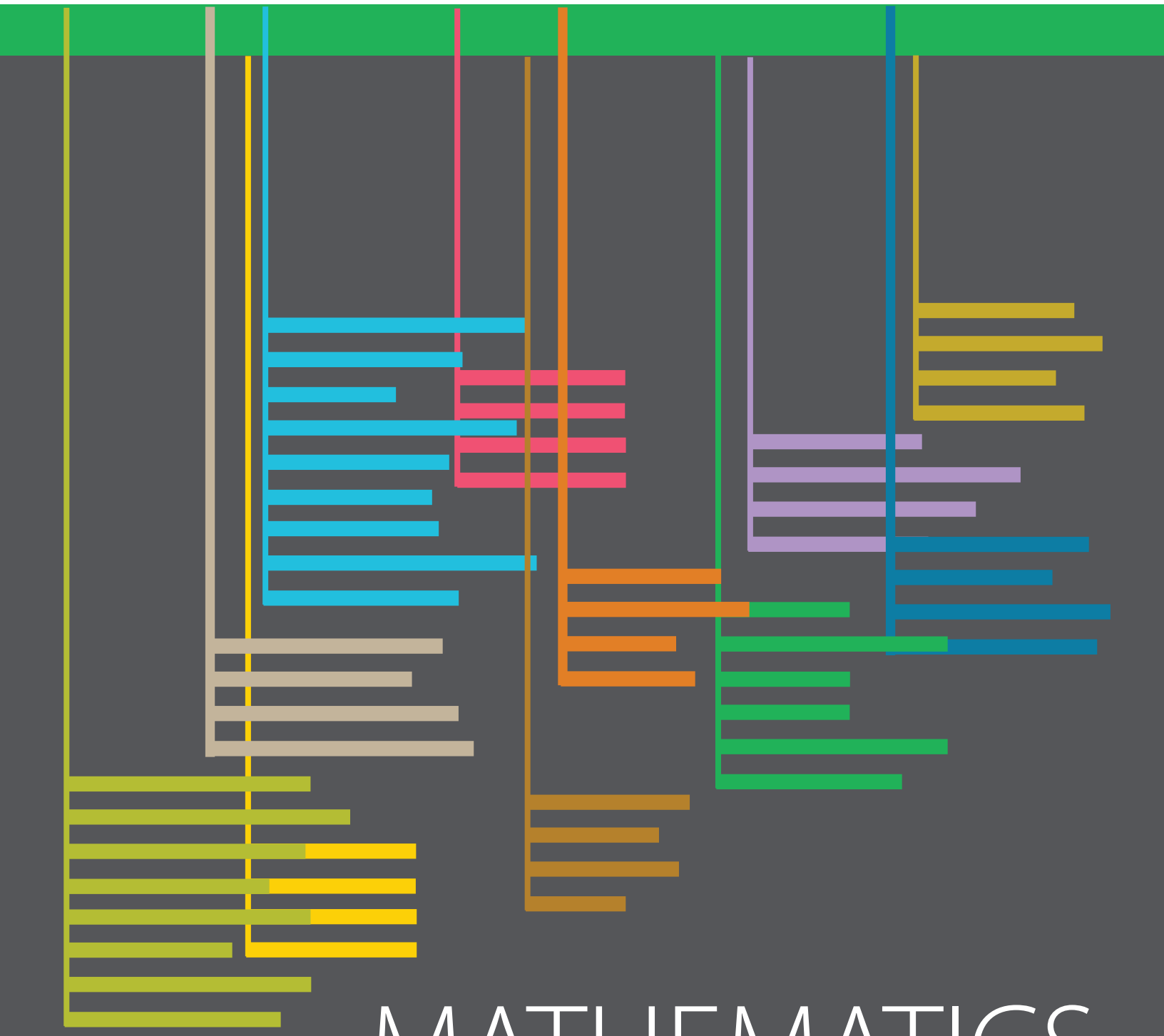

MATHEMATICS
Elementary (K–5)

UTAH STATE OFFICE OF EDUCATION 250 EAST 500 SOUTH P.O. BOX 144200 SALT LAKE CITY, UTAH 84114-4200 SYDNEE DICKSON, INTERIM STATE SUPERINTENDENT OF PUBLIC INSTRUCTION

Utah STATE OFFICE of Education

UTAH CORE STATE STANDARDS
for
MATHEMATICS
ELEMENTARY LEVELS (K–5)

Adopted August 2010

by the

Utah State Board of Education

Revised April 2016

The Utah State Board of Education, in January of 1984, established policy requiring the identification of specific core standards to be met by all K–12 students in order to graduate from Utah's secondary schools. The Utah State Board of Education regularly updates the Utah Core Standards, while parents, teachers, and local school boards continue to control the curriculum choices that reflect local values.

The Utah Core Standards are aligned to scientifically based content standards. They drive high quality instruction through statewide comprehensive expectations for all students. The standards outline essential knowledge, concepts, and skills to be mastered at each grade level or within a critical content area. The standards provide a foundation for ensuring learning within the classroom.

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3/2016

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INTRODUCTION

Organization of the Standards

The Utah Core Standards are organized into **strands**, which represent significant areas of learning within content areas. Depending on the core area, these strands may be designated by time periods, thematic principles, modes of practice, or other organizing principles.

Within each strand are **standards**. A standard is an articulation of the demonstrated proficiency to be obtained. A standard represents an essential element of the learning that is expected. While some standards within a strand may be more comprehensive than others, all standards are essential for mastery.

Understanding Mathematics

These standards define what students should understand and be able to do in their study of mathematics. Asking a student to understand something means asking a teacher to assess whether the student has understood it. But what does mathematical understanding look like? One hallmark of mathematical understanding is the ability to justify, in a way appropriate to the student's mathematical maturity, why a particular mathematical statement is true or where a mathematical rule comes from. Mathematical understanding and procedural skill are equally important, and both are assessable using mathematical tasks of sufficient richness.

The standards set grade-specific standards but do not dictate curriculum or teaching methods, nor do they define the intervention methods or materials necessary to support students who are well below or well above grade-level expectations. It is also beyond the scope of the Standards to define the full range of supports appropriate for English language learners and for students with special needs. At the same time, all students must have the opportunity to learn and meet the same high standards if they are to access the knowledge and skills necessary in their post-school lives. The standards should be read as allowing for the widest possible range of students to participate fully from the outset, along with appropriate accommodations to ensure maximum participation of students with special education needs. No set of grade-specific standards can fully reflect the great variety in abilities, needs, learning rates, and achievement levels of students in any given classroom. However, the standards do provide clear signposts along the way to the goal of college and career readiness for all students.

What students can learn at any particular grade level depends upon what they have learned before. Ideally then, each standard in this document might have been phrased in the form, "Students who already know… should next come to learn ..." Grade placements for specific topics have been made on the basis of state and international comparisons and the collective experience and collective professional judgment of educators, researchers and mathematicians. Learning opportunities will continue to vary across schools and school systems, and educators should make every effort to meet the needs of individual students based on their current understanding.

Utah STATE OFFICE of Education

UTAH CORE STATE STANDARDS

for

MATHEMATICS ELEMENTARY COURSES

(K–5)

Mathematics | Kindergarten

In kindergarten, instructional time should focus on two critical areas: (1) representing, relating, and operating on whole numbers, initially with sets of objects; (2) describing shapes and space. More learning time in kindergarten should be devoted to number than to other topics.

(1) Students will use numbers, including written numerals, to represent quantities and to solve quantitative problems, such as counting objects in a set; counting out a given number of objects; comparing sets or numerals; and modeling simple joining and separating situations with sets of objects, or eventually with equations such as 5 + 2 = 7 and 7 – 2 = 5. (Kindergarten students should see addition and subtraction equations, and student writing of equations in kindergarten is encouraged, but is not required.) Students will choose, combine, and apply effective strategies for answering quantitative questions, including quickly recognizing the cardinalities of small sets of objects, counting and producing sets of given sizes, counting the number of objects in combined sets, or counting the number of objects that remain in a set after some are taken away.

(2) Students will describe their physical world using geometric ideas *(for example, shape, orientation, spatial relations)* and vocabulary. They will identify, name, and describe basic two-dimensional shapes, such as squares, triangles, circles, rectangles, and hexagons, presented in a variety of ways *(for example, with different sizes and orientations)*, as well as three-dimensional shapes such as cubes, cones, cylinders, and spheres. They will use basic shapes and spatial reasoning to model objects in their environment and to construct more complex shapes.

Strand: MATHEMATICAL PRACTICES (K.MP)

The Standards for Mathematical Practice in Kindergarten describe mathematical habits of mind that teachers should seek to develop in their students. Students will become mathematically proficient in engaging with mathematical content and concepts as they learn, experience, and apply these skills and attitudes **(Standards K.MP.1–8).**

- **Standard K.MP.1 Make sense of problems and persevere in solving them.** Explain the meaning of a problem, look for entry points to begin work on the problem, and plan and choose a solution pathway. When a solution pathway does not make sense, look for another pathway that does. Explain connections between various solution strategies and representations. Upon finding a solution, look back at the problem to determine whether the solution is reasonable and accurate, often checking answers to problems using a different method or approach.

- **Standard K.MP.2 Reason abstractly and quantitatively.** Make sense of quantities and their relationships in problem situations. Contextualize quantities and operations by using images or stories. Decontextualize a given situation and represent it symbolically. Interpret symbols as having meaning, not just as directions to carry out a procedure. Know and flexibly use different properties of operations, numbers, and geometric objects.

- **Standard K.MP.3 Construct viable arguments and critique the reasoning of others.** Use stated assumptions, definitions, and previously established results to construct arguments. Explain and justify the mathematical reasoning underlying a strategy, solution, or conjecture by using concrete referents such as objects, drawings, diagrams, and actions. Listen to or read the arguments of others, decide whether they make sense, ask useful questions to clarify or improve the arguments, and build on those arguments.

- **Standard K.MP.4 Model with mathematics.** Identify the mathematical elements of a situation and create a mathematical model that shows the relationships among them. Identify important quantities in a contextual situation, use mathematical models to show the relationships of those quantities, analyze the relationships, and draw conclusions. Models may be verbal, contextual, visual, symbolic, or physical.

- **Standard K.MP.5 Use appropriate tools strategically.** Consider the tools that are available when solving a mathematical problem, whether in a real-world or mathematical context. Choose tools that are relevant and useful to the problem at hand, such as physical objects, drawings, diagrams, physical tools, technologies, or mathematical tools such as estimation or a particular strategy or algorithm.

- **Standard K.MP.6 Attend to precision.** Communicate precisely to others by crafting careful explanations that communicate mathematical reasoning by referring specifically to each important mathematical element, describing the relationships among them, and connecting their words clearly to representations. Calculate accurately and efficiently, and use clear and concise notation to record work.

- **Standard K.MP.7 Look for and make use of structure.** Recognize and apply the structures of mathematics, such as patterns, place value, the properties of operations, or the flexibility of numbers. See complicated things as single objects or as being composed of several objects.

- **Standard K.MP.8 Look for and express regularity in repeated reasoning.** Notice repetitions in mathematics when solving multiple related problems. Use observations and reasoning to find shortcuts or generalizations. Evaluate the reasonableness of intermediate results.

Strand: COUNTING AND CARDINALITY (K.CC)

Know number names and the counting sequence **(Standards K.CC.1–3).** Count to tell the number of objects **(Standards K.CC. 4–5).** Identify and compare quantities of objects and numerals **(Standards K.CC.6–7).**

- **Standard K.CC.1** Count to 100 by ones and by tens.

- **Standard K.CC.2** Count forward beginning from a given number within the known sequence (instead of having to begin at 1).

- **Standard K.CC.3** Read and write numbers using base ten numerals from 0 to 20. Represent a number of objects with a written numeral, in or out of sequence (0 represents a count of no objects).

- **Standard K.CC.4** Understand the relationship between numbers and quantities; connect counting to cardinality.

  a. When counting objects, say the numbers in the standard order. Pair each quantity of objects with one and only one number, and each number with the correct quantity of objects.

  b. Understand that the last number said represents the number of objects counted. The number of objects is the same regardless of their arrangement or the order in which they were counted.

  c. Understand that each successive number refers to a quantity that is one greater than the previous number.

- **Standard K.CC.5** Use counting to answer questions about "how many." *For example, 20 or fewer objects arranged in a line, a rectangular array, or circle; 10 or fewer objects in a scattered configuration.* Using a number from 1–20, count out that many objects.

- **Standard K.CC.6** Use matching or counting strategies to identify whether the number of objects in one group is greater than, less than, or equal to the number of objects in another group. Include groups with up to ten objects.

- **Standard K.CC.7** Compare two numbers between 1 and 10 presented as written numerals using "greater than," "less than," or "equal to."

Strand: OPERATIONS AND ALGEBRAIC THINKING (K.OA)

Understand addition as putting together and adding to, and understand subtraction as taking apart and taking from **(Standards K.OA.1–5).**

- **Standard K.OA.1** Represent addition and subtraction with objects, fingers, mental images, simple drawings, or sounds. *For example, use clapping, act out situations, and use verbal explanations, expressions, or equations*.
- **Standard K.OA.2** Solve addition and subtraction word problems within 10. Use objects or drawings to represent the problem.
- **Standard K.OA.3** Decompose numbers less than or equal to 10 into pairs in more than one way by using objects or drawings. Record each decomposition by a drawing or equation. *For example, 5 = 2 + 3 and 5 = 4 + 1*.
- **Standard K.OA.4** Make sums of 10 using any number from 1 to 9. *For example, 2 + 8 = 10.* Use objects or drawings to represent and record the answer.
- **Standard K.OA.5** Fluently add and subtract using numbers within 5.

Strand: NUMBER AND OPERATIONS IN BASE TEN (K.NBT)

Compose and decompose numbers 11–19 to gain foundations for place value **(Standard K.NBT.1).**

- **Standard K.NBT.1** Compose and decompose numbers from 11–19 into ten ones and some further ones. Use objects or drawings and record each composition or decomposition by a drawing or equation. *For example, 18 = 10 + 8.* Understand that these numbers are composed of ten ones and one, two, three, four, five, six, seven, eight, or nine ones.

Strand: MEASUREMENT AND DATA (K.MD)

Describe and compare measurable attributes of objects **(Standards K.MD.1–2)** and classify objects and count the number of objects in each category **(Standard K.MD.3).**

- **Standard K.MD.1** Describe measurable attributes of objects, such as length or weight. Describe several measurable attributes of a single object.
- **Standard K.MD.2** Directly compare two objects with a measurable attribute in common, to see which object has "more of"/"less of" the attribute, and describe the difference. *For example, directly compare the length of two pencils and describe one as shorter or longer.*
- **Standard K.MD.3** Classify objects into given categories; count the numbers of objects in each category and sort the categories by count. Limit the category counts to less than or equal to 10.

Strand: GEOMETRY (K.G)

Identify and describe shapes, including squares, circles, triangles, rectangles, hexagons, cubes, cones, cylinders, and spheres **(Standards K.G.1–3).** Analyze, compare, create, and compose shapes **(Standards K.G.4–6).**

- **Standard K.G.1** Describe objects in the environment using names of shapes, and describe the relative positions of these objects using terms such as *above*, *below*, *beside*, *in front of*, *behind*, and *next to*.
- **Standard K.G.2** Correctly name shapes regardless of their orientations or overall sizes.
- **Standard K.G.3** Identify shapes as two-dimensional ("flat") or three-dimensional ("solid").
- **Standard K.G.4** Analyze, compare, and sort two- and three-dimensional shapes and objects, in different sizes and orientations, using informal language to describe their similarities, differences, and other attributes *(for example, color, size, shape, number of sides)*.
- **Standard K.G.5** Model and create shapes from components such as sticks and clay balls.
- **Standard K.G.6** Compose simple shapes to form larger shapes. *For example, "Can you join these two triangles with full sides touching to make a rectangle?"*

Mathematics | Grade 1

In Grade 1, instructional time should focus on four critical areas: (1) developing understanding of addition, subtraction, and strategies for addition and subtraction within 20; (2) developing understanding of whole number relationships and place value, including grouping in tens and ones; (3) developing understanding of linear measurement and measuring lengths as iterating length units; and (4) reasoning about attributes of, and composing and decomposing geometric shapes.

(1) Students will develop strategies for adding and subtracting whole numbers based on their prior work with small numbers. They will use a variety of models, including discrete objects and length-based models *(for example, cubes connected to form lengths)*, to model add-to, take-from, put-together, and take-apart; compare situations to develop meaning for the operations of addition and subtraction; and develop strategies to solve arithmetic problems with these operations. Students will understand connections between counting and addition and subtraction *(for example, adding two is the same as counting on two)*. They will use properties of addition to add whole numbers and to create and use increasingly sophisticated strategies based on these properties *(for example, "making tens")* to solve addition and subtraction problems within 20. By comparing a variety of solution strategies, children will build their understanding of the relationship between addition and subtraction.

(2) Students will develop, discuss, and use efficient, accurate, and generalizable methods to add within 100 and subtract multiples of 10. They will compare whole numbers (at least to 100) to develop understanding of and solve problems involving their relative sizes. They will think of whole numbers between 10 and 100 in terms of tens and ones (especially recognizing the numbers 11 to 19 as composed of a ten and some ones). Through activities that build number sense, they will understand the order of the counting numbers and their relative magnitudes.

(3) Students will develop an understanding of the meaning and processes of measurement, including underlying concepts such as iterating (the mental activity of building up the length of an object with equal-sized units) and the transitivity principle for indirect measurement.

(4) Students will compose and decompose plane or solid figures *(for example, put two triangles together to make a quadrilateral)* and build understanding of part-whole relationships, as well as the properties of the original and composite shapes. As they combine shapes, they will recognize them from different perspectives and orientations, describe their geometric attributes, and determine how they are alike and different to develop the background for measurement and for initial understandings of properties such as congruence and symmetry.

Strand: MATHEMATICAL PRACTICES (1.MP)

The Standards for Mathematical Practice in first grade describe mathematical habits of mind that teachers should seek to develop in their students. Students become mathematically proficient in engaging with mathematical content and concepts as they learn, experience, and apply these skills and attitudes **(Standards 1.MP. 1–8).**

- **Standard 1.MP.1 Make sense of problems and persevere in solving them.** Explain the meaning of a problem, look for entry points to begin work on the problem, and plan and choose a solution pathway. When a solution pathway does not make sense, look for another pathway that does. Explain connections between various solution strategies and representations. Upon finding a solution, look back at the problem to determine whether the solution is reasonable and accurate, often checking answers to problems using a different method or approach.

- **Standard 1.MP.2 Reason abstractly and quantitatively.** Make sense of quantities and their relationships in problem situations. Contextualize quantities and operations by using images or stories. Decontextualize a given situation and represent it symbolically. Interpret symbols as having meaning, not just as directions to carry out a procedure. Know and flexibly use different properties of operations, numbers, and geometric objects.

- **Standard 1.MP.3 Construct viable arguments and critique the reasoning of others.** Use stated assumptions, definitions, and previously established results to construct arguments. Explain and justify the mathematical reasoning underlying a strategy, solution, or conjecture by using concrete referents such as objects, drawings, diagrams, and actions. Listen to or read the arguments of others, decide whether they make sense, ask useful questions to clarify or improve the arguments, and build on those arguments.

- **Standard 1.MP.4 Model with mathematics.** Identify the mathematical elements of a situation and create a mathematical model that shows the relationships among them. Identify important quantities in a contextual situation, use mathematical models to show the relationships of those quantities, analyze the relationships, and draw conclusions. Models may be verbal, contextual, visual, symbolic, or physical.

- **Standard 1.MP.5 Use appropriate tools strategically.** Consider the tools that are available when solving a mathematical problem, whether in a real-world or mathematical context. Choose tools that are relevant and useful to the problem at hand, such as physical objects, drawings, diagrams, physical tools, technologies, or mathematical tools, such as estimation or a particular strategy or algorithm.

- **Standard 1.MP.6 Attend to precision.** Communicate precisely to others by crafting careful explanations that communicate mathematical reasoning by referring specifically to each important mathematical element, describing the relationships among them, and connecting their words clearly to representations. Calculate accurately and efficiently, and use clear and concise notation to record work.

- **Standard 1.MP.7 Look for and make use of structure.** Recognize and apply the structures of mathematics such as patterns, place value, the properties of operations, or the flexibility of numbers. See complicated things as single objects or as being composed of several objects.

- **Standard 1.MP.8 Look for and express regularity in repeated reasoning.** Notice repetitions in mathematics when solving multiple related problems. Use observations and reasoning to find shortcuts or generalizations. Evaluate the reasonableness of intermediate results.

Strand: OPERATIONS AND ALGEBRAIC THINKING (1.OA)

Represent and solve problems involving addition and subtraction within 20 **(Standards 1.OA. 1–2, 1.OA.5–6).** Understand and apply properties of operations and the relationship between addition and subtraction **(Standards 1.OA.3–4).** Work with addition and subtraction equations **(Standards 1.OA.7–8).**

- **Standard 1.OA.1** Use addition and subtraction within 20 to solve word problems involving situations of adding to, taking from, putting together, taking apart, and comparing, with unknowns in all positions. *For example, use objects, drawings, and equations with a symbol for the unknown number to represent the problem.*

- **Standard 1.OA.2** Solve word problems that call for addition of three whole numbers whose sum is less than or equal to 20. *For example, use objects, drawings, and equations with a symbol for the unknown number to represent the problem.*

- **Standard 1.OA.3** Apply properties of operations as strategies to add and subtract. *For example: If 8 + 3 = 11 is known, then 3 + 8 = 11 is also known. (Commutative property of addition.) To add 2 + 6 + 4, the second two numbers can be added to make a ten, so 2 + 6 + 4 = 2 + 10 =12. (Associative property of addition.)* First grade students need not use formal terms for these properties.

- **Standard 1.OA.4** Understand subtraction as an unknown-addend problem. *For example, subtract 10 – 8 by finding the number that makes 10 when added to 8.*

- **Standard 1.OA.5** Relate counting to addition and subtraction. *For example, by counting on 2 to add 2.*

- **Standard 1.OA.6** Add and subtract within 20.

  a. Use strategies such as counting on; making ten *(for example, 8 + 6 = 8 + 2 + 4 = 10 + 4 = 14)*; decomposing a number leading to a ten *(for example, 13 - 4 = 13 – 3 – 1 = 10 – 1 = 9)*; using the relationship between addition and subtraction *(for example, knowing that 8 + 4 = 12, one knows 12 – 8 = 4)*; and creating equivalent but easier or known sums (*for example, adding 6 + 7 by creating the known equivalent 6 + 6 + 1 = 12 + 1 = 13)*.

  b. By the end of Grade 1, demonstrate fluency for addition and subtraction within 10.

- **Standard 1.OA.7** Understand the meaning of the equal sign, and determine whether equations involving addition and subtraction are true or false. *For example, which of the following equations are true and which are false? 6 = 6, 7 = 8 – 1, 5 + 2 = 2 + 5, 4 + 1 = 5 + 2.*

- **Standard 1.OA.8** Determine the unknown whole number in an addition or subtraction equation relating three whole numbers. *For example, determine the unknown number that makes the equation true in each of the equations 8 + ? = 11, 5 = ? – 3, 6 + 6 = ?*

Strand: NUMBER AND OPERATIONS IN BASE TEN (1.NBT)

Extend the counting sequence **(Standard 1.NBT.1).** Understand place value **(Standards 1.NBT.2–3).** Use place value understanding and properties of operations to add and subtract **(Standards 1.NBT.4–6).**

- **Standard 1.NBT.1** Count to 120, starting at any number less than 120. In this range, read and write numerals and represent a number of objects with a written numeral.

- **Standard 1.NBT.2** Understand that the two digits of a two-digit number represent amounts of tens and ones. Understand the following as special cases:
  - **a.** 10 can be thought of as a bundle of ten ones, called a "ten."
  - **b.** The numbers from 11 to 19 are composed of a ten and one, two, three, four, five, six, seven, eight, or nine ones.
  - **c.** The numbers 10, 20, 30, 40, 50, 60, 70, 80, 90 refer to one, two, three, four, five, six, seven, eight, or nine tens (and 0 ones).

- **Standard 1.NBT.3** Compare two two-digit numbers based on meanings of the tens and ones digits, recording the results of comparisons with the symbols >, =, and <.

- **Standard 1.NBT.4** Add within 100, including adding a two-digit number and a one-digit number, and adding a two-digit number and a multiple of 10, using concrete models or drawings and strategies based on place value, properties of operations, and/or the relationship between addition and subtraction; relate the strategy to a written method and explain the reasoning used. Understand that in adding two-digit numbers, one adds tens to tens and ones to ones, and that it is sometimes necessary to compose a ten.

- **Standard 1.NBT.5** Given a two-digit number, mentally find 10 more or 10 less than the number, without having to count; explain the reasoning used.

- **Standard 1.NBT.6** Subtract multiples of 10 in the range 10–90 from multiples of 10 in the range 10–90 (positive or zero differences), using concrete models or drawings and strategies based on place value, properties of operations, and/or the relationship between addition and subtraction; relate the strategy to a written method and explain the reasoning used.

Strand: MEASUREMENT AND DATA (1.MD)

Measure lengths indirectly and by iterating length units **(Standards 1.MD.1–2).** Tell and write time **(Standard 1.MD.3)**. Represent and interpret data **(Standard 1.MD.4).** Identify the value of coins **(Standard 1.MD.5).**

- **Standard 1.MD.1** Order three objects by length; compare the lengths of two objects indirectly by using a third object.
- **Standard 1.MD.2** Express the length of an object as a whole number of length units, by laying multiple copies of a shorter object (the length unit) end to end; understand that the length measurement of an object is the number of same-size length units that span it with no gaps or overlaps. *Limit to contexts where the object being measured is spanned by a whole number of length units with no gaps or overlaps.*
- **Standard 1.MD.3** Tell and write time in hours and half-hours using analog and digital clocks.
- **Standard 1.MD.4** Organize, represent, and interpret data with up to three categories; ask and answer questions about the total number of data points, how many in each category, and how many more or less are in one category than in another.
- **Standard 1.MD.5** Identify the values of pennies, nickels, dimes and quarters, and know their comparative values. *(For example, a dime is of greater value than a nickel.)* Use appropriate notation to designate a coin's value. *(For example, 5¢.)*

Strand: GEOMETRY (1.G)

Reason with shapes and their attributes **(Standards 1.G.1–3).**

- **Standard 1.G.1** Distinguish between defining attributes *(for example, triangles are closed and three-sided)* versus non-defining attributes *(for example, color, orientation, overall size)*; build and draw shapes that possess defining attributes.
- **Standard 1.G.2** Compose shapes.
  - **a.** Compose two-dimensional shapes (rectangles, squares, trapezoids, triangles, half-circles, and quarter-circles) to create a composite shape, and compose new shapes from the composite shape.
  - **b.** Compose three-dimensional shapes (cubes, right rectangular prisms, right circular cones, and right circular cylinders) to create a composite shape, and compose new shapes from the composite shape. First grade students do not need to learn formal names such as "right rectangular prism."
- **Standard 1.G.3** Partition circles and rectangles into two and four equal shares; describe the shares using the words halves, fourths, and quarters; and use the phrases *half of*, *fourth of*, and *quarter of*. Describe the whole as two or four of the shares. Understand that, for these examples, decomposing into more equal shares creates smaller shares.

Mathematics | Grade 2

In Grade 2, instructional time should focus on four critical areas: (1) extending understanding of base-ten notation; (2) building fluency with addition and subtraction; (3) using standard units of measure; and (4) describing and analyzing shapes.

(1) Students will extend their understanding of the base-ten system. This includes ideas of counting in fives, tens, and multiples of hundreds, tens, and ones, as well as number relationships involving these units, including comparing. Students will understand multi-digit numbers (up to 1,000) written in base-ten notation, recognizing that the digits in each place represent amounts of thousands, hundreds, tens, or ones *(for example, 853 is 8 hundreds + 5 tens + 3 ones)*.

(2) Students will use their understanding of addition to develop fluency with addition and subtraction within 100. They will solve problems within 1,000 by applying their understanding of models for addition and subtraction, and they develop, discuss, and use efficient, accurate, and generalizable methods to compute sums and differences of whole numbers in base-ten notation, using their understanding of place value and the properties of operations. They will select and accurately apply methods that are appropriate for the context and the numbers involved to mentally calculate sums and differences for numbers with only tens or only hundreds.

(3) Students will recognize the need for standard units of measure (such as centimeter and inch) and they will use rulers and other measurement tools with the understanding that linear measure involves an iteration of units. They will recognize that the smaller the unit, the more iterations they need to cover a given length.

(4) Students will describe and analyze shapes by examining their sides and angles. Students will investigate, describe, and reason about decomposing and combining shapes to make other shapes. Through building, drawing, and analyzing two- and three-dimensional shapes, students will develop a foundation for understanding area, volume, congruence, similarity, and symmetry in later grades.

Strand: MATHEMATICAL PRACTICES (2.MP)

The Standards for Mathematical Practice in Second Grade describe mathematical habits of mind that teachers should seek to develop in their students. Students become mathematically proficient in engaging with mathematical content and concepts as they learn, experience, and apply these skills and attitudes **(Standards 2.MP.1–6).**

- **Standard 2.MP.1 Make sense of problems and persevere in solving them.** Explain the meaning of a problem, look for entry points to begin work on the problem, and plan and choose a solution pathway. When a solution pathway does not make sense, look for another pathway that does. Explain connections between various solution strategies and representations. Upon finding a solution, look back at the problem to determine whether the solution is reasonable and accurate, often checking answers to problems using a different method or approach.

- **Standard 2.MP.2 Reason abstractly and quantitatively.** Make sense of quantities and their relationships in problem situations. Contextualize quantities and operations by using images or stories. Decontextualize a given situation and represent it symbolically. Interpret symbols as having meaning, not just as directions to carry out a procedure. Know and flexibly use different properties of operations, numbers, and geometric objects.

- **Standard 2.MP.3 Construct viable arguments and critique the reasoning of others.** Use stated assumptions, definitions, and previously established results to construct arguments. Explain and justify the mathematical reasoning underlying a strategy, solution, or conjecture by using concrete referents such as objects, drawings, diagrams, and actions. Listen to or read the arguments of others, decide whether they make sense, ask useful questions to clarify or improve the arguments, and build on those arguments.

- **Standard 2.MP.4 Model with mathematics.** Identify the mathematical elements of a situation and create a mathematical model that shows the relationships among them. Identify important quantities in a contextual situation, use mathematical models to show the relationships of those quantities, analyze the relationships, and draw conclusions. Models may be verbal, contextual, visual, symbolic, or physical.

- **Standard 2.MP.5 Use appropriate tools strategically.** Consider the tools that are available when solving a mathematical problem, whether in a real-world or mathematical context. Choose tools that are relevant and useful to the problem at hand, such as drawings, diagrams, technologies, and physical objects and tools, as well as mathematical tools such as estimation or a particular strategy or algorithm.

- **Standard 2.MP.6 Attend to precision.** Communicate precisely to others by crafting careful explanations that communicate mathematical reasoning by referring specifically to each important mathematical element, describing the relationships among them, and connecting their words clearly to representations. Calculate accurately and efficiently, and use clear and concise notation to record work.

- **Standard 2.MP.7 Look for and make use of structure.** Recognize and apply the structures of mathematics such as patterns, place value, the properties of operations, or the flexibility of numbers. See complicated things as single objects or as being composed of several objects.

- **Standard 2.MP.8 Look for and express regularity in repeated reasoning.** Notice repetitions in mathematics when solving multiple related problems. Use observations and reasoning to find shortcuts or generalizations. Evaluate the reasonableness of intermediate results.

Strand: OPERATIONS AND ALGEBRAIC THINKING (2.OA)

Represent and solve problems involving addition and subtraction **(Standard 2.OA.1).** Fluently add and subtract within 20 **(Standard 2.OA.2)** and work with equal groups of objects to gain foundations for multiplication **(Standards 2.OA.3–4).**

- **Standard 2.OA.1** Use addition and subtraction within 100 to solve one- and two-step word problems involving situations of adding to, taking from, putting together, taking apart, and comparing with unknowns in all positions, *for example, by using drawings and equations with a symbol for the unknown number to represent the problem*.

- **Standard 2.OA.2** Fluently add and subtract within 20.

  a. Add and subtract within 20 using mental strategies such as counting on; making ten *(for example, 8 + 6 = 8 + 2 + 4 = 10 + 4 = 14)*; decomposing a number leading to a ten *(for example, 13 – 4 = 13 – 3 – 1 = 10 – 1 = 9)*; using the relationship between addition and subtraction *(for example, knowing that 8 + 4 = 12, one knows 12 – 8 = 4)*; and creating equivalent but easier or known sums *(for example, adding 6 + 7 by creating the known equivalent 6 + 6 + 1 = 12 + 1 = 13)*.

  b. By the end of Grade 2, know from memory all sums of two one-digit numbers.

- **Standard 2.OA.3** Determine whether a group of objects (up to 20) has an odd or even number of members, (*for example, by pairing objects or counting them by twos*). Write an equation to express an even number as a sum of two equal addends.

- **Standard 2.OA.4** Use addition to find the total number of objects arranged in rectangular arrays with up to 5 rows and up to 5 columns; write an equation to express the total as a sum of equal addends.

Strand: NUMBER AND OPERATIONS IN BASE TEN (2.NBT)

Understand place value **(Standards 2.NBT.1–4).** Use place value understanding and properties of operations to add and subtract **(Standards 2.NBT.5–9).**

- **Standard 2.NBT.1** Understand that the three digits of a three-digit number represent amounts of hundreds, tens, and ones; *for example, 706 equals 7 hundreds, 0 tens, and 6 ones*. Understand the following as special cases:

a. 100 can be thought of as a bundle of ten tens called a "hundred."

b. The numbers 100, 200, 300, 400, 500, 600, 700, 800, 900 refer to one, two, three, four, five, six, seven, eight, or nine hundreds (and 0 tens and 0 ones).

- **Standard 2.NBT.2** Count within 1,000; skip-count by fives, tens, and hundreds.
- **Standard 2.NBT.3** Read and write numbers to 1,000 using base-ten numerals, number names, and expanded form.
- **Standard 2.NBT.4** Compare two three-digit numbers based on meanings of the hundreds, tens, and ones digits, using >, =, and < symbols to record the results of comparisons.
- **Standard 2.NBT.5** Fluently add and subtract within 100 using strategies based on place value, properties of operations, and/or the relationship between addition and subtraction.
- **Standard 2.NBT.6** Add up to four two-digit numbers using strategies based on place value and properties of operations.
- **Standard 2.NBT.7** Add and subtract within 1,000 using concrete models or drawings and strategies based on place value, properties of operations, and/or the relationship between addition and subtraction; relate the strategy to a written method. Understand that in adding or subtracting three-digit numbers, one adds or subtracts hundreds and hundreds, tens and tens, and ones and ones, and that it is sometimes necessary to compose or decompose tens or hundreds.
- **Standard 2.NBT.8** Mentally add 10 or 100 to a given number 100-900, and mentally subtract 10 or 100 from a given number 100-900.
- **Standard 2.NBT.9** Explain why addition and subtraction strategies work, using place value and the properties of operations. Explanations may be supported by drawings or objects.

Strand: MEASUREMENT AND DATA (2.MD)

Measure and estimate lengths in standard units **(Standards 2.MD.1–4)** and relate addition and subtraction to length **(Standards 2.MD.5–6).** Work with time and money **(Standards 2.MD.7–8)**. Represent and interpret data **(Standards 2.MD.9–10)**.

- **Standard 2.MD.1** Measure the length of an object by selecting and using appropriate tools such as rulers, yardsticks, meter sticks, and measuring tapes.
- **Standard 2.MD.2** Measure the length of an object twice, using length units of different lengths for the two measurements; describe how the two measurements relate to the size of the unit chosen.
- **Standard 2.MD.3** Estimate lengths using units of inches, feet, centimeters, and meters.

- **Standard 2.MD.4** Measure to determine how much longer one object is than another, expressing the length difference in terms of a standard length unit. *For example, after measuring a pencil and a crayon, a student uses the measurements to determine that the pencil is two inches longer than the crayon.*

- **Standard 2.MD.5** Use addition and subtraction within 100 to solve word problems involving lengths that are given in the same units. *For example, use drawings (such as drawings of rulers) and equations with a symbol for the unknown number to represent the problem.*

- **Standard 2.MD.6** Represent whole numbers as lengths from 0 on a number line diagram with equally spaced points corresponding to the numbers 0, 1, 2… Represent whole-number sums and differences within 100 on a number line diagram.

- **Standard 2.MD.7** Tell and write time from analog and digital clocks to the nearest five minutes, using a.m. and p.m.

- **Standard 2.MD.8** Solve word problems involving dollar bills, quarters, dimes, nickels, and pennies, using $ and ¢ symbols appropriately. *For example, if you have 2 dimes and 3 pennies, how many cents do you have?*

- **Standard 2.MD.9** Generate measurement data by measuring lengths of several objects to the nearest whole unit, or by making repeated measurements of the same object. Show the measurements by making a line plot, where the horizontal scale is marked off in whole-number units.

- **Standard 2.MD.10** Draw a picture graph and a bar graph (with single-unit scale) to represent a data set with up to four categories. Solve simple put-together, take-apart, and comparison problems using information presented in a bar graph.

Strand: GEOMETRY (2.G)

Reason with shapes and their attributes **(Standards 2.G.1–3).**

- **Standard 2.G.1** Recognize and draw shapes having specified attributes, such as a given number of angles or a given number of equal faces. Sizes are compared directly or visually, not compared by measuring. Identify triangles, quadrilaterals, pentagons, hexagons, and cubes.

- **Standard 2.G.2** Partition a rectangle into rows and columns of same-size squares and count to find the total number of squares.

- **Standard 2.G.3** Partition circles and rectangles into two, three, or four equal shares; describe the shares using the words *halves, thirds, half of, a third of,* etc.; and describe the whole as two halves, three thirds, or four fourths. Recognize that equal shares of identical wholes need not have the same shape.

Mathematics | Grade 3

In Grade 3, instructional time should focus on four critical areas: (1) developing understanding of multiplication and division and strategies for multiplication and division within 100; (2) developing understanding of fractions, especially unit fractions (fractions with numerator 1); (3) developing understanding of the structure of rectangular arrays and of area; and (4) describing and analyzing two-dimensional shapes.

(1) Students will develop an understanding of the meanings of multiplication and division of whole numbers through activities and problems involving equal-sized groups, arrays, and area models; multiplication is finding an unknown product, and division is finding an unknown factor in these situations. For equal-sized group situations, division can require finding the unknown number of groups or the unknown group size. Students will use properties of operations to calculate products of whole numbers, using increasingly sophisticated strategies based on these properties to solve multiplication and division problems involving single-digit factors. By comparing a variety of solution strategies, students learn the relationship between multiplication and division.

(2) Students will develop an understanding of fractions, beginning with unit fractions. Students will view fractions in general as being built out of unit fractions, and they will use fractions along with visual fraction models to represent parts of a whole. Students understand that the size of a fractional part is relative to the size of the whole. *For example, 1/2 of the paint in a small bucket could be less paint than 1/3 of the paint in a larger bucket, but 1/3 of a ribbon is longer than 1/5 of the same ribbon because when the ribbon is divided into 3 equal parts, the parts are longer than when the ribbon is divided into 5 equal parts.* Students will be able to use fractions to represent numbers equal to, less than, and greater than one. They will solve problems that involve comparing fractions by using visual fraction models and strategies based on noticing equal numerators or denominators.

(3) Students will recognize area as an attribute of two-dimensional regions. They will measure the area of a shape by finding the total number of same-size units of area required to cover the shape without gaps or overlaps, a square with sides of unit length being the standard unit for measuring area. Students will understand that rectangular arrays can be decomposed into identical rows or into identical columns. By decomposing rectangles into rectangular arrays of squares, students will connect area to multiplication and justify using multiplication to determine the area of a rectangle.

(4) Students will describe, analyze, and compare properties of two-dimensional shapes. They will compare and classify shapes by their sides and angles, and connect these with definitions of shapes. Students will also relate their fraction work to geometry by expressing the area of part of a shape as a unit fraction of the whole.

Strand: MATHEMATICAL PRACTICES (3.MP)

The Standards for Mathematical Practice in Third Grade describe mathematical habits of mind that teachers should seek to develop in their students. Students become mathematically proficient in engaging with mathematical content and concepts as they learn, experience, and apply these skills and attitudes **(Standards 3.MP.1–8).**

- **Standard 3.MP.1 Make sense of problems and persevere in solving them.** Explain the meaning of a problem, look for entry points to begin work on the problem, and plan and choose a solution pathway. When a solution pathway does not make sense, look for another pathway that does. Explain connections between various solution strategies and representations. Upon finding a solution, look back at the problem to determine whether the solution is reasonable and accurate, often checking answers to problems using a different method or approach.

- **Standard 3.MP.2 Reason abstractly and quantitatively.** Make sense of quantities and their relationships in problem situations. Contextualize quantities and operations by using images or stories. Decontextualize a given situation and represent it symbolically. Interpret symbols as having meaning, not just as directions to carry out a procedure. Know and flexibly use different properties of operations, numbers, and geometric objects.

- **Standard 3.MP.3 Construct viable arguments and critique the reasoning of others.** Use stated assumptions, definitions, and previously established results to construct arguments. Explain and justify the mathematical reasoning underlying a strategy, solution, or conjecture by using concrete referents such as objects, drawings, diagrams, and actions. Listen to or read the arguments of others, decide whether they make sense, ask useful questions to clarify or improve the arguments, and build on those arguments.

- **Standard 3.MP.4 Model with mathematics.** Identify the mathematical elements of a situation and create a mathematical model that shows the relationships among them. Identify important quantities in a contextual situation, use mathematical models to show the relationships of those quantities, analyze the relationships, and draw conclusions. Models may be verbal, contextual, visual, symbolic, or physical.

- **Standard 3.MP.5 Use appropriate tools strategically.** Consider the tools that are available when solving a mathematical problem, whether in a real-world or mathematical context. Choose tools that are relevant and useful to the problem at hand, such as drawings, diagrams, technologies, and physical objects and tools, as well as mathematical tools such as estimation or a particular strategy or algorithm.

- **Standard 3.MP.6 Attend to precision.** Communicate precisely to others by crafting careful explanations that communicate mathematical reasoning by referring specifically to each important mathematical element, describing the relationships among them, and connecting their words clearly to representations. Calculate accurately and efficiently, and use clear and concise notation to record work.

- **Standard 3.MP.7 Look for and make use of structure.** Recognize and apply the structures of mathematics such as patterns, place value, the properties of operations, or the flexibility of numbers. See complicated things as single objects or as being composed of several objects.

- **Standard 3.MP.8 Look for and express regularity in repeated reasoning.** Notice repetitions in mathematics when solving multiple related problems. Use observations and reasoning to find shortcuts or generalizations. Evaluate the reasonableness of intermediate results.

Strand: OPERATIONS AND ALGEBRAIC THINKING (3.OA)

Represent and solve problems involving multiplication and division within 100 **(Standards 3.OA.1–4 and Standard 3.OA.7).** Demonstrate understanding of the properties of multiplication and the relationship between multiplication and division **(Standards 3.OA.5–6).** Use the four operations to identify and explain patterns in arithmetic **(Standards 3.OA.8–9).**

- **Standard 3.OA.1** Interpret the products of whole numbers, such as interpreting 5×7 as the total number of objects in 5 groups of 7 objects each. *For example, describe a context in which a total number of objects can be expressed as 5×7.*

- **Standard 3.OA.2** Interpret whole-number quotients of whole numbers. *For example, interpret 56 ÷ 8 as the number of objects in each share when 56 objects are partitioned equally into eight shares (partitive), or as a number of shares when 56 objects are partitioned into equal shares of eight objects each (quotative).*

- **Standard 3.OA.3** Use multiplication and division within 100 to solve word problems in situations involving equal groups, arrays, and measurement quantities. *For example, use drawings and equations with a symbol for the unknown number to represent the problem.*

- **Standard 3.OA.4** Determine the unknown whole number in a multiplication or division equation relating three whole numbers. *For example, determine the unknown number—product, factor, quotient, dividend, or divisor—that makes the equation true in each of the equations 8 x ? = 48, 5 = ? ÷ 3, and 6 x 6 = ?*

- **Standard 3.OA.5** Apply properties of operations as strategies to multiply and divide. *For example: If 6 x 4 = 24 is known, then 4 x 6 = 24 is also known (commutative property of multiplication). 3 x 5 x 2 can be found by 3 x 5 = 15, then 15 x 2 = 30, or by 5 x 2 = 10, then 3 x 10 = 30 (associative property of multiplication). Knowing that 8 x 5 = 40 and 8 x 2 = 16, one can find 8 x 7 as 8 x (5 + 2) = (8 x 5) + (8 x 2) = 40 + 16 = 56 (distributive property).* (Third grade students may, but need not, use formal terms for these properties.)

- **Standard 3.OA.6** Understand division as an unknown-factor problem. Understand the relationship between multiplication and division (multiplication and division are inverse operations). *For example, find 32 ÷ 8 by finding the number that makes 32 when multiplied by 8.*

- **Standard 3.OA.7** Fluently multiply and divide.
  - **a.** Fluently multiply and divide within 100, using strategies such as the relationship between multiplication and division or properties of operations. *(For example, knowing that 8 x 5 = 40, one knows 40 ÷ 5 = 8.)*
  - **b.** By the end of Grade 3, know from memory all products of two one-digit numbers.
- **Standard 3.OA.8** Solve two-step word problems.
  - **a.** Solve two-step word problems using the four operations. Know how to perform operations in the conventional order when there are no parentheses to specify a particular order (Order of Operations). (Limit to problems posed with whole numbers and having whole number answers.)
  - **b.** Represent two-step problems using equations with a letter standing for the unknown quantity. Create accurate equations to match word problems.
  - **c.** Assess the reasonableness of answers using mental computation and estimation strategies, including rounding.
- **Standard 3.OA.9** Identify arithmetic patterns (including patterns in the addition table or multiplication table), and explain them using properties of operations. *For example, observe that four times a number is always even, and explain why four times a number can be decomposed into two equal addends.*

Strand: NUMBER AND OPERATIONS IN BASE TEN (3.NBT)

Use place value understanding and properties of operations to perform multi-digit arithmetic. A range of algorithms may be used **(Standards 3.NBT.1–3)**.

- **Standard 3.NBT.1** Use place value understanding to round whole numbers to the nearest 10 or 100.
- **Standard 3.NBT.2** Fluently add and subtract within 1,000 using strategies and algorithms based on place value, properties of operations, and/or the relationship between addition and subtraction.
- **Standard 3.NBT.3** Multiply one-digit whole numbers by multiples of 10 in the range 10–90 *(for example, 9 x 80 and 5 x 60)* using strategies based on place value and properties of operations.

Strand: NUMBER AND OPERATIONS—FRACTIONS (3.NF)

Develop understanding of fractions as numbers. Denominators are limited to 2, 3, 4, 6, and 8 in third grade.

- **Standard 3.NF.1** Understand that a unit fraction has a numerator of one and a non-zero denominator.

a. Understand a fraction 1/*b* as the quantity formed by one part when a whole is partitioned into *b* equal parts.

b. Understand a fraction *a/b* as the quantity formed by a parts of size 1/*b*. *For example: 1/4 + 1/4 + 1/4 = 3/4.*

■ **Standard 3.NF.2** Understand a fraction as a number on the number line; represent fractions on a number line diagram.

a. Represent a fraction 1/*b* on a number line diagram by defining the interval from 0 to 1 as the whole and partitioning it into *b* equal parts. Recognize that each part has size 1/*b* and that the endpoint of the part based at 0 locates the number 1/*b* on the number line.

b. Represent a fraction *a/b* on a number line diagram by marking off *a* lengths 1/*b* from 0. Recognize that the resulting interval has size *a/b* and that its endpoint locates the number *a/b* on the number line.

■ **Standard 3.NF.3** Explain equivalence of fractions in special cases, and compare fractions by reasoning about their size.

a. Understand two fractions as equivalent if they are the same size, or the same point on a number line.

b. Recognize and generate simple equivalent fractions, such as 1/2 = 2/4, 4/6 = 2/3. *Explain why the fractions are equivalent by using a visual fraction model, for example*.

c. Express whole numbers as fractions, and recognize fractions that are equivalent to whole numbers. *For example, express 3 in the form 3 = 3/1; recognize that 6/1 = 6; locate 4/4 and 1 at the same point of a number line diagram.*

d. Compare two fractions with the same numerator or the same denominator by reasoning about their size. Recognize that comparisons are valid only when the two fractions refer to the same whole. *Record the results of comparisons with the symbols >, =, or <, and justify the conclusions, for example, by using a visual fraction model.*

Strand: MEASUREMENT AND DATA (3.MD)

Solve problems involving measurement and estimation of intervals of time, liquid volumes, and masses of objects. **(Standards 3.MD.1–2).** Represent and interpret data **(Standards 3.MD.3–4).** Understand concepts of area and relate area to multiplication and addition **(Standards 3.MD.5–7).** Recognize perimeter as an attribute of plane figures and distinguish between linear and area measures **(Standard 3.MD.8).**

■ **Standard 3.MD.1** Tell and write time to the nearest minute and measure time intervals in minutes. Solve word problems involving addition and subtraction of time intervals in minutes, for example, by representing the problem on a number line diagram.

■ **Standard 3.MD.2** Measure and estimate liquid volumes and masses of objects using standard units of grams (g), kilograms (kg), milliliters (ml), and liters (l). (Excludes compound

units such as cubic centimeters [cc or cm3] and finding the geometric volume of a container.) *Add, subtract, multiply, or divide to solve one-step word problems involving masses of objects or volumes of liquids that are given in the same units, for example, by using drawings (such as a beaker with a measurement scale) to represent the problem. (Excludes multiplicative comparison problems.)*

- **Standard 3.MD.3** Draw a scaled picture graph and a scaled bar graph to represent a data set with several categories. Solve one- and two-step "how many more" and "how many less" problems using information presented in scaled bar graphs. *For example, draw a bar graph in which each square in the bar graph might represent five pets.*

- **Standard 3.MD.4** Generate measurement data by measuring lengths using rulers marked with halves and fourths of an inch. Show the data by making a line plot where the horizontal scale is marked off in appropriate units-whole numbers, halves, or quarters.

- **Standard 3.MD.5** Recognize area as an attribute of plane figures and understand concepts of area measurement.

  a. A square with side length one unit, called "a unit square," is said to have "one square unit" of area, and can be used to measure area.

  b. A plane figure which can be covered without gaps or overlaps by *n* unit squares is said to have an area of *n* square units.

- **Standard 3.MD.6** Measure area by counting unit squares (square centimeters, square meters, square inches, square feet, and improvised units).

- **Standard 3.MD.7** Relate area to the operations of multiplication and addition (refer to 3.OA.5).

  a. Find the area of a rectangle with whole-number side lengths by tiling it, and show that the area is the same as would be found by multiplying the side lengths.

  b. Multiply side lengths to find areas of rectangles with whole-number side lengths in the context of solving real-world and mathematical problems, and represent whole-number products as rectangular areas in mathematical reasoning.

  c. Use tiling to show in a concrete case that the area of a rectangle with whole-number side lengths a and $b + c$ is the sum of $a \times b$ and $a \times c$. Use area models to represent the distributive property in mathematical reasoning.

  d. Recognize area as additive. Find areas of rectilinear figures by decomposing them into non- overlapping rectangles and adding the areas of the non-overlapping parts, applying this technique to solve real-world problems.

- **Standard 3.MD.8** Solve real-world and mathematical problems involving perimeters of polygons, including finding the perimeter given the side lengths, finding an unknown side length, and exhibiting rectangles with the same perimeter and different areas or with the same area and different perimeters.

Strand: GEOMETRY (3.G)

Reason with shapes and their attributes **(Standards 3.G.1–2).**

- **Standard 3.G.1** Understand that shapes in different categories *(for example, rhombuses, rectangles, and others)* may share attributes *(for example, having four sides)*, and that the shared attributes can define a larger category *(for example, quadrilaterals)*. Recognize rhombuses, rectangles, and squares as examples of quadrilaterals, and draw examples of quadrilaterals that do not belong to any of these subcategories.

- **Standard 3.G.2** Partition shapes into parts with equal areas. Express the area of each part as a unit fraction of the whole. *For example, partition a shape into four parts with equal area, and describe the area of each part as 1/4 of the area of the shape.*

Mathematics | Grade 4

In Grade 4, instructional time should focus on three critical areas: (1) developing understanding and fluency with multi-digit multiplication, and developing understanding of dividing to find quotients involving multi-digit dividends; (2) developing an understanding of fraction equivalence, addition and subtraction of fractions with like denominators, and multiplication of fractions by whole numbers; and (3) understanding that geometric figures can be analyzed and classified based on their properties, such as having parallel sides, perpendicular sides, particular angle measures, and symmetry.

(1) Students will generalize their understanding of place value to 1,000,000, understanding the relative sizes of numbers in each place. They will apply their understanding of models for multiplication (equal-sized groups, arrays, area models), place value, and properties of operations, in particular the distributive property, as they develop, discuss, and use efficient, accurate, and generalizable methods to compute products of multi-digit whole numbers. Depending on the numbers and the context, they will select and accurately apply appropriate methods to estimate or mentally calculate products. They will develop fluency with efficient procedures for multiplying whole numbers, understand and explain why the procedures work based on place value and properties of operations, and use them to solve problems. Students will apply their understanding of models for division, place value, properties of operations, and the relationship of division to multiplication as they develop, discuss, and use efficient, accurate, and generalizable procedures to find quotients involving multi-digit dividends. They will select and accurately apply appropriate methods to estimate and mentally calculate quotients, and interpret remainders based upon the context.

(2) Students will develop understanding of fraction equivalence and operations with fractions. They will recognize that two different fractions can be equal *(for example, 15/9 = 5/3)*, and they will develop methods for generating and recognizing equivalent fractions. Students will extend previous understandings about how fractions are built from unit fractions, composing fractions from unit fractions, decomposing fractions into unit fractions, and using the meaning of fractions and the meaning of multiplication to multiply a fraction by a whole number.

(3) Students will describe, analyze, compare, and classify two-dimensional shapes. Through building, drawing, and analyzing two-dimensional shapes, students will deepen their understanding of properties of two-dimensional objects and their use in solving problems involving symmetry.

Mathematical PRACTICES (4.MP)

The Standards for Mathematical Practice in Fourth Grade describe mathematical habits of mind that teachers should seek to develop in their students. Students become mathematically proficient in engaging with mathematical content and concepts as they learn, experience, and apply these skills and attitudes **(Standards 4.MP.1–8).**

- **Standard 4.MP.1 Make sense of problems and persevere in solving them.** Explain the meaning of a problem, look for entry points to begin work on the problem, and plan and choose a solution pathway. When a solution pathway does not make sense, look for another pathway that does. Explain connections between various solution strategies and representations. Upon finding a solution, look back at the problem to determine whether the solution is reasonable and accurate, often checking answers to problems using a different method or approach.

- **Standard 4.MP.2 Reason abstractly and quantitatively.** Make sense of quantities and their relationships in problem situations. Contextualize quantities and operations by using images or stories. Decontextualize a given situation and represent it symbolically. Interpret symbols as having meaning, not just as directions to carry out a procedure. Know and flexibly use different properties of operations, numbers, and geometric objects.

- **Standard 4.MP.3 Construct viable arguments and critique the reasoning of others.** Use stated assumptions, definitions, and previously established results to construct arguments. Explain and justify the mathematical reasoning underlying a strategy, solution, or conjecture by using concrete referents such as objects, drawings, diagrams, and actions. Listen to or read the arguments of others, decide whether they make sense, ask useful questions to clarify or improve the arguments, and build on those arguments.

- **Standard 4.MP.4 Model with mathematics.** Identify the mathematical elements of a situation and create a mathematical model that shows the relationships among them. Identify important quantities in a contextual situation, use mathematical models to show the relationships of those quantities, analyze the relationships, and draw conclusions. Models may be verbal, contextual, visual, symbolic, or physical.

- **Standard 4.MP.5 Use appropriate tools strategically.** Consider the tools that are available when solving a mathematical problem, whether in a real-world or mathematical context. Choose tools that are relevant and useful to the problem at hand, such as drawings, diagrams, technologies, and physical objects and tools, as well as mathematical tools such as estimation or a particular strategy or algorithm.

- **Standard 4.MP.6 Attend to precision.** Communicate precisely to others by crafting careful explanations that communicate mathematical reasoning by referring specifically to each important mathematical element, describing the relationships among them, and connecting their words clearly to representations. Calculate accurately and efficiently, and use clear and concise notation to record work.

- **Standard 4.MP.7 Look for and make use of structure.** Recognize and apply the structures of mathematics such as patterns, place value, the properties of operations, or the flexibility of numbers. See complicated things as single objects or as being composed of several objects.

- **Standard 4.MP.8 Look for and express regularity in repeated reasoning.** Notice repetitions in mathematics when solving multiple related problems. Use observations and reasoning to find shortcuts or generalizations. Evaluate the reasonableness of intermediate results.

Strand: OPERATIONS AND ALGEBRAIC THINKING (4.OA)

Use the four operations with whole numbers (addition, subtraction, multiplication, and division) to solve problems **(Standards 4.OA.1–3).** Gain familiarity with factors and multiples **(Standard 4.OA.4).** Generate and analyze numeric and shape patterns **(Standard 4.OA.5).** Demonstrate complete fluency with products of one-digit numbers **(Standard 4.OA.6).**

- **Standard 4.OA.1** Interpret a multiplication equation as a comparison *(for example, interpret 35 = 5 x 7 as a statement that 35 is 5 times as many as 7 and 7 times as many as 5)*. Represent verbal statements of multiplicative comparisons as multiplication equations.

- **Standard 4.OA.2** Multiply or divide to solve word problems involving multiplicative comparison, *for example, by using drawings and equations with a symbol for the unknown number to represent the problem, distinguishing multiplicative comparison from additive comparison*.

- **Standard 4.OA.3** Solve multi-step word problems posed with whole numbers and having whole-number answers using the four operations, including problems in which remainders must be interpreted.

  a. Represent these problems using equations with a letter standing for the unknown quantity.

  b. Assess the reasonableness of answers using mental computation and estimation strategies, including rounding.

- **Standard 4.OA.4** Find all factor pairs for a whole number in the range 1-100. Recognize that a whole number is a multiple of each of its factors. Determine whether a given whole number in the range 1-100 is a multiple of a given one-digit number. Determine whether a given whole number in the range 1-100 is prime or composite.

- **Standard 4.OA.5** Generate a number or shape pattern that follows a given rule. Identify apparent features of the pattern that were not explicit in the rule itself. *For example, given the rule "add 3" and the starting number 1, generate terms in the resulting sequence and observe that the terms appear to alternate between odd and even numbers. Explain informally why the numbers will continue to alternate in this way.*

Strand: NUMBER AND OPERATIONS IN BASE TEN (4.NBT)

Generalize place value understanding for multi-digit whole numbers by analyzing patterns, writing whole numbers in a variety of ways, making comparisons, and rounding **(Standards 4.NBT.1–3).** Use place value understanding and properties of operations to perform multi-digit addition, subtraction, multiplication, and division using a one-digit divisor **(Standards 4.NBT.4–6).** Expectations in this strand are limited to whole numbers less than or equal to 1,000,000.

- **Standard 4.NBT.1** Recognize that in a multi-digit whole number, a digit in one place represents ten times what it represents in the place to its right. *For example, recognize that 700 ÷ 70 = 10 by applying concepts of place value and division.*
- **Standard 4.NBT.2** Read and write multi-digit whole numbers using base-ten numerals, number names, and expanded form. Compare two multi-digit numbers based on meanings of the digits in each place, using >, =, and < symbols to record the results of comparisons.
- **Standard 4.NBT.3** Use place value understanding to round multi-digit whole numbers to any place.
- **Standard 4.NBT.4** Fluently add and subtract multi-digit whole numbers using the standard algorithm.
- **Standard 4.NBT.5** Multiply a whole number of up to four digits by a one-digit whole number, and multiply two two-digit numbers, using strategies based on place value and the properties of operations. Illustrate and explain the calculation by using equations, rectangular arrays, and/or area models.
- **Standard 4.NBT.6** Find whole-number quotients and remainders with up to four-digit dividends and one-digit divisors, using strategies based on place value, the properties of operations, and/or the relationship between multiplication and division. Illustrate and explain the calculation by using equations, rectangular arrays, and/or area models.

Strand: NUMBER AND OPERATIONS—FRACTIONS (4.NF)

Extend understanding of equivalence and ordering of fractions **(Standards 4.NF.1–2).** Build fractions from unit fractions by applying and extending previous understandings of operations on whole numbers **(Standards 4.NF.3–4).** Understand decimal notation to the hundredths and compare decimal fractions with denominators of 10 and 100 **(Standards 4.NF.5–7).** Denominators for fourth grade are limited to 2, 3, 4, 5, 6, 8, 10, 12, and 100.

- **Standard 4.NF.1** Explain why a fraction a/b is equivalent to a fraction (n x a)/(n x b) by using visual fraction models, with attention to how the number and size of the parts differ even though the two fractions themselves are the same size. Use this principle to recognize and generate equivalent fractions.
- **Standard 4.NF.2** Compare two fractions with different numerators and different denominators, *for example, by creating common denominators or numerators, or by comparing to a*

benchmark fraction such as 1/2. Recognize that comparisons are valid only when the two fractions refer to the same whole. Record the results of comparisons with symbols >, =, or <, and justify the conclusions, *for example, by using a visual fraction model*.

■ **Standard 4.NF.3** Understand a fraction *a/b* with *a* >1 as a sum of fractions 1/*b*. In other words, any fraction is a sum of unit fractions.

a. Understand addition and subtraction of fractions as joining and separating parts referring to the same whole.

b. Decompose a fraction into a sum of fractions with the same denominator in more than one way, recording each decomposition by an equation. Justify decompositions, *for example, by using a visual fraction model*. *For example, 3/8 = 1/8 + 1/8 + 1/8; 3/8 = 1/8 + 2/8; 2 1/8 = 1 + 1 + 1/8; 2 1/8 = 8/8 + 8/8 + 1/8.*

c. Add and subtract mixed numbers with like denominators, *for example, by replacing each mixed number with an equivalent fraction, and/or by using properties of operations and the relationship between addition and subtraction*. *For example, 3 1/4 + 2 1/4 = 13/4 + 9/4 = 22/4; 3 1/4 + 2 1/4 = (3+ 2) + (1/4 + 1/4) = 5 + 2/4 = 5 2/4, which is equivalent to 22/4.*

d. Solve word problems involving addition and subtraction of fractions referring to the same whole and having like denominators, *for example, by using visual fraction models and equations to represent the problem*.

■ **Standard 4.NF.4** Apply and extend previous understandings of multiplication to multiply a fraction by a whole number.

a. Understand a fraction *a/b* as a multiple of 1/*b*. *For example, use a visual fraction model to represent 5/4 as the product 5 x (1/4), recording the conclusion by the equation 5/4 = 5 x (1/4).*

b. Understand a multiple of *a/b* as a multiple of 1/*b*, and use this understanding to multiply a fraction by a whole number. *For example, use a visual fraction model to express 3 x (2/5) as 6 x (1/5), recognizing this product as 6/5. (In general,* n *x (*a/b*) = (*n x a*)/*b*).*

c. Solve word problems involving multiplication of a fraction by a whole number (*for example, by using visual fraction models and equations to represent the problem*). *For example, if each person at a party will eat 3/8 of a pound of roast beef, and there will be five people at the party, how many pounds of roast beef will be needed? Between what two whole numbers does your answer lie?*

■ **Standard 4.NF.5** Express a fraction with denominator 10 as an equivalent fraction with denominator 100, and use this technique to add two fractions with respective denominators 10 and 100. *For example, express 3/10 as 30/100, and add 3/10 + 4/100 = 34/100.*

■ **Standard 4.NF.6** Use decimal notation for fractions with denominators 10 or 100. *For example, rewrite 0.62 as 62/100, describe a length as 0.62 meters; locate 0.62 on a number line diagram.*

■ **Standard 4.NF.7** Compare two decimals to hundredths by reasoning about their size. Recognize that comparisons are valid only when the two decimals refer to the same whole. Record the results of comparisons with the symbols >, =, or <, and justify the conclusions, *for example, by using a visual model*.

Strand: MEASUREMENT AND DATA (4.MD)

Solve problems involving measurement and conversion of measurements from a larger unit to a smaller unit **(Standards 4.MD.1–2).** Apply knowledge of area and perimeter to solve real-world and mathematical problems **(Standard 4.MD.3).** Represent and interpret data through the use of a line plot **(Standard 4.MD.4).** Understand various concepts of angles and angle measurement **(Standard 4.MD.5–7).**

■ **Standard 4.MD.1** Know relative sizes of measurement units within each system of units (standard and metric), including kilometers, meters, and centimeters; liters and milliliters; kilograms and grams; pounds and ounces; hours, minutes, and seconds. Within a single system of measurement, express measurements in a larger unit in terms of a smaller unit. Record measurement equivalents in a two-column table. *For example, know that one foot is 12 times as long as one inch. Express the length of a four-foot snake as 48 inches. Know that one meter is 100 times as long as one centimeter. Generate a conversion table for feet and inches listing the number pairs (1, 12), (2, 24), (3, 36)…*

■ **Standard 4.MD.2** Use the four operations to solve word problems involving distances, intervals of time, liquid volumes, masses of objects, and money.

- **a.** Include problems involving simple fractions or decimals, and problems that require expressing measurements given in a larger unit in terms of a smaller unit.
- **b.** Represent measurement quantities using diagrams such as number line diagrams that feature a measurement scale.

■ **Standard 4.MD.3** Apply the area and perimeter formulas for rectangles in real-world and mathematical problems. *For example, find the width of a rectangular room given the area of the flooring and the length, by viewing the area formula as a multiplication equation with an unknown factor.*

■ **Standard 4.MD.4** Make a line plot to display a data set of measurements in fractions of a unit (halves, quarters, and eighths). Solve problems involving addition and subtraction with like denominators of fractions by using information presented in line plots. *For example, use a line plot to find and interpret the difference in length between the longest and shortest pencils in a classroom.*

■ **Standard 4.MD.5** Recognize angles as geometric figures that are formed wherever two rays share a common endpoint, and understand concepts of angle measurement.

- **a.** Understand that an angle is measured with reference to a circle with its center at the common endpoint of the rays, by considering the fraction of the circular arc between the points where the two rays intersect the circle. An angle that turns through 1/360

of a circle is called a "one-degree angle," and can be used to measure other angles.

b. Understand that an angle that turns through *n* one-degree angles is said to have an angle measure of n degrees.

- **Standard 4.MD.6** Measure angles in whole-number degrees using a protractor. Sketch angles of specified measure.
- **Standard 4.MD.7** Recognize angle measure as additive.
  - **a.** Understand that when an angle is decomposed into non-overlapping parts, the angle measure of the whole is the sum of the angle measures of the parts.
  - **b.** Solve addition and subtraction problems to find unknown angles on a diagram in real-world and mathematical problems, *for example by using an equation with a symbol for the unknown angle measure*.

Strand: GEOMETRY (4.G)

Draw and identify lines and angles, as well as classify shapes by properties of their lines and angles **(Standards 4.G.1–3).**

- **Standard 4.G.1** Draw points, lines, line segments, rays, angles (right, acute, and obtuse), and perpendicular and parallel lines. Identify these in two-dimensional figures.
- **Standard 4.G.2** Classify two-dimensional figures based on the presence or absence of parallel or perpendicular lines, or the presence or absence of angles of a specified size. Recognize right triangles as a category, and identify right triangles.
- **Standard 4.G.3** Recognize a line of symmetry for a two-dimensional figure as a line across the figure such that the figure can be folded along the line into matching parts. Identify line-symmetric figures and draw lines of symmetry.

Mathematics | Grade 5

In Grade 5, instructional time should focus on three critical areas: (1) developing fluency with addition and subtraction of fractions, and developing understanding of the multiplication of fractions and of division of fractions in limited cases (unit fractions divided by whole numbers and whole numbers divided by unit fractions); (2) extending division to two-digit divisors, integrating decimal fractions into the place value system and developing understanding of operations with decimals to hundredths, and developing fluency with whole number and decimal operations; and (3) developing understanding of volume.

(1) Students will apply their understanding of fractions and fraction models to represent the addition and subtraction of fractions with unlike denominators as equivalent calculations with like denominators. They will develop fluency in calculating sums and differences of fractions, and make reasonable estimates of them. Students will also use the meaning of fractions, of multiplication and division, and the relationship between multiplication and division to understand and explain why the procedures for multiplying and dividing fractions make sense. (Note: This is limited to the case of dividing unit fractions by whole numbers and whole numbers by unit fractions.)

(2) Students will develop understanding of why division procedures work based on the meaning of base-ten numerals and properties of operations. They will finalize fluency with multi-digit addition, subtraction, multiplication, and division. They will apply their understandings of models for decimals, decimal notation, and properties of operations to add and subtract decimals to hundredths. They will develop fluency in these computations, and make reasonable estimates of their results. Students will use the relationship between decimals and fractions, as well as the relationship between finite decimals and whole numbers (e.g., a finite decimal multiplied by an appropriate power of 10 is a whole number), to understand and explain why the procedures for multiplying and dividing finite decimals make sense. They will compute products and quotients of decimals to hundredths efficiently and accurately.

(3) Students will recognize volume as an attribute of three-dimensional space. They will understand that volume can be measured by finding the total number of same-size units of volume required to fill the space without gaps or overlaps. They will understand that a one-unit-by-one-unit-by-one-unit cube is the standard unit for measuring volume. They will select appropriate units, strategies, and tools for solving problems that involve estimating and measuring volume. They will decompose three-dimensional shapes and find volumes of right rectangular prisms by viewing them as decomposed into layers of arrays of cubes. They will measure necessary attributes of shapes in order to determine volumes to solve real-world and mathematical problems.

Strand: MATHEMATICAL PRACTICES (5.MP)

The Standards for Mathematical Practice in Fifth Grade describe mathematical habits of mind that teachers should seek to develop in their students. Students become mathematically proficient in engaging with mathematical content and concepts as they learn, experience, and apply these skills and attitudes.

- **Standard 5.MP.1 Make sense of problems and persevere in solving them.** Explain the meaning of a problem, look for entry points to begin work on the problem, and plan and choose a solution pathway. When a solution pathway does not make sense, look for another pathway that does. Explain connections between various solution strategies and representations. Upon finding a solution, look back at the problem to determine whether the solution is reasonable and accurate, often checking answers to problems using a different method or approach.

- **Standard 5.MP.2 Reason abstractly and quantitatively.** Make sense of quantities and their relationships in problem situations. Contextualize quantities and operations by using images or stories. Decontextualize a given situation and represent it symbolically. Interpret symbols as having meaning, not just as directions to carry out a procedure. Know and flexibly use different properties of operations, numbers, and geometric objects.

- **Standard 5.MP.3 Construct viable arguments and critique the reasoning of others.** Use stated assumptions, definitions, and previously established results to construct arguments. Explain and justify the mathematical reasoning underlying a strategy, solution, or conjecture by using concrete referents such as objects, drawings, diagrams, and actions. Listen to or read the arguments of others, decide whether they make sense, ask useful questions to clarify or improve the arguments, and build on those arguments.

- **Standard 5.MP.4 Model with mathematics.** Identify the mathematical elements of a situation and create a mathematical model that shows the relationships among them. Identify important quantities in a contextual situation, use mathematical models to show the relationships of those quantities, analyze the relationships, and draw conclusions. Models may be verbal, contextual, visual, symbolic, or physical.

- **Standard 5.MP.5 Use appropriate tools strategically.** Consider the tools that are available when solving a mathematical problem, whether in a real-world or mathematical context. Choose tools that are relevant and useful to the problem at hand, such as drawings, diagrams, technologies, and physical objects and tools, as well as mathematical tools such as estimation or a particular strategy or algorithm.

- **Standard 5.MP.6 Attend to precision.** Communicate precisely to others by crafting careful explanations that communicate mathematical reasoning by referring specifically to each important mathematical element, describing the relationships among them, and connecting their words clearly to representations. Calculate accurately and efficiently, and use clear and concise notation to record work.

- **Standard 5.MP.7 Look for and make use of structure.** Recognize and apply the structures of mathematics such as patterns, place value, the properties of operations, or the flexibility of numbers. See complicated things as single objects or as being composed of several objects.

- **Standard 5.MP.8 Look for and express regularity in repeated reasoning.** Notice repetitions in mathematics when solving multiple related problems. Use observations and reasoning to find shortcuts or generalizations. Evaluate the reasonableness of intermediate results.

Strand: OPERATIONS AND ALGEBRAIC THINKING (5.OA)

Write and interpret numerical expressions **(Standards 5.OA.1–2),** and analyze patterns and relationships **(Standard 5.OA.3).**

- **Standard 5.OA.1** Use parentheses, brackets, or braces in numerical expressions, and evaluate expressions with these symbols.

- **Standard 5.OA.2** Write and interpret simple numerical expressions.

  a. Write simple expressions that record calculations with numbers. *For example, use 2 x (8+7) to express the calculation "add 8 and 7, then multiply by 2."*

  b. Interpret numerical expressions without evaluating them. *For example, use conceptual understanding of multiplication to interpret 3 x (18939 + 921) as being three times as large as 18932 + 921 without calculating the indicated sum or product.*

- **Standard 5.OA.3** Generate two numerical patterns using two given rules. Identify apparent relationships between corresponding terms. Form ordered pairs consisting of corresponding terms from the two patterns, and graph the ordered pairs on a coordinate plane. *For example, given the rule "add 3" and the starting number 0, and given the rule "add 6" and the starting number 0, generate terms in the resulting sequences, and observe that the terms in one sequence are twice the corresponding terms in the other sequence. Explain informally why this is so.*

Strand: NUMBER AND OPERATIONS IN BASE TEN (5.NBT)

Understand the place value system **(Standards 5.NBT.1–4).** Perform operations with multi-digit whole numbers and with decimals to hundredths **(Standards 5.NBT.5–7).**

- **Standard 5.NBT.1** Recognize that in a multi-digit number, a digit in one place represents 10 times as much as it represents in the place to its right and 1/10 of what it represents in the place to its left.

- **Standard 5.NBT.2** Explain patterns in the number of zeros of the product when multiplying a number by powers of 10, and explain patterns in the placement of the decimal point when a decimal is multiplied or divided by a power of 10. Use whole-number exponents to denote powers of 10.

- **Standard 5.NBT.3** Read, write, and compare decimals to thousandths.
  - **a.** Read and write decimals to thousandths using base-ten numerals, number names, and expanded form. *For example, 347.392 = 3 x 100 + 4 x 10 + 7 x 1 + 3 x (1/10) + 9 x (1/100) + 2 x (1/1000).*
  - **b.** Compare two decimals to thousandths based on meanings of the digits in each place, using >, =, and < symbols to record the results of comparisons.
- **Standard 5.NBT.4** Use place value understanding to round decimals to any place.
- **Standard 5.NBT.5** Fluently multiply multi-digit whole numbers using the standard algorithm.
- **Standard 5.NBT.6** Find whole-number quotients of whole numbers with up to four-digit dividends and two-digit divisors, using strategies based on place value, the properties of operations, and/or the relationship between multiplication and division. Illustrate and explain the calculation by using equations, rectangular arrays, and/or area models.
- **Standard 5.NBT.7** Add, subtract, multiply, and divide decimals to hundredths, using concrete models or drawings and strategies based on place value, properties of operations, and/or the relationship between addition and subtraction; relate the strategy to a written method and explain the reasoning used. In this standard, dividing decimals is limited to a whole number dividend with a decimal divisor or a decimal dividend with a whole number divisor. Compare the value of the quotient on the basis of the values of the dividend and divisor.

Strand: NUMBER AND OPERATIONS—FRACTIONS (5.NF)

Use equivalent fractions as a strategy to add and subtract fractions **(Standards 5.NF.1–2).** Apply and extend previous understandings of multiplication and division to multiply and divide fractions **(Standards 5.NF.3–7).**

- **Standard 5.NF.1** Add and subtract fractions with unlike denominators (including mixed numbers) by replacing given fractions with equivalent fractions in such a way as to produce an equivalent sum or difference of fractions with like denominators. *For example, 2/3 + 5/4 = 8/12 + 15/12 = 23/12. (In general,* a/b + c/d = (ad + bc)/bd.*)*
- **Standard 5.NF.2** Solve real-world problems involving addition and subtraction of fractions referring to the same whole, including cases of unlike denominators by, *for example, using visual fraction models or equations to represent the problem*. Use benchmark fractions and number sense of fractions to estimate mentally and assess the reasonableness of answers. *For example, recognize 2/5 + 1/2 = 3/7 as an incorrect result, by observing that 3/7 < 1/2.*
- **Standard 5.NF.3** Interpret a fraction as division of the numerator by the denominator (*a/b = a ÷ b*). Solve real-world problems involving division of whole numbers leading to answers in the form of fractions or mixed numbers, through the use of visual fraction

models or equations to represent the problem. *For example, interpret 3/4 as the result of dividing three by four, noting that 3/4 multiplied by four equals three, and that when three wholes are shared equally among four people each person has a share of size 3/4. If nine people want to share a 50-pound sack of rice equally by weight, how many pounds of rice should each person get? Between what two whole numbers does your answer lie?*

- **Standard 5.NF.4** Apply and extend previous understandings of multiplication to multiply a fraction or whole number by a fraction.

  a. Interpret the product (*a/b*) x *q* as a parts of a partition of q into b equal parts; equivalently, as the result of a sequence of operations *a* x *q* ÷ *b* using a visual fraction model. *For example, use a fraction model to show (2/3) x 4 = 8/3, and create a story context for this equation. Do the same with (2/3) x (4/5) = 8/15. (In general,* (a/b) *x* (c/d) = ac/bd.*)*

  b. Find the area of a rectangle with fractional side lengths by tiling it with unit squares of the appropriate unit fraction side lengths, and show that the area is the same as would be found by multiplying the side lengths. Multiply fractional side lengths to find areas of rectangles, and represent fraction products as rectangular areas.

- **Standard 5.NF.5** Interpret multiplication as scaling.

  a. Compare the size of a product to the size of one factor on the basis of the size of the other factor, without performing the indicated multiplication. *For example, the products of expressions such as 5 x 3 or ½ x 3 can be interpreted in terms of a quantity, three, and a scaling factor, five or ½. Thus in addition to knowing that 5 x 3 = 15, they can also say that 5 x 3 is five times as big as three, without evaluating the product. Likewise they see ½ x 3 as half the size of three.*

  b. Explain why multiplying a given number by a fraction greater than one results in a product greater than the given number (recognizing multiplication by whole numbers greater than one as a familiar case); explain why multiplying a given number by a fraction less than one results in a product smaller than the given number; and relate the principle of fraction equivalence. *For example, 6/10 = (2x3)/(2x5). In general,* a/b = *(*n *x* a*)/(*n *x* b*) has the effect of multiplying* a/b *by one*.

- **Standard 5.NF.6** Solve real-world problems involving multiplication of fractions and mixed numbers, *for example, by using visual fraction models or equations to represent the problem*.

- **Standard 5.NF.7** Apply and extend previous understandings of division to divide unit fractions by whole numbers and whole numbers by unit fractions. Use strategies to divide fractions by reasoning about the relationship between multiplication and division. Division of a fraction by a fraction is not a requirement at this grade.

  a. Interpret division of a unit fraction by a non-zero whole number, and compute such quotients. *For example, create a story context for (1/3) ÷ 4, and use a visual fraction model to show the quotient. Use the relationship between multiplication and division to explain that (1/3) ÷ 4 = 1/12 because (1/12) x 4 = 1/3.*

b. Interpret division of a whole number by a unit fraction, and compute such quotients. *For example, create a story context for 4 ÷ (1/5), and use a visual fraction model to show the quotient. Use the relationship between multiplication and division to explain that 4 ÷ (1/5) = 20 because 20 x (1/5) = 4.*

c. Solve real-world problems involving division of unit fractions by non-zero whole numbers and division of whole numbers by unit fractions, *for example, by using visual fraction models and equations to represent the problem. For example, how much chocolate will each person get if three people share 1/2 lb. of chocolate equally? How many 1/3-cup servings are in two cups of raisins?*

Strand: MEASUREMENT AND DATA (5.MD)

Convert like measurement units within a given measurement system **(Standard 5.MD.1).** Represent and interpret data **(Standard 5.MD.2).** Understand concepts of geometric measurement and volume, as well as how multiplication and addition relate to volume **(Standard 5.MD.3).**

- **Standard 5.MD.1** Convert among different-sized standard measurement units within a given measurement system *(for example, convert 5 cm to 0.05 m)*; use these conversions in solving multi-step, real-world problems.
- **Standard 5.MD.2** Make a line plot to display a data set of measurements in fractions of a unit (halves, quarters, eighths). Use operations on fractions for this grade to solve problems involving information presented in line plots. *For example, given graduated cylinders with different measures of liquid in each, find the amount of liquid each cylinder would contain if the total amount in all the cylinders were redistributed equally.*
- **Standard 5.MD.3** Recognize volume as an attribute of solid figures and understand concepts of volume measurement.
  - **a.** A cube with side length one unit, called a "unit cube," is said to have "one cubic unit" of volume, and can be used to measure volume.
  - **b.** A solid figure which can be packed without gaps or overlaps using *n* unit cubes is said to have a volume of *n* cubic units.
- **Standard 5.MD.4** Measure volumes by counting unit cubes, using cubic cm, cubic in., cubic ft., and improvised units.
- **Standard 5.MD.5** Relate volume to the operations of multiplication and addition and solve real-world and mathematical problems involving volume.
  - **a.** Find the volume of a right rectangular prism with whole-number side lengths by packing it with unit cubes, and show that the volume is the same as would be found by multiplying the edge lengths, equivalently by multiplying the height by the area of the base. Represent threefold whole-number products as volumes, *for example, to represent the associative property of multiplication*.

b. Apply the formulas $V = l \times w \times h$ *and* $V = b \times h$ for rectangular prisms to find volumes of right rectangular prisms with whole-number edge lengths in the context of solving real-world and mathematical problems.

c. Recognize volume as additive. Find volumes of solid figures composed of two non-overlapping right rectangular prisms by adding the volumes of the non-overlapping parts, applying this technique to solve real-world problems.

Strand: GEOMETRY (5.G)

Graph points on the coordinate plane to solve real-world and mathematical problems in quadrant one **(Standards 5.G.1–2).** Classify two-dimensional figures into categories based on their properties. **(Standards 5.G.3–4).**

■ **Standard 5.G.1** Compose and understand the coordinate plane.

a. Use a pair of perpendicular number lines, called axes, to define a coordinate system, with the intersection of the lines (the origin) arranged to coincide with the zero on each line and a given point in the plane located by using an ordered pair of numbers, called its coordinates.

b. Using quadrant one on the coordinate plane, understand that the first number in a coordinate pair indicates how far to travel from the origin in the direction of the horizontal axis, and the second number indicates how far to travel in the direction of the vertical axis, with the convention that the names of the two axes and the coordinates correspond (*x*-axis and *x*-coordinate, *y*-axis and *y*-coordinate).

■ **Standard 5.G.2** Represent real-world and mathematical problems by graphing points in the first quadrant of the coordinate plane, and interpret coordinate values of points in the context of the situation.

■ **Standard 5.G.3** Understand that attributes belonging to a category of two-dimensional figures also belong to all subcategories of that category. *For example, all rectangles have four right angles and all squares are rectangles, so all squares have four right angles.*

■ **Standard 5.G.4** Classify two-dimensional figures in a hierarchy based on properties.

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