **Standard SII.MP.7 Look for and make use of structure.** Look closely at mathematical relationships to identify the underlying structure by recognizing a simple structure within a more complicated structure. See complicated things, such as some algebraic expressions, as single objects or as being composed of several objects. *For example, see* $5 - 3(x - y)^2$ *as 5 minus a positive number times a square and use that to realize that its value cannot be more than 5 for any real numbers* x *and* y.

- **Standard SII.MP.8 Look for and express regularity in repeated reasoning.** Notice if reasoning is repeated, and look for both generalizations and shortcuts. Evaluate the reasonableness of intermediate results by maintaining oversight of the process while attending to the details.

Strand: NUMBER AND QUANTITY—The Real Number System (N.RN)

Extend the properties of exponents to rational exponents **(Standards N.RN.1–2).** Use properties of rational and irrational numbers **(Standard N.RN. 3).**

- **Standard N.RN.1** Explain how the definition of the meaning of rational exponents follows from extending the properties of integer exponents to those values, allowing for a notation for radicals in terms of rational exponents. *For example, we define* $5^{1/3}$ *to be the cube root of 5 because we want* $(5^{1/3})^3 = 5^{(1/3)3}$ *to hold, so* $(5^{1/3})^3$ *must equal 5.*

- **Standard N.RN.2** Rewrite expressions involving radicals and rational exponents using the properties of exponents.

- **Standard N.RN.3** Explain why sums and products of rational numbers are rational, that the sum of a rational number and an irrational number is irrational, and that the product of a nonzero rational number and an irrational number is irrational. Connect to physical situations (e.g., finding the perimeter of a square of area 2).

Strand: NUMBER AND QUANTITY—The Complex Number System (N.CN)

Perform arithmetic operations with complex numbers **(Standards N.CN.1–2).** Use complex numbers in polynomial identities and equations **(Standards N.CN.7–9).**

- **Standard N.CN.1** Know there is a complex number i such that $i^2 = -1$, and every complex number has the form $a + bi$ with a and b real.

- **Standard N.CN.2** Use the relation $i^2 = -1$ and the commutative, associative, and distributive properties to add, subtract, and multiply complex numbers. Limit to multiplications that involve i^2 as the highest power of i.

- **Standard N.CN.7** Solve quadratic equations with real coefficients that have complex solutions.

- **Standard N.CN.8** Extend polynomial identities to the complex numbers. Limit to quadratics with real coefficients. *For example, rewrite* $x^2 + 4$ *as* $(x + 2i)(x - 2i)$.

■ **Standard N.CN.9** Know the Fundamental Theorem of Algebra; show that it is true for quadratic polynomials.

Strand: ALGEBRA—Seeing Structure in Expression (A.SSE)

Interpret the structure of expressions **(Standards A.SSE.1–2).** Write expressions in equivalent forms to solve problems, balancing conceptual understanding and procedural fluency in work with equivalent expressions **(Standard A.SSE.3).**

■ **Standard A.SSE.1** Interpret quadratic and exponential expressions that represent a quantity in terms of its context.★

a. Interpret parts of an expression, such as terms, factors, and coefficients.

b. Interpret increasingly more complex expressions by viewing one or more of their parts as a single entity. Exponents are extended from the integer exponents to rational exponents focusing on those that represent square or cube roots.

■ **Standard A.SSE.2** Use the structure of an expression to identify ways to rewrite it. For example, see $x^4 - y^4$ as $(x^2)^2 - (y^2)^2$, thus recognizing it as a difference of squares that can be factored as $(x^2 - y^2)(x^2 + y^2)$.

■ **Standard A.SSE.3** Choose and produce an equivalent form of an expression to reveal and explain properties of the quantity represented by the expression. *For example, development of skill in factoring and completing the square goes hand in hand with understanding what different forms of a quadratic expression reveal.* ★

a. Factor a quadratic expression to reveal the zeros of the function it defines.

b. Complete the square in a quadratic expression to reveal the maximum or minimum value of the function it defines.

c. Use the properties of exponents to transform expressions for exponential functions. *For example, the expression* 1.15^t *can be rewritten as* $(1.15^{1/12})^{12t} \approx 1.012^{12t}$ *to reveal the approximate equivalent monthly interest rate if the annual rate is 15%.*

Strand: ALGEBRA—Arithmetic With Polynomials and Rational Expressions (A.APR)

Perform arithmetic operations on polynomials. Focus on polynomial expressions that simplify to forms that are linear or quadratic in a positive integer power of *x* **(Standard A.APR.1).**

■ **Standard A.APR.1** Understand that polynomials form a system analogous to the integers, namely, they are closed under the operations of addition, subtraction, and multiplication; add, subtract, and multiply polynomials.

Strand: ALGEBRA—Creating Equations (A.CED)

Create equations that describe numbers or relationships. Extend work on linear and exponential equations to quadratic equations **(Standards A.CED.1–2, 4).**

- **Standard A.CED.1** Create equations and inequalities in one variable and use them to solve problems. Include equations arising from linear and quadratic functions, and simple rational and exponential functions.

- **Standard A.CED.2** Create equations in two or more variables to represent relationships between quantities; graph equations on coordinate axes with labels and scales.

- **Standard A.CED.4** Rearrange formulas to highlight a quantity of interest, using the same reasoning as in solving equations; extend to formulas involving squared variables. *For example, rearrange the formula for the volume of a cylinder* $V = \pi r^2 h$.

Strand: ALGEBRA—Reasoning With Equations and Inequalities (A.REI)

Solve equations and inequalities in one variable **(Standard A.REI.4).** Solve systems of equations. Extend the work of systems to include solving systems consisting of one linear and one nonlinear equation **(Standard A.REI.7).**

- **Standard A.REI.4** Solve quadratic equations in one variable.

  a. Use the method of completing the square to transform any quadratic equation in x into an equation of the form $(x - p)^2 = q$ that has the same solutions. Derive the quadratic formula from this form.

  b. Solve quadratic equations by inspection (e.g., for $x^2 = 49$), taking square roots, completing the square, the quadratic formula and factoring, as appropriate to the initial form of the equation. Recognize when the quadratic formula gives complex solutions and write them as $a \pm bi$ for real numbers a and b.

- **Standard A.REI.7** Solve a simple system consisting of a linear equation and a quadratic equation in two variables algebraically and graphically. *For example, find the points of intersection between the line* $y = -3x$ *and the circle* $x^2 + y^2 = 3$.

Strand: FUNCTIONS—Interpret Functions (F.IF)

Interpret quadratic functions that arise in applications in terms of a context **(Standards F.IF.4–6).** Analyze functions using different representations **(Standards F.IF.7–9).**

- **Standard F.IF.4** For a function that models a relationship between two quantities, interpret key features of graphs and tables in terms of the quantities, and sketch graphs showing key features given a verbal description of the relationship. *Key features include intercepts; intervals where the function is increasing, decreasing, positive, or negative; relative maximums and minimums; symmetries; and end behavior.*★

- **Standard F.IF.5** Relate the domain of a function to its graph and, where applicable, to the quantitative relationship it describes. Focus on quadratic functions; compare with linear and exponential functions. *For example, if the function* h(n) *gives the number of person-hours it takes to assemble n engines in a factory, then the positive integers would be an appropriate domain for the function.*★

- **Standard F.IF.6** Calculate and interpret the average rate of change of a function (presented symbolically or as a table) over a specified interval. Estimate the rate of change from a graph.★

- **Standard F.IF.7** Graph functions expressed symbolically and show key features of the graph, by hand in simple cases and using technology for more complicated cases.★

  a. Graph linear and quadratic functions and show intercepts, maxima, and minima.

  b. Graph piecewise-defined functions and absolute value functions. Compare and contrast absolute value and piecewise-defined functions with linear, quadratic, and exponential functions. Highlight issues of domain, range, and usefulness when examining piecewise-defined functions.

- **Standard F.IF.8** Write a function defined by an expression in different but equivalent forms to reveal and explain different properties of the function.

  a. Use the process of factoring and completing the square in a quadratic function to show zeros, extreme values, and symmetry of the graph, and interpret these in terms of a context.

  b. Use the properties of exponents to interpret expressions for exponential functions. *For example, identify percent rate of change in functions such as* $y = (1.02)^{t}$, $y = (0.97)^{t}$, $y = (1.01)^{12t}$, $y = (1.2)^{t/10}$, *and classify them as representing exponential growth or decay.*

- **Standard F.IF.9** Compare properties of two functions, each represented in a different way (algebraically, graphically, numerically in tables, or by verbal descriptions). Extend work with quadratics to include the relationship between coefficients and roots, and that once roots are known, a quadratic equation can be factored. *For example, given a graph of one quadratic function and an algebraic expression for another, say which has the larger maximum.*

Strand: FUNCTIONS—Building Functions (F.BF)

Build a function that models a relationship between two quantities **(Standard F.BF.1).** Build new functions from existing functions **(Standard F.BF.3).**

- **Standard F.BF.1** Write a quadratic or exponential function that describes a relationship between two quantities.★

  a. Determine an explicit expression, a recursive process, or steps for calculation from a context.

b. Combine standard function types using arithmetic operations. *For example, build a function that models the temperature of a cooling body by adding a constant function to a decaying exponential, and relate these functions to the model.*

■ **Standard F.BF.3** Identify the effect on the graph of replacing *f(x) by f(x) + k, k f(x), f(kx), and f(x + k)* for specific values of *k* (both positive and negative); find the value of *k* given the graphs. Focus on quadratic functions and consider including absolute value functions. Experiment with cases and illustrate an explanation of the effects on the graph using technology. Include recognizing even and odd functions from their graphs and algebraic expressions for them.

Strand: FUNCTIONS—Linear, Quadratic, and Exponential Models (F.LE)

Construct and compare linear, quadratic, and exponential models and solve problems **(Standard F.LE.3).**

■ **Standard F.LE.3** Observe using graphs and tables that a quantity increasing exponentially eventually exceeds a quantity increasing linearly, quadratically, or (more generally) as a polynomial function. Compare linear and exponential growth to quadratic growth.

Strand: FUNCTIONS—Trigonometric Functions (F.TF)

Prove and apply trigonometric identities. Limit θ to angles between 0 and 90 degrees. Connect with the Pythagorean Theorem and the distance formula **(Standard F.TF.8).**

■ **Standard F.TF.8** Prove the Pythagorean identity $\sin^2(\theta) + \cos^2(\theta) = 1$ and use it to find sin (θ), cos (θ), or tan (θ), given sin (θ), cos (θ), or tan (θ), and the quadrant of the angle.

Strand: GEOMETRY—Congruence (G.CO)

Prove geometric theorems. Encourage multiple ways of writing proofs, such as narrative paragraphs, flow diagrams, two-column format, and diagrams without words. Focus on the validity of the underlying reasoning while exploring a variety of formats for expressing that reasoning **(Standards G.CO.9–11).**

■ **Standard G.CO.9** Prove theorems about lines and angles. *Theorems include: vertical angles are congruent; when a transversal crosses parallel lines, alternate interior angles are congruent and corresponding angles are congruent; points on a perpendicular bisector of a line segment are exactly those equidistant from the segment's endpoints.*

■ **Standard G.CO.10** Prove theorems about triangles. *Theorems include: measures of interior angles of a triangle sum to 180°; base angles of isosceles triangles are congruent; the segment joining midpoints of two sides of a triangle is parallel to the third side and half the length; the medians of a triangle meet at a point.*

■ **Standard G.CO.11** Prove theorems about parallelograms. *Theorems include: opposite sides are congruent, opposite angles are congruent, the diagonals of a parallelogram bisect each other, and conversely, rectangles are parallelograms with congruent diagonals.*

Strand: GEOMETRY—Similarity, Right Triangles, and Trigonometry (G.SRT)

Understand similarity in terms of similarity transformations **(Standards G.SRT.1–3).** Prove theorems involving similarity **(Standards G.SRT.4–5).** Define trigonometric ratios and solve problems involving right triangles **(Standards G.SRT.6–8).**

■ **Standard G.SRT.1** Verify experimentally the properties of dilations given by a center and a scale factor.

- **a.** A dilation takes a line not passing through the center of the dilation to a parallel line, and leaves a line passing through the center unchanged.
- **b.** The dilation of a line segment is longer or shorter in the ratio given by the scale factor.

■ **Standard G.SRT.2** Given two figures, use the definition of similarity in terms of similarity transformations to decide whether they are similar; explain using similarity transformations the meaning of similarity for triangles as the equality of all corresponding pairs of angles and the proportionality of all corresponding pairs of sides.

■ **Standard G.SRT.3** Use the properties of similarity transformations to establish the AA criterion for two triangles to be similar.

■ **Standard G.SRT.4** Prove theorems about triangles. *Theorems include: a line parallel to one side of a triangle divides the other two proportionally and conversely; the Pythagorean Theorem (proved using triangle similarity).*

■ **Standard G.SRT.5** Use congruence and similarity criteria for triangles to solve problems and to prove relationships in geometric figures.

■ **Standard G.SRT.6** Understand that by similarity, side ratios in right triangles are properties of the angles in the triangle, leading to definitions of trigonometric ratios for acute angles.

■ **Standard G.SRT.7** Explain and use the relationship between the sine and cosine of complementary angles.

■ **Standard G.SRT.8** Use trigonometric ratios and the Pythagorean Theorem to solve right triangles in applied problems.

Strand: GEOMETRY—Circles (G.C)

Understand and apply theorems about circles **(Standard G.C.1–4).** Find arc lengths and areas of sectors of circles. Use this as a basis for introducing the radian as a unit of measure. It is not intended that it be applied to the development of circular trigonometry in this course **(Standard G.C.5).**

■ **Standard G.C.1** Prove that all circles are similar.

- **Standard G.C.2** Identify and describe relationships among inscribed angles, radii, and chords. *Relationships include the relationship between central, inscribed, and circumscribed angles; inscribed angles on a diameter are right angles; the radius of a circle is perpendicular to the tangent where the radius intersects the circle.*
- **Standard G.C.3** Construct the inscribed and circumscribed circles of a triangle, and prove properties of angles for a quadrilateral inscribed in a circle.
- **Standard G.C.4** Construct a tangent line from a point outside a given circle to the circle.
- **Standard G.C.5** Derive, using similarity, the fact that the length of the arc intercepted by an angle is proportional to the radius, and define the radian measure of the angle as the constant of proportionality; derive the formula for the area of a sector.

Strand: GEOMETRY—Expressing Geometric Properties With Equations (G.GPE)

Translate between the geometric description and the equation for a conic section **(Standard G.GPE.1).** Use coordinates to prove simple geometric theorems algebraically. Include simple proofs involving circles (**Standard G.GPE.4**). Use coordinates to prove simple geometric theorems algebraically **(Standard G.GPE.6).**

- **Standard G.GPE.1** Derive the equation of a circle of given center and radius using the Pythagorean Theorem; complete the square to find the center and radius of a circle given by an equation.
- **Standard G.GPE.4** Use coordinates to prove simple geometric theorems algebraically. *For example, prove or disprove that a figure defined by four given points in the coordinate plane is a rectangle; prove or disprove that the point (1, $\sqrt{3}$) lies on the circle centered at the origin and containing the point (0, 2).*
- **Standard G.GPE.6** Find the point on a directed line segment between two given points that partitions the segment in a given ratio.

Strand: GEOMETRY—Geometric Measurement and Dimension (G.GMD)

Explain volume formulas and use them to solve problems **(Standards G.GMD.1, 3).**

- **Standard G.GMD.1** Give an informal argument for the formulas for the circumference of a circle, area of a circle, volume of a cylinder, pyramid, and cone. Informal arguments for area formulas can make use of the way in which area scale under similarity transformations: when one figure in the plane results from another by applying a similarity transformation with scale factor k, its area is k^2 times the area of the first. *Use dissection arguments, Cavalieri's principle, and informal limit arguments.*
- **Standard G.GMD.3** Use volume formulas for cylinders, pyramids, cones, and spheres to solve problems. Informal arguments for volume formulas can make use of the way in which volume scale under similarity transformations: when one figure results from

another by applying a similarity transformation, volumes of solid figures scale by k^3 under a similarity transformation with scale factor k. ★

Strand: STATISTICS—Interpreting Categorical and Quantitative Data (S.ID)

Summarize, represent, and interpret data on two categorical or quantitative variables **(Standard S.ID.5).**

- **Standard S.ID.5** Summarize categorical data for two categories in two-way frequency tables. Interpret relative frequencies in the context of the data (including joint, marginal, and condition reltive frequencies). Recognize possible associations and trends in the date.

Strand: STATISTICS—Conditional Probability and the Rules of Probability (S.CP)

Understand independence and conditional probability and use them to interpret data **(Standards S.CP.1, 4–5).** Use the rules of probability to compute probabilities of compound events in a uniform probability model **(Standard S.CP.6).**

- **Standard S.CP.1** Describe events as subsets of a sample space (the set of outcomes) using characteristics (or categories) of the outcomes, or as unions, intersections, or complements of other events ("or," "and," "not").
- **Standard S.CP.4** Construct and interpret two-way frequency tables of data when two categories are associated with each object being classified. Use the two-way table as a sample space to decide if events are independent and to approximate conditional probabilities. *For example, collect data from a random sample of students in your school on their favorite subject among math, science, and English. Estimate the probability that a randomly selected student from your school will favor science given that the student is in tenth grade. Do the same for other subjects and compare the results.*
- **Standard S.CP.5** Recognize and explain the concepts of conditional probability and independence in everyday language and everyday situations. *For example, compare the chance of having lung cancer if you are are a smoker with the chance of being a smoker if you have lung cancer.*
- **Standard S.CP.6** Find the conditional probability of A given B as the fraction of B's outcomes that also belong to A, and interpret the answer in terms of the model.

SECONDARY MATHEMATICS II—HONORS STANDARDS

Strand: NUMBER AND QUANTITY—Complex Number System (N.CN)

Perform arithmetic operations with complex numbers **(Standard N.CN.3).** Represent complex numbers and their operations on the complex plane **(Standards N.CN.4–5).**

- **Standard N.CN.3** Find the conjugate of a complex number; use conjugates to find moduli and quotients of complex numbers.
- **Standard N.CN.4** Represent complex numbers on the complex plane in rectangular form, and explain why the rectangular form of a given complex number represents the same number.
- **Standard N.CN.5** Represent addition, subtraction, and multiplication geometrically on the complex plane; use properties of this representation for computation. *For example,* $(-1 + \sqrt{3}\,i)3 = 8$ *because* $(-1 + \sqrt{3}\,i)$ *has modulus 2 and argument 120°.*

Strand: ALGEBRA—Reasoning With Equations and Inequalities (A.REI)

Solve systems of equations **(Standards A.REI.8–9).**

- **Standard A.REI.8** Represent a system of linear equations as a single-matrix equation in a vector variable.
- **Standard A.REI.9** Find the inverse of a matrix if it exists, and use it to solve systems of linear equations (using technology for matrices of dimension 3 x 3 or greater).

Strand: FUNCTIONS—Interpreting Functions (F.IF)

Analyze functions using different representations **(Standards F.IF.10–11).**

- **Standard F.IF.10** Use sigma notation to represent the sum of a finite arithmetic or geometric series.
- **Standard F.IF.11** Represent series algebraically, graphically, and numerically.

Strand: GEOMETRY—Expressing Geometric Properties With Equations (G-GPE)

Translate between the geometric description and the equation for a conic section **(Standards G.GPE.2–3).**

- **Standard G.GPE.2** Derive the equation of a parabola given a focus and directrix.
- **Standard G.GPE.3** Derive the equations of ellipses and hyperbolas given the foci, using the fact that the sum or difference of distances from the foci is constant.

Strand: STATISTICS AND PROBABILITY—Conditional Probability and the Rules of Probability (S.CP)

Understand independence and conditional probability and use them to interpret data **(Standards S.CP.2–3).** Use the rules of probability to compute probabilities of compound events in a uniform probability model **(Standards S.CP.7–8).**

- **Standard S.CP.2** Understand that two events A and B are independent if the probability of A and B occurring together is the product of their probabilities, and use this characterization to determine if they are independent.
- **Standard S.CP.3** Understand the conditional probability of *A* given *B* as *P(A and B)/P(B)*, and interpret independence of A and *B* as saying that the conditional probability of *B* given *A* is the same as the probability of *B*.
- **Standard S.CP.7** Apply the Addition Rule, $P(A \text{ or } B) = P(A) + P(B) - P(A \text{ and } B)$, and interpret the answer in terms of the model.
- **Standard S.CP.8** Apply the general Multiplication Rule in a uniform probability model, $P(A \text{ and } B) = P(A)P(B|A) = P(B)P(A|B)$, and interpret the answer in terms of the model.

SECONDARY MATHEMATICS III

IN SECONDARY MATHEMATICS III students pull together and apply the accumulation of learning that they have from their previous courses, with content grouped into four critical areas, organized into units. They apply methods from probability and statistics to draw inferences and conclusions from data. Students expand their repertoire of functions to include polynomial, rational, and radical functions. They expand their study of right triangle trigonometry to include general triangles. And, finally, students bring together all of their experience with functions and geometry to create models and solve contextual problems. The Mathematical Practice Standards apply throughout each course and, together with the content standards, prescribe that students experience mathematics as a coherent, useful, and logical subject that makes use of their ability to make sense of problem situations.

CRITICAL AREA 1: Students see how the visual displays and summary statistics they learned in earlier grades relate to different types of data and to probability distributions. They identify different ways of collecting data—including sample surveys, experiments, and simulations—and the role that randomness and careful design play in the conclusions that can be drawn.

CRITICAL AREA 2: This area develops the structural similarities between the system of polynomials and the system of integers. Students draw on analogies between polynomial arithmetic and base-ten computation, focusing on properties of operations, particularly the distributive property. Students connect multiplication of polynomials with multiplication of multi-digit integers, and division of polynomials with long division of integers. Students identify zeros of polynomials and make connections between zeros of polynomials and solutions of polynomial equations. The unit culminates with the fundamental theorem of algebra. Rational numbers extend the arithmetic of integers by allowing division by all numbers except 0. Similarly, rational expressions extend the arithmetic of polynomials by allowing division by all polynomials except the zero polynomial. A central theme of this unit is that the arithmetic of rational expressions is governed by the same rules as the arithmetic of rational numbers.

CRITICAL AREA 3: Students develop the Laws of Sines and Cosines in order to find missing measures of general (not necessarily right) triangles. They are able to distinguish whether three given measures (angles or sides) define 0, 1, 2, or infinitely many triangles. This discussion of general triangles opens up the idea of trigonometry applied beyond the right triangle—that is, at least to obtuse angles. Students build on this idea to develop the notion of radian measure for angles and extend the domain of the trigonometric functions to all real numbers. They apply this knowledge to model simple periodic phenomena.

CRITICAL AREA 4: Students synthesize and generalize what they have learned about a variety of function families. They extend their work with exponential functions to include solving exponential equations with logarithms. They explore the effects of transformations on graphs of diverse functions, including functions arising in an application, in order to abstract the general principle that transformations on a graph always have the same effect regardless of the type of the underlying functions. They identify appropriate types of functions to model a situation, adjust parameters to improve the model, and compare models by analyzing the appropriateness of the fit and making judgments about the domain over which a model is a good fit. The description of modeling as "the process of choosing and using mathematics and statistics to analyze empirical situations, to understand them better, and to make decisions" is at the heart of this area. The narrative discussion and diagram of the modeling cycle should be considered when knowledge of functions, statistics, and geometry is applied in a modeling context.

Strand: MATHEMATICAL PRACTICES (MP)

The Standards for Mathematical Practice in Secondary Mathematics III describe mathematical habits of mind that teachers should seek to develop in their students. Students become mathematically proficient in engaging with mathematical content and concepts as they learn, experience, and apply these skills and attitudes **(Standards MP.1–8).**

- **Standard SIII.MP.1 Make sense of problems and persevere in solving them.** Explain the meaning of a problem and look for entry points to its solution. Analyze givens, constraints, relationships, and goals. Make conjectures about the form and meaning of the solution, plan a solution pathway, and continually monitor progress asking, "Does this make sense?" Consider analogous problems, make connections between multiple representations, identify the correspondence between different approaches, look for trends, and transform algebraic expressions to highlight meaningful mathematics. Check answers to problems using a different method.

- **Standard SIII.MP.2 Reason abstractly and quantitatively.** Make sense of the quantities and their relationships in problem situations. Translate between context and algebraic representations by contextualizing and decontextualizing quantitative relationships. This includes the ability to decontextualize a given situation, representing it algebraically and manipulating symbols fluently as well as the ability to contextualize algebraic representations to make sense of the problem.

- **Standard SIII.MP.3 Construct viable arguments and critique the reasoning of others.** Understand and use stated assumptions, definitions, and previously established results in constructing arguments. Make conjectures and build a logical progression of statements to explore the truth of their conjectures. Justify conclusions and communicate them to others. Respond to the arguments of others by listening, asking clarifying questions, and critiquing the reasoning of others.

- **Standard SIII.MP.4 Model with mathematics.** Apply mathematics to solve problems arising in everyday life, society, and the workplace. Make assumptions and approximations, identifying important quantities to construct a mathematical model. Routinely interpret mathematical results in the context of the situation and reflect on whether the results make sense, possibly improving the model if it has not served its purpose.

- **Standard SIII.MP.5 Use appropriate tools strategically.** Consider the available tools and be sufficiently familiar with them to make sound decisions about when each tool might be helpful, recognizing both the insight to be gained as well as the limitations. Identify relevant external mathematical resources and use them to pose or solve problems. Use tools to explore and deepen their understanding of concepts.

- **Standard SIII.MP.6 Attend to precision.** Communicate precisely to others. Use explicit definitions in discussion with others and in their own reasoning. They state the meaning of the symbols they choose. Specify units of measure and label axes to clarify the correspondence with quantities in a problem. Calculate accurately and efficiently, express numerical answers with a degree of precision appropriate for the problem context.

- **Standard SIII.MP.7 Look for and make use of structure.** Look closely at mathematical relationships to identify the underlying structure by recognizing a simple structure within a more complicated structure. See complicated things, such as some algebraic expressions, as single objects or as being composed of several objects. *For example, see* $5 - 3(x - y)^2$ *as 5 minus a positive number times a square and use that to realize that its value cannot be more than 5 for any real numbers* x *and* y.

- **Standard SIII.MP.8 Look for and express regularity in repeated reasoning.** Notice if reasoning is repeated, and look for both generalizations and shortcuts. Evaluate the reasonableness of intermediate results by maintaining oversight of the process while attending to the details.

Strand: NUMBER AND QUANTITY—The Complex Number System (N.CN)

Use complex numbers in polynomial identities and equations. Build on work with quadratic equations in Secondary Mathematics II **(Standards N.CN.8–9).**

- **Standard N.CN.8** Extend polynomial identities to the complex numbers. *For example, rewrite* $x^2 + 4$ *as* $(x + 2i)(x - 2i)$.

- **Standard N.CN.9** Know the Fundamental Theorem of Algebra; show that it is true for quadratic polynomials. Limit to polynomials with real coefficients.

Strand: ALGEBRA—Seeing Structures in Expressions (A.SSE)

Interpret the structure of expressions. Extend to polynomial and rational expressions **(Standards A.SSE.1–2).** Write expressions in equivalent forms to solve problems **(Standard A.SSE.4).**

- **Standard A.SSE.1** Interpret polynomial and rational expressions that represent a quantity in terms of its context.★
  - **a.** Interpret parts of an expression, such as terms, factors, and coefficients.
  - **b.** Interpret complex expressions by viewing one or more of their parts as a single entity. *For example, examine the behavior of* $P(1+r/n)^{nt}$ *as* n *becomes large.*

- **Standard A.SSE.2** Use the structure of an expression to identify ways to rewrite it. *For example, see* $x^4 - y^4$ *as* $(x^2)^2 - (y^2)^2$, *thus recognizing it as a difference of squares that can be factored as* $(x^2 - y^2)(x^2 + y^2)$.

- **Standard A.SSE.4** Understand the formula for the sum of a series and use the formula to solve problems.
  - **a.** Derive the formula for the sum of an arithmetic series.
  - **b.** Derive the formula for the sum of a geometric series, and use the formula to solve problems. Extend to infinite geometric series. *For example, calculate mortgage payments.*★

Strand: ALGEBRA—Arithmetic With Polynomials and Rational Expressions (A.APR)

Perform arithmetic operations on polynomials, extending beyond the quadratic polynomials **(Standard A.APR.1).** Understand the relationship between zeros and factors of polynomials **(Standards A.APR.2–3).** Use polynomial identities to solve problems **(Standards A.APR.4–5).** Rewrite rational expressions **(Standards A.APR.6–7).**

- **Standard A.APR.1** Understand that all polynomials form a system analogous to the integers, namely, they are closed under the operations of addition, subtraction, and multiplication; add, subtract, and multiply polynomials.
- **Standard A.APR.2** Know and apply the Remainder Theorem: For a polynomial $p(x)$ and a number a, the remainder on division by $x - a$ is $p(a)$, so $p(a) = 0$ if and only if $(x - a)$ is a factor of $p(x)$.
- **Standard A.APR.3** Identify zeros of polynomials when suitable factorizations are available, and use the zeros to construct a rough graph of the function defined by the polynomial.
- **Standard A.APR.4** Prove polynomial identities and use them to describe numerical relationships. *For example, the polynomial identity* $(x^2 + y^2)^2 = (x^2 - y^2)^2 + (2xy)^2$ *can be used to generate Pythagorean triples.*
- **Standard A.APR.5** Know and apply the Binomial Theorem for the expansion of $(x + y)^n$ in powers of x and y for a positive integer n, where x and y are any numbers. *For example, with coefficients determined by Pascal's Triangle.*
- **Standard A.APR.6** Rewrite simple rational expressions in different forms; write $a(x)/b(x)$ in the form $q(x) + r(x)/b(x)$, where $a(x)$, $b(x)$, $q(x)$, and $r(x)$ are polynomials with the degree of $r(x)$ less than the degree of $b(x)$, using inspection, long division or, for the more complicated examples, a computer algebra system.
- **Standard A.APR.7** Understand that rational expressions form a system analogous to the rational numbers, closed under addition, subtraction, multiplication, and division by a nonzero rational expression; add, subtract, multiply, and divide rational expressions.

Strand: ALGEBRA: CREATING EQUATIONS (A.CED)

Create equations that describe numbers or relationships, using all available types of functions to create such equations **(Standards A.CED.1–4).**

- **Standard A.CED.1** Create equations and inequalities in one variable and use them to solve problems. Include equations arising from linear and quadratic functions, and simple rational and exponential functions.
- **Standard A.CED.2** Create equations in two or more variables to represent relationships between quantities; graph equations on coordinate axes with labels and scales.
- **Standard A.CED.3** Represent constraints by equations or inequalities, and by systems of equations and/or inequalities, and interpret solutions as viable or non-viable options in

a modeling context. *For example, maximizing the volume of a box for a given surface area while drawing attention to the practical domain.*

- **Standard A.CED.4** Rearrange formulas to highlight a quantity of interest, using the same reasoning as in solving equations. *For example, rearrange the compound interest formula to solve for* t: $A = P(1+r/n)^{nt}$

Strand: ALGEBRA: REASONING WITH EQUATIONS AND INEQUALITIES (A.REI)

Understand solving equations as a process of reasoning and explain the reasoning **(Standard A.REI.2).** Represent and solve equations and inequalities graphically **(Standard A.REI.11).**

- **Standard A.REI.2** Solve simple rational and radical equations in one variable, and give examples showing how extraneous solutions may arise.

- **Standard A.REI.11** Explain why the *x*-coordinates of the points where the graphs of the equations $y = f(x)$ and $y = g(x)$ intersect are the solutions of the equation $f(x) = g(x)$; find the solutions approximately, for example, using technology to graph the functions, make tables of values, or find successive approximations. Include cases where $f(x)$ and/or $g(x)$ are linear, polynomial, rational, absolute value, exponential, and logarithmic functions.★

Strand: FUNCTIONS—Interpreting Functions (F.IF)

Interpret functions that arise in applications in terms of a context **(Standards F.IF.4–6).** Analyze functions using different representations **(Standards F.IF.7–9).**

- **Standard F.IF.4** For a function that models a relationship between two quantities, interpret key features of graphs and tables in terms of the quantities, and sketch graphs showing key features given a verbal description of the relationship. *Key features include intercepts; intervals where the function is increasing, decreasing, positive, or negative; relative maximums and minimums; symmetries; end behavior; and periodicity.*★

- **Standard F.IF.5** Relate the domain of a function to its graph and, where applicable, to the quantitative relationship it describes. *For example, if the function* h(n) *gives the number of person-hours it takes to assemble* n *engines in a factory, then the positive integers would be an appropriate domain for the function.*★

- **Standard F.IF.6** Calculate and interpret the average rate of change of a function (presented symbolically or as a table) over a specified interval. Estimate the rate of change from a graph.★

- **Standard F.IF.7** Graph functions expressed symbolically and show key features of the graph, by hand in simple cases and using technology for more complicated cases.★

    b. Graph square root, cube root, and piecewise-defined functions, including step functions and absolute value functions. Compare and contrast square root, cubed root, and step functions with all other functions.

c. Graph polynomial functions, identifying zeros when suitable factorizations are available, and showing end behavior.

d. Graph rational functions, identifying zeros and asymptotes when suitable factorizations are available, and showing end behavior.

e. Graph exponential and logarithmic functions, showing intercepts and end behavior; and trigonometric functions, showing period, midline, and amplitude.

■ **Standard F.IF.8** Write a function defined by an expression in different but equivalent forms to reveal and explain different properties of the function.

■ **Standard F.IF.9** Compare properties of two functions each represented in a different way (algebraically, graphically, numerically in tables, or by verbal descriptions). *For example, given a graph of one quadratic function and an algebraic expression for another, say which has the larger maximum.*

Strand: FUNCTIONS—Building Functions (F.BF)

Build a function that models a relationship between two quantities. Develop models for more complex or sophisticated situations **(Standards F.BF.1).** Build new functions from existing functions **(Standards F.BF.3–4).**

■ **Standard F.BF.1** Write a function that describes a relationship between two quantities.★

b. Combine standard function types using arithmetic operations. *For example, build a function that models the temperature of a cooling body by adding a constant function to a decaying exponential, and relate these functions to the model.*

■ **Standard F.BF.3** Identify the effect on the graph of replacing $f(x)$ by $f(x) + k$, $k\,f(x)$, $f(kx)$, and $f(x + k)$ for specific values of k (both positive and negative); find the value of k given the graphs. Note the effect of multiple transformations on a single function and the common effect of each transformation across function types. Include functions defined only by a graph. Experiment with cases and illustrate an explanation of the effects on the graph using technology. Include recognizing even and odd functions from their graphs and algebraic expressions for them.

■ **Standard F.BF.4** Find inverse functions.

a. Solve an equation of the form $f(x) = c$ for a simple function f that has an inverse and write an expression for the inverse. Include linear, quadratic, exponential, logarithmic, rational, square root, and cube root functions. *For example,* $f(x) = 2x^3$ *or* $f(x) = (x+1)/(x-1)$ *for* $x \neq 1$.

Strand: FUNCTIONS—Linear, Quadratic, and Exponential Models (F.LE)

Construct and compare linear, quadratic, and exponential models and solve problems **(Standards F.LE.3–4).** Interpret expressions for functions in terms of the situation it models.

Introduce $f(x) = e^x$ as a model for continuous growth **(Standard F.LE.5).**

- **Standard F.LE.3** Observe using graphs and tables that a quantity increasing exponentially eventually exceeds a quanitity increasing linearly, quadratically, or (more generally) as a polynomial function.
- **Standard F.LE.4** For exponential models, express as a logarithm the solution to $ab^{ct} = d$ where *a, c*, and *d* are numbers and the base *b* is 2, 10, or *e*; evaluate the logarithm using technology. Include the relationship between properties of logarithms and properties of exponents, such as the connection between the properties of exponents and the basic logarithm property that log $xy = \log x + \log y$.
- **Standard F.LE.5** Interpret the parameters in a linear, quadratic, or exponential function in terms of a context.

Strand: FUNCTIONS—Trigonometric Functions (F.TF)

Extend the domain of trigonometric functions using the unit circle **(Standards F.TF.1–3).** Model periodic phenomena with trigonometric functions **(Standards F.TF.5–7).**

- **Standard F.TF.1** Understand radian measure of an angle as the length of the arc on the unit circle subtended by the angle.
- **Standard F.TF.2** Explain how the unit circle in the coordinate plane enables the extension of trigonometric functions to all real numbers, interpreted as radian measures of angles traversed counterclockwise around the unit circle.
- **Standard F.TF.3** Use special triangles to determine geometrically the values of sine, cosine, tangent for $\pi/3$, $\pi/4$ and π6, and use the unit circle to express the values of sine, cosine, and tangent for $\pi - x$, $\pi + x$, and $2\pi - x$ in terms of their values for *x*, where *x* is any real number.
- **Standard F.TF.5** Choose trigonometric functions to model periodic phenomena with specified amplitude, frequency, and midline.★
- **Standard F.TF.7** Use inverse functions to solve trignometric equations that arise in modeling context; evaluate the solutions using technology and interpret them in terms of context. Limit solutions to a given interval. ★

Strand: GEOMETRY—Similarity, Right Triangles, and Trigonometry (G.SRT)

Apply trigonometry to general triangles. With respect to the general case of the Laws of Sines and Cosines, the definitions of sine and cosine must be extended to obtuse angles **(Standards G.SRT.9–11).**

- **Standard G.SRT.9** Derive the formula $A = ½\, ab \sin(C)$ for the area of a triangle by drawing an auxiliary line from a vertex perpendicular to the opposite side.
- **Standard G.SRT.10** Prove the Laws of Sines and Cosines and use them to solve problems.

- **Standard G.SRT.11** Understand and apply the Law of Sines and the Law of Cosines to find unknown measurements in right and non-right triangles (e.g., surveying problems, resultant forces).

Strand: GEOMETRY—Geometric Measurement and Dimension (G.GMD)

Visualize relationships between two-dimensional and three-dimensional objects **(Standards G.MD.4).**

- **Standard G.GMD.4** Identify the shapes of two-dimensional cross-sections of three-dimensional objects, and identify three-dimensional objects generated by rotations of two-dimensional objects.

Strand: GEOMETRY—Modeling With Geometry (G.MG)

Apply geometric concepts in modeling situations **(Standards G.MG.1–3).**

- **Standard G.MG.1** Use geometric shapes, their measures, and their properties to describe objects (e.g., modeling a tree trunk or a human torso as a cylinder).★
- **Standard G.MG.2** Apply concepts of density based on area and volume in modeling situations (e.g., persons per square mile, BTUs per cubic foot).★
- **Standard G.MG.3** Apply geometric methods to solve design problems (e.g., designing an object or structure to satisfy physical constraints or minimize cost; working with typographic grid systems based on ratios).★

Strand: STATISTICS—Interpreting Categorical and Quantitative Data (S.ID)

Summarize, represent, and interpret data on a single count or measurement variable. While students may have heard of the normal distribution, it is unlikely that they will have prior experience using it to make specific estimates. Build on students' understanding of data distributions to help them see how the normal distribution uses area to make estimates of frequencies (which can be expressed as probabilities). Emphasize that only some data are well described by a normal distribution **(Standard S.ID.4).**

- **Standard S.ID.4** Use the mean and standard deviation of a data set to fit it to a normal distribution and to estimate population percentages. Recognize that there are data sets for which such a procedure is not appropriate. Use calculators, spreadsheets, and tables to estimate areas under the normal curve.

Strand: STATISTICS—Making Inferences and Justifying Conclusions (S.IC)

Understand and evaluate random processes underlying statistical experiments **(Standard S.IC.1).** Draw and justify conclusions from sample surveys, experiments, and observational studies. In earlier grades, students are introduced to different ways of collecting data and use graphical displays and summary statistics to make comparisons. These ideas are revisited with a focus on how the way in which data is collected determines the scope and nature of the conclusions that can be drawn from that data. The concept of statistical significance is developed informally through simulation as meaning a result that is unlikely to have occurred solely as a result of random selection in sampling or random assignment in an experiment. For S.IC.4, focus on the variability of results from experiments—that is, focus on statistics as a way of dealing with, not eliminating, inherent randomness **(Standards S.IC.3–4, 6).**

- **Standard S.IC.1** Understand that statistics allow inferences to be made about population parameters based on a random sample from that population.
- **Standard S.IC.3** Recognize the purposes of and differences among sample surveys, experiments, and observational studies; explain how randomization relates to each.
- **Standard S.IC.4** Use data from a sample survey to estimate a population mean or proportion; develop a margin of error through the use of simulation models for random sampling.
- **Standard S.IC.6** Evaluate reports based on data.

SECONDARY MATHEMATICS III—HONORS STANDARDS

Strand: NUMBER AND QUANTITY—Complex Number System (N.CN)

Perform arithmetic operations with complex numbers **(Standard N.CN.3).** Represent complex numbers and their operations on the complex plane **(Standard N.CN.4–6).** Use complex numbers in polynomial identities and equations **(Standard N.CN.10).**

- **Standard N.CN.3** Find the conjugate of a complex number; use conjugates to find moduli and quotients of complex numbers.
- **Standard N.CN.4** Represent complex numbers on the complex plane in rectangular form and polar form (including real and imaginary numbers), and explain why the rectangular form of a given complex number represents the same number.
- **Standard N.CN.5** Represent addition, subtraction, multiplication, and conjugation of complex numbers geometrically on the complex plane; use properties of this representation for computation. *For example, (-1 + √3 i)3 = 8 because (-1 + √3 i) has modulus 2 and argument 120°.*
- **Standard N.CN.6** Calculate the distance between numbers in the complex plane as the modulus of the difference, and the midpoint of a segment as the average of the numbers at its endpoints.
- **Standard N.CN.10** Multiply complex numbers in polar form and use DeMoivre's Theorem to find roots of complex numbers.

Strand: FUNCTIONS—Interpreting Functions (F.IF)

Analyze functions using different representations **(Standard F.IF.7, d and f).**

- **Standard F.IF.7** Graph functions expressed symbolically, and show key features of the graph, by hand in simple cases and using technology for more complicated cases.
  - **d.** Graph rational functions, identifying zeros, asymptotes, and point discontinuities when suitable factorizations are available, and showing end behavior.
  - **f.** Define a curve parametrically and draw its graph.

Strand: FUNCTIONS—Building Functions (F.BF).

Build a function that models a relationship between two quantities **(Standard F.BF.1.c).** Build new functions from existing functions **(Standards F.BF.4, b,c,d–5).**

- **Standard F.BF.1** Write a function that describes a relationship between two quantities.
  - **c.** Compose functions. *For example, if* T(y) *is the temperature in the atmosphere as a function of height, and* h(t) *is the height of a weather balloon as a function of time, then* T(h(t)) *is the temperature at the location of the weather balloon as a function of time.*

- **Standard F.BF.4** Find inverse functions.
  - **b.** Verify by composition that one function is the inverse of another.
  - **c.** Read values of an inverse function from a graph or a table, given that the function has an inverse.
  - **d.** Produce an invertible function from a non-invertible function by restricting the domain.
- **Standard F.BF.5** Understand the inverse relationship between exponents and logarithms, and use this relationship to solve problems involving logarithms and exponents.

Strand: FUNCTIONS—Trigonometric Functions (F.TF)

Extend the domain of trigonometric functions using the unit circle **(Standard T.FT.4).** Model periodic phenomena with trigonometric functions **(Standards T.FT.6–7).** Prove and apply trigonometric identities **(Standard T.FT.9).**

- **Standard F.TF.4** Use the unit circle to explain symmetry (odd and even) and periodicity of trigonometric functions.
- **Standard F.TF.6** Understand that restricting a trigonometric function to a domain on which it is always increasing or always decreasing allows its inverse to be constructed.
- **Standard F.TF.7** Use the inverse functions to solve trigonometric equations that arise in the modeling contexts; evaluate the solutions using technology, and interpret them in terms of the context.
- **Standard F.TF.9** Prove the addition and subtraction formulas for sine, cosine, and tangent, and use them to solve problems.

Strand: GEOMETRY—Geometric Measurement and Dimension (G.GMD)

Explain volume formulas and use them to solve problems **(Standard G.GMD.2).**

- **Standard G.GMD.2** Give an informal argument using Cavalieri's principle for the formulas for the volume of a sphere and other solid figures.

Strand: STATISTICS AND PROBABILITY—Conditional Probability and the Rules of Probability (S.CP)

Use the rules of probability to compute probabilities of compound events in a uniform probability model **(Standard S.CP.9).**

- **Standard S.CP.9** Use permutations and combinations to compute probabilities of compound events and solve problems.

PRECALCULUS

Number and Quantity

- **Vector and Matrix Quantities**
- **Complex Number System**

Algebra

- **Reasoning With Equations and Inequalities**

Functions

- **Interpreting Functions**
- **Building Functions**
- **Trigonometric Functions**

Geometry

- **Geometric Measurement and Dimension**
- **Expressing Geometric Properties with Equations**

Statistics

- **Conditional Probability and the Rules of Probability**

MATHEMATICAL PRACTICES

1. Make sense of problems and persevere in solving them.
2. Reason abstractly and quantitatively.
3. Construct viable arguments and critique the reasoning of others.
4. Model with mathematics.
5. Use appropriate tools strategically.
6. Attend to precision.
7. Look for and make use of structure.
8. Look for and express regularity in repeated reasoning.

Strand: MATHEMATICAL PRACTICES (MP)

The Standards for Mathematical Practice in PreCalculus describe mathematical habits of mind that teachers should seek to develop in their students. Students become mathematically proficient in engaging with mathematical content and concepts as they learn, experience, and apply these skills and attitudes **(Standards MP.1–8).**

- **Standard P.MP.1 Make sense of problems and persevere in solving them.** Explain the meaning of a problem and look for entry points to its solution. Analyze givens, constraints, relationships, and goals. Make conjectures about the form and meaning of the solution, plan a solution pathway, and continually monitor progress asking, "Does this make sense?" Consider analogous problems, make connections between multiple representations, identify the correspondence between different approaches, look for trends, and transform algebraic expressions to highlight meaningful mathematics. Check answers to problems using a different method.

- **Standard P.MP.2 Reason abstractly and quantitatively.** Make sense of the quantities and their relationships in problem situations. Translate between context and algebraic representations by contextualizing and decontextualizing quantitative relationships. This includes the ability to decontextualize a given situation, representing it algebraically and manipulating symbols fluently as well as the ability to contextualize algebraic representations to make sense of the problem.

- **Standard P.MP.3 Construct viable arguments and critique the reasoning of others.** Understand and use stated assumptions, definitions, and previously established results in constructing arguments. Make conjectures and build a logical progression of statements to explore the truth of their conjectures. Justify conclusions and communicate them to others. Respond to the arguments of others by listening, asking clarifying questions, and critiquing the reasoning of others.

- **Standard P.MP.4 Model with mathematics.** Apply mathematics to solve problems arising in everyday life, society, and the workplace. Make assumptions and approximations, identifying important quantities to construct a mathematical model. Routinely interpret mathematical results in the context of the situation and reflect on whether the results make sense, possibly improving the model if it has not served its purpose.

- **Standard P.MP.5 Use appropriate tools strategically.** Consider the available tools and be sufficiently familiar with them to make sound decisions about when each tool might be helpful, recognizing both the insight to be gained as well as the limitations. Identify relevant external mathematical resources and use them to pose or solve problems. Use tools to explore and deepen their understanding of concepts.

- **Standard P.MP.6 Attend to precision.** Communicate precisely to others. Use explicit definitions in discussion with others and in their own reasoning. They state the meaning of the symbols they choose. Specify units of measure and label axes to clarify the correspondence with quantities in a problem. Calculate accurately and efficiently, express numerical answers with a degree of precision appropriate for the problem context.

- **Standard P.MP.7 Look for and make use of structure.** Look closely at mathematical relationships to identify the underlying structure by recognizing a simple structure within a more complicated structure. See complicated things, such as some algebraic expressions, as single objects or as being composed of several objects. *For example, see* $5 - 3(x - y)^2$ *as 5 minus a positive number times a square and use that to realize that its value cannot be more than 5 for any real numbers* x *and* y.

- **Standard P.MP.8 Look for and express regularity in repeated reasoning.** Notice if reasoning is repeated, and look for both generalizations and shortcuts. Evaluate the reasonableness of intermediate results by maintaining oversight of the process while attending to the details.

Strand: NUMBER AND QUANTITY—Vector and Matrix Quantities (N.VM)

Represent and model with vector quantities **(Standards 1–3).** Perform operations on vectors **(Standards 4–5).** Perform operations on matrices and use matrices in applications **(Standards 6–13).**

- **Standard N.VM.1** Recognize vector quantities as having both magnitude and direction. Represent vector quantities by directed line segments, and use appropriate symbols for vectors and their magnitudes (e.g., *v, |v|, ||v||, v).*

- **Standard N.VM.2** Find the components of a vector by subtracting the coordinates of an initial point from the coordinates of a terminal point.

- **Standard N.VM.3** Solve problems involving velocity and other quantities that can be represented by vectors.

- **Standard N.VM. 4** Add and subtract vectors.

    a. Add vectors end to end, component-wise, and by the parallelogram rule. Understand that the magnitude of a sum of two vectors is typically not the sum of the magnitudes.

    b. Given two vectors in magnitude and direction form, determine the magnitude and direction of their sum.

    c. Understand vector subtraction *v – w as v + (–w)*, where *–w* is the additive inverse of *w*, with the same magnitude as *w* and pointing in the opposite direction. Represent vector subtraction graphically by connecting the tips in the appropriate order, and perform vector subtraction component-wise.

- **Standard N.VM.5** Multiply a vector by a scalar.

    a. Represent scalar multiplication graphically by scaling vectors and possibly reversing their direction; perform scalar multiplication component-wise, e.g., *as* $c(v_x, v_y) = (cv_x, cv_y)$.

b. Compute the magnitude of a scalar multiple cv using $||cv|| = |c|v$. Compute the direction of cv knowing that when $|c|v \neq 0$, the direction of cv is either along v (for $c > 0$) or against vs (for $c < 0$).

■ **Standard N.VM.6** Use matrices to represent and manipulate data, e.g., to represent payoffs or incidence relationships in a network.

■ **Standard N.VM.7** Multiply matrices by scalars to produce new matrices, e.g., as when all of the payoffs in a game are doubled.

■ **Standard N.VM.8** Add, subtract, and multiply matrices of appropriate dimensions.

■ **Standard N.VM.9** Understand that, unlike multiplication of numbers, matrix multiplication for square matrices is not a commutative operation, but still satisfies the associative and distributive properties.

■ **Standard N.VM.10** Understand that the zero and identity matrices play a role in matrix addition and multiplication similar to the role of 0 and 1 in the real numbers. The determinant of a square matrix is nonzero if and only if the matrix has a multiplicative inverse.

■ **Standard N.VM.11** Multiply a vector (regarded as a matrix with one column) by a matrix of suitable dimensions to produce another vector. Work with matrices as transformations of vectors.

■ **Standard N.VM.12** Work with 2×2 matrices as transformations of the plane, and interpret the absolute value of the determinant in terms of area.

■ **Standard N.VM.13** Solve systems of linear equations up to three variables using matrix row reduction.

Strand: NUMBER AND QUANTITY—Complex Number Systems (N.CN)

Perform arithmetic operations with complex numbers **(Standard 3).** Represent complex numbers and their operations on the complex plane **(Standards 4–6).** Use complex numbers in polynomial identities and equations **(Standard 10).**

■ **Standard N.CN.3** Find the conjugate of a complex number; use conjugates to find moduli and quotients of complex numbers.

■ **Standard N.CN.4** Represent complex numbers on the complex plane in rectangular and polar form (including real and imaginary numbers), and explain why the rectangular and polar forms of a given complex number represent the same number.

■ **Standard N.CN.5** Represent addition, subtraction, multiplication, and conjugation of complex numbers geometrically on the complex plane; use properties of this representation for computation. *For example,* $(-1 + \sqrt{3}\,i)^3 = 8$*, because* $(-1 + \sqrt{3}\,i)$ *has modulus 2 and argument 120°.*

- **Standard N.CN.6** Calculate the distance between numbers in the complex plane as the modulus of the difference, and the midpoint of a segment as the average of the numbers at its endpoints.

- **Standard N.CN.10** Multiply complex numbers in polar form and use DeMoivre's Theorem to find roots of complex numbers.

Strand: ALGEBRA: REASONING WITH EQUATIONS AND INEQUALITIES (A.REI)

Solve systems of equations **(Standards 8–9).**

8. Represent a system of linear equations as a single matrix equation in a vector variable.

9. Find the inverse of a matrix, if it exists, and use it to solve systems of linear equations (using technology for matrices of dimension 3 x 3 or greater).

Strand: FUNCTIONS—Interpreting Functions (F.IF)

Analyze functions using different representations **(Standard 7, 10–11).**

- **Standard F.IF.7** Graph functions expressed symbolically, and show key features of the graph, by hand in simple cases and using technology for more complicated cases.

  d. Graph rational functions, identifying zeros, asymptotes, and point discontinuities when suitable factorizations are available, and showing end behavior.

  f. Define a curve parametrically and draw its graph.

- **Standard F.IF.10** Use sigma notation to represent the sum of a finite arithmetic or geometric series.

- **Standard F.IF.11** Represent series algebraically, graphically, and numerically.

Strand: FUNCTIONS—Building Functions (F.BF)

Build a function that models a relationship between two quantities **(Standard 1).** Build new functions from existing functions **(Standard 4–5).**

- **Standard F.BF.1** Write a function that describes a relationship between two quantities.

  c. Compose functions. *For example, if* T(y) *is the temperature in the atmosphere as a function of height, and* h(t) *is the height of a weather balloon as a function of time, then* T(h(t)) *is the temperature at the location of the weather balloon as a function of time.*

- **Standard F.BF.4** Find inverse functions.

  b. Verify by composition that one function is the inverse of another.

c. Read values of an inverse function from a graph or a table, given that the function has an inverse.

d. Produce an invertible function from a non-invertible function by restricting the domain.

■ **Standard F.BF.5** Understand the inverse relationship between exponents and logarithms and use this relationship to solve problems involving logarithms and exponents.

Strand: FUNCTIONS—Trigonometric Functions (F.TF)

Extend the domain of trigonometric functions using the unit circle **(Standard 4).** Model periodic phenomena with trigonometric functions **(Standard 6–7).** Prove and apply trigonometric identities **(Standard 9).**

■ **Standard F.TF.4** Use the unit circle to explain symmetry (odd and even) and periodicity of trigonometric functions.

■ **Standard F.TF.6** Understand that restricting a trigonometric function to a domain on which it is always increasing or always decreasing allows its inverse to be constructed.

■ **Standard F.TF.7** Use inverse functions to solve trigonometric equations that arise in modeling contexts; evaluate the solutions using technology, and interpret them in terms of the context

■ **Standard F.TF.9** Prove the addition and subtraction formulas for sine, cosine, and tangent, and use them to solve problems.

Strand: GEOMETRY—Geometric Measurement and Dimension (G.GMD)

Explain volume formulas and use them to solve problems **(Standard 2).**

■ **Standard G.GMD.2** Give an informal argument using Cavalieri's principle for the formulas for the volume of a sphere and other solid figures.

Strand: GEOMETRY—Expressing Geometric Properties With Equations (G.GPE)

Translate between the geometric description and the equation for a conic section **(Standards 2–3).**

■ **Standard G.GPE.2** Derive the equation of a parabola given a focus and a directrix.

■ **Standard G.GPE.3** Derive the equations of ellipses and hyperbolas given the foci, using the fact that the sum or difference of distances from the foci is constant.

Strand: STATISTICS—Conditional Probability and the Rules of Probability (S.CP)

Understand independence and conditional probability and use them to interpret data **(Standards 2–3).** Use the rules of probability to compute probabilities of compound events in a uniform probability model **(Standards 7–9).**

- **Standard S.CP.2** Understand that two events *A* and *B* are independent if the probability of *A* and *B* occurring together is the product of their probabilities, and use this characterization to determine if they are independent.
- **Standard S.CP.3** Understand the conditional probability of *A* given *B* as *P(A and B)/P(B)*, and interpret independence of *A* and *B* as saying that the conditional probability of *B* given *A* is the same as the probability of *B*.
- **Standard S.CP.7** Apply the Addition Rule, *P(A or B) = P(A) + P(B) – P(A and B)*, and interpret the answer in terms of the model.
- **Standard S.CP.8** Apply the general Multiplication Rule in a uniform probability model, *P(A andB) = P(A)P(B|A) = P(B)P(A|B)*, and interpret the answer in terms of the model.
- **Standard S.CP.9** Use permutations and combinations to compute probabilities of compound events and solve problems.

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