Introductory Calculus

Pre-requisite: Pre-calculus or Math 1050

Introductory Calculus is a year-long course where students will develop a conceptual understanding of limits, derivatives, and integrals through discovery and applications. Students will study polynomial, radical, exponential, logarithmic, and rational functions with an emphasis on graphical analysis preparatory to the study of limits. Students will compare the concept of slope with the formal definition of the derivative as well as develop and apply different techniques of integration to model and solve a variety of problems. Students will survey the basic topics of calculus and demonstrate their application in a wide variety of fields. Students will explore calculus concepts through inquiry using technology and develop explicit methods to use calculus in applications.

Standard I: Students will develop a conceptual understanding of limits and the characteristics of functions.

Objective 1: Analyze and perform applications using characteristics of functions, including polynomial, radical, rational, piece-wise, power, exponential, and logarithmic functions.

a. Sketch graphs showing key features of functions.
b. Perform transformations of functions graphically and algebraically by changing one or more parameters in $y = af(x)$, $y = f(ax)$, $y = f(x) + a$, and $y = f(x + a)$ for both positive and negative *a*.
c. Describe and analyze local and global behavior of functions, including increasing, decreasing, asymptotic and end behavior.
d. Recognize even and odd functions from their graphs and algebraic expressions for them.
e. Identify the domain, range, and other attributes of families of functions.

Objective 2: Demonstrate an understanding of the nature of limits.

a. Estimate the limit of a function at a point using graphs and tables.
b. Calculate limits using algebra.
c. Calculate limits involving infinity.
d. Use the asymptotic behavior of a function to identify limits.

Objective 3: Understand the concept of continuity as a property of functions.

a. Understand the definition of continuity as it applies to functions.
b. Understand the role of limits in continuity.
c. Demonstrate an understanding of the implications of continuity on the graphs of functions, including use of the Intermediate Value Theorem and Extreme Value Theorem.

Mathematical Language and Symbols Students Should Use:

polynomial, radical, rational, piece-wise, power, exponential, logarithmic, *f(x)*, $\lim_{x \to n} f(x)$, asymptote, even function, odd function, limit, continuity, Intermediate Value Theorem, Extreme Value Theorem.

Standard II: Students will understand the derivative as an instantaneous rate of change and will use it to analyze curves, optimize values, and model rates of change in applied contexts.

Objective 1: Develop an understanding of derivatives graphically, numerically, and analytically.

a. Interpret derivatives as a limit of average rates or as an instantaneous rate of change in various contexts.
b. Define derivatives as the limit of the difference quotient.
c. Estimate, calculate, and interpret the derivative of a function.
d. Approximate a slope using local linearity, secant lines and tangent lines.
e. Discover and understand the implications of continuity for differentiability.

Objective 2: Manipulate and simplify derivatives using properties.

a. Find derivatives of polynomial functions using the power rule.
b. Understand and use derivative rules for sums, products, and quotients of functions.
c. Calculate the derivative of a composite function using the chain rule.
d. Find derivatives by implicit differentiation with respect to a specified variable.

Objective 3: Solve application problems involving polynomial, exponential, and logarithmic models, including applications in business, economics, and physics.

a. Construct a framework for the application of derivative by solving various real-life problems.
b. Use optimization techniques to maximize and/or minimize functions.
c. Graph functions to illustrate curves.
d. Explain the meaning of the second derivative.
e. Model rates of change, including related rates problems.

Mathematical Language and Symbols Students Should Use:

rate of change, derivative, differentiable, power rule, product rule, quotient rule, composite function, chain rule, local linearity, implicit differentiation, $\frac{\Delta y}{\Delta x}, \frac{dy}{dx}$

Standard III: Students will develop and apply different techniques of integration to model and solve a variety of problems.

Objective 1: Use various numerical methods to approximate definite integrals of functions represented as equations, graphs, and tables.

a. Understand the definite integral of a function as the area under the graph of that function between two points.
b. Approximate the area under the curve by using areas of familiar geometric shapes.
c. Approximate the area under the curve by using left, right, and midpoint Riemann sums.
d. Approximate the area under the curve by using trapezoidal sums.
e. Recognize the definite integral as a limit of Riemann sums.

Objective 2: Use the Fundamental Theorem of Calculus to find definite integrals and to solve differential equations.

a. Understand antidifferentiation and the indefinite integral in terms of reversing the operation of differentiation.
b. Understand that the antiderivative is unique up to a constant.
c. Relate antiderivatives and definite integrals by the Fundamental Theorem of Calculus.
d. Use an initial condition to find a specific antiderivative.
e. Create slope fields from differential equations and use them to draw possible antiderivatives.
f. Understand properties of integrals.

Objective 3: Model, solve, and interpret applications of antiderivatives.

a. Approximate and evaluate change in various contexts numerically, algebraically, and graphically.
b. Use integration techniques to solve geometric problems, including finding the area between two curves.
c. Use integration techniques to solve problems in physics.

Mathematical Language and Symbols Students Should Use:

average value, definite integral, Riemann sum, antiderivative, Fundamental Theorem of Calculus, $\int_a^b f(x)$