Utah State Board of Education
Elementary Mathematics Endorsement Course Framework
Mathematics for Teaching K-8 - Data Analysis and Problem-Solving

Course Description: This course will develop a firm problem-solving foundation. Using skills and strategies applied in mathematical contexts practicing teachers will learn to think, work with others, present solutions orally to the whole class, and write up detailed solutions. This course will also provide practicing teachers a deeper understanding of probability and data representation and analysis. Special attention in this course will be given to children's typical error patterns, problem solving strategies, interpreting and assessing students' work and learning, and integration of the NCTM process standards and the Utah Intended Learning Outcomes (ILOs).

Course Objectives:
During this course students will:

- Select appropriate strategies to solve a problem.
- Solve challenging mathematical problems in groups and individually.
- Experience the joy and creativity inherent in mathematical problem solving.
- Write problem solving summaries, communicate orally solution processes and conclusions, and improve collaboration skills.
- Communicate data analysis and problem-solving strategies orally, visually, and in writing, as well as facilitate effective discourse in a positive mathematics learning environment.
- Collect and organize data using tally marks, tables, pictographs, bar graphs, line graphs, frequency tables, line plots, stem-and-leaf plots, circle graphs, scatter plots, histograms, and box-and-whisker plots.
- Select and interpret measures of central tendency (e.g. mean, median, and mode, including the impact of outliers).
- Select and interpret measures of dispersion (e.g. range, variance, standard deviation, percentiles).
- Identify and apply concepts of probability including: likely, unlikely, certain, impossible, sample space, experimental and theoretical, and recognition of probability as a value between 0 and 1.
- Conduct experiments with and without replacement and compare theoretical and experimental probabilities.
- Analyze misrepresentation and misleading data that exists in the real-world, in order to become informed "consumers" of data.

Data Analysis and Problem Solving is a unique course. The goal is to present a variety of non-standard mathematical problems which will challenge participants to think, explore, communicate, and present solutions. It could be organized many ways, but should establish the teacher more as a mentor than a lecturer. Students should feel safe following whatever thinking path they launch and allowed to self-correct without intervention. Problems should progress in difficulty as the course progresses to ensure students feel confident in their abilities at the start, grow in tenacity and ability, and feel challenged. It is ideal if problems toward the end of the course require multiple strategies.

A tried and proven approach is:

1. Introduce students to a problem solving strategy, about one per week.
2. Use class time to practice that strategy in groups.
3. Assign one "Problem of the Day" (using the week's strategy) to be presented at the next class.
4. Assign five homework problems per week requiring a variety of solution strategies.
5. Require students to write a problem solving explanation for each homework problem. (The explanation should be organized and concise. It should include the thoughts, assumptions, and understandings students developed as they solved the problem. It should include the answer in a complete sentence.)
6. Require students to present solutions orally to the whole class, and write up detailed solutions.
7. Homework problems can be graded by using a rubric.

Topics:

Problem Solving

- Draw a diagram
- Make a systematic list
- Eliminate possibilities
- Use matrix logic
- Look for a pattern
- Guess and check
- Identify sub-problems
- Analyze the units
- Solve an easier related problem
- Create a physical representation
- Draw Venn diagrams
- Convert to algebra
- Evaluate finite differences

Data Analysis and Probability

- Formulating appropriate questions for data analysis research
- Tools for collecting and organizing data:

    - Tally marks, tables, pictographs, bar graphs, line graphs, frequency tables, line plots, stem-and-leaf plots, circle graphs, scatter plots, histograms, and box-and-whisker plots.
- Measures of Central Tendency: what they are, when to use them, how to interpret
    - mean, median, and mode, including the impact of outliers.
- Measures of Dispersion: what they are, when to use them, how to interpret them
    - range, variance, standard deviation, percentiles.
- Concepts of Probability
    - likely, unlikely, certain, impossible, sample space, experimental and theoretical, and recognition of probability as a value between 0 and 1.
- Theoretical and Experimental Probabilities.

Pedagogy

The purpose of the Elementary Mathematics Endorsement courses is to ensure that practicing teachers gain the mathematical content knowledge needed to teach mathematical concepts to students in the elementary grades. Teachers must also, however, know how to transfer that content knowledge and the conceptual understandings inherent in the content to students. An understanding of sound pedagogical practice is essential to that transfer. Following are suggested pedagogical concepts and strategies that should be infused into the courses to aid teachers in student instruction. These concepts should never be taught in isolation, but should be modeled throughout the courses. It is not necessary nor intended that all the following concepts be infused in every course, but they should be covered in their entirety in the series.

Knowledge of mathematics learners:

- Employ prior knowledge, skills, dispositions, beliefs and conceptions, to clarify misconceptions and confusions around particular topics;
- Employ knowledge of cognition, child development, and learning theory;
- Use knowledge of learners culturally and socially to know what motivates and inspires them.

Knowledge of effective mathematics teaching practices for student and teacher learning:

- Design, select and adapt worthwhile tasks and sequences of examples that support a particular learning goal, including tasks to be solved by teachers themselves, and others for which they try to anticipate children's solutions, and design responses.
- Develop sensitivity with the careful use of technical language, attending to both mathematical integrity and usability by learners.

- Construct and evaluate multiple representations of a mathematical idea or process, and establish correspondences between representations.
- Use questions to effectively probe mathematical understanding and make productive use of responses.
- Develop learners' skills at clear and coherent public mathematical communication in a classroom setting, using a variety of resources and media.
- Model and cultivate mathematical practices – questioning, representing, communicating, problem solving, inquiring and conjecturing, making connections, reasoning and proving, self-assessing.
- Use various instructional applications of technology and be able to make judicious, mathematically and pedagogically grounded use of them.
- Construct and evaluate mathematical contexts or story problems for various mathematical situations and processes.
- Analyze and evaluate student work, and design appropriate responses.
- Develop skillful and flexible use of grouping strategies – whole group, small group, partner, and individual – in support of learning goals.
- Know the resources for managing the diversities – cultural, linguistic, gender, socioeconomic, developmental – of the classroom, and ways to skillfully deploy them in support of the learning of all students.
- Understand and address diversity in planning and implementing instruction and in helping other teachers foster mathematics classroom environments that support the learning of all students.
- Understand and use various cultural approaches to learning mathematics.

Knowledge of mathematics curriculum and assessment:

- Know common developmental paths (learning trajectories) related to foundational mathematical topics (e.g., place value, rational numbers and equivalence); understand these developmental paths, and use them to sequence activities to build learning environments that are developmentally appropriate and effective.
- Select, use, adapt and determine the suitability of mathematics curricula and teaching materials (e.g., textbooks, technology, manipulatives) for particular learning goals.
- Be cognizant of a formative assessment cycle (administering a formative assessment task, analyzing student work from the task, using that analysis to enhance teacher knowledge, and designing and teaching reengagement lessons) and resources.
- Provide appropriate interpretations of assessment results, and communicate results (in context) to specific individuals and groups (e.g., students, parents, caregivers, colleagues, administrators, policymakers, community members).

(Standards for Elementary Mathematics Specialists, Association of Mathematics Teacher Educators, 2009)

Possible Assignments:

- Data analysis, probability, and problem solving math homework.
- Content exams, performance tasks, and homework for determining participants' mathematical knowledge for teaching on the topic of data analysis and problem-solving.
- Mixed practice problem-solving sessions, communicate strategy selected and solution.
- A data-based investigation into a classroom question (relating data to instruction).
- Mathematical problem solving tasks: Strategically selected mathematical tasks focused on developing problem solving strategies and exposing important ideas about data analysis.
- Clinical interview: Design an interview to determine student understanding of data analysis and problem solving, including interview questions with extensions to press for student understanding. Videotape and conduct the interview. Analyze the video using a rubric that assesses both student understanding and the interview process.
- Differentiated lesson: Select one of the course objectives. Develop a lesson to meet the objective for the whole group. Then develop differentiation activities for at least three populations (e.g., gifted students, English Language Learners, Students with Disabilities).
- Action research project: Action research is a particular approach to research that aims to improve practice by addressing real-world situations and needs through focused, reflective practice and the application of interventions or procedures that are designed for improving performance. Develop a classroom research project about teaching concepts of mathematics for your own classroom. Consult the mathematics education literature, citing at least three sources from peer-reviewed journals. Develop a research question which could be a new teaching, assessment, or curriculum strategy for rational numbers in your class. Collect data, and analyze the results.
- Case study: Write a case presenting the mathematical thinking of a student or group of students. Be sure to include details in your narrative, such as student dialogue, your questions, and what you were thinking as you listened to the students. Analyze the student thinking and discuss the questions that are raised for you in the students' mathematical thinking.
- Lesson study cycle: Work with a group to develop a task-based lesson including anticipated student responses, questions for classroom discourse, and formative assessment. One person in the group will teach the lesson while other members of the group observe student thinking. Group members refine the lesson in response to the observation data and then teach and analyze the lesson in their own classes.
- Reflections on your own mathematical understanding: Keep a reflections log that discusses: a mathematical idea from the session that is new or important for you, a question you have, and an application of the mathematical idea for your classroom.

Suggested Texts:

Beckman, Sybilla. (2008). *Mathematics for Elementary Teachers* (*2nd Edition*). Reading, MA: Addison Wesley.

Van De Walle, John A. et. al., (2009). *Elementary and Middle School Mathematics, Teaching Developmentally (7th Edition).* Boston, MA: Allyn & Bacon.

Johnson, K; Herr, T. & Kysh, J. (2008). *Crossing the River with Dogs: Problem Solving for College Students.* Hoboken, NJ: Wiley.

Best, Joel (2004). *Damned Lies and Statistics .* Berkeley, CA: University of California Press.

Carpenter, Thomas P. et. al., (2003). *Thinking Mathematically: Integrating Arithmetic and Algebra in Elementary School*. Chicago, IL: Heinemann.

Schifter, Deborah, et. al., (2007). *Reasoning Algebraically About Operations Casebook: A Collaborative Project by the Staff and Participants of Teaching to the Big Ideas (Developing Mathematical Ideas).* Palo Alto, CA: Dale Seymour Publications.

Schifter, Deborah, et. al., (2008). *Developing Mathematical Ideas Patterns, Functions, and Change Casebook.* Palo Alto, CA: Dale Seymour Publications.

Patrick, David, (2005). *Introduction to Counting & Probability (The Art of Problem Solving).* Alpine, CA: AoPS, Inc.

Parker, Thomas H. and Baldridge, Scott J. (2005). *Elementary Geometry for Teachers*. Portland, OR: Sefton-Ash Publishers Chapter 10.