Utah State Board of Education
Elementary Mathematics Endorsement Course Framework
Mathematics for Teaching K-8 - Algebraic Reasoning

Course Description: To provide practicing teachers a deeper understanding of algebraic expressions, equations, functions, real numbers, and instructional strategies to facilitate the instruction of this content for elementary students.

Course Objectives:
During this course students will:

- Demonstrate in-depth knowledge of problem-solving and reasoning using algebraic notation and functions.
- Use a variety of tools, technology, and mathematical representations to explore and model algebra and functions concepts.
- Make connections among mathematical topics, concepts, and real-world situations.
- Communicate algebraic ideas orally, visually, and in writing, as well as facilitate effective discourse related to these topics in a positive mathematics learning environment.
- Examine the variety of ways in which children learn the fundamentals of algebra, functions, problem-solving strategies, and how they construct an understanding of generalization, and use of variables.

Topics:

Represent and Analyze Mathematical Situations and Structures Using Algebraic Symbols

- Understand the meaning of equivalent forms of expressions, equations, inequalities, and relations.
- Use symbolic algebra to represent and explain mathematical relationships.
- Judge the meaning, utility and reasonableness of the results of symbol manipulations, including those carried out by technology.
    - Factors and factoring of polynomials using area models and connecting to whole number multiplication.
    - Rational exponents and the laws of exponents.
    - Relate place value to polynomials.
    - Commutative, associative, and distributive properties.
    - Recognize the difference between an equation and an expression.

Real Number System

- Understand numbers, ways of representing numbers, relations among numbers, and numbers systems.
    - Fractional exponents.
    - Real number line.
    - Rational and irrational numbers.

Equations

- Write equivalent forms of equations, inequalities, and systems of equations and solve them with fluency – mentally or with paper and pencil in simple cases, using technology in all cases.
    - Solving linear equations.
    - Solving problems with linear equations and inequalities and their graphs.
    - Graphing and solving systems of linear equations and inequalities.
    - Quadratic equations.

Patterns and Functions

- Generalize patterns using explicitly defined and recursively defined functions.
- Understand relations and functions and select, convert flexibly among, and use various representations for them.
- Analyze functions of one variable by investigating rates of change, intercepts, zeros, asymptotes, and local and global behavior.
- Understand and perform transformations such as arithmetically combing, composing and inverting commonly used functions, using technology to perform such operation on more-complicated symbolic expressions.
- Understand and compare the properties of classes of functions.
- Identify essential quantitative relationships in a situation and determine the class or classes of functions that might model the relations.
- Draw reasonable conclusions about a situation being modeled.
    - Concept of variables.
    - Arithmetic and geometric sequences.
    - Function notation.
    - Observe a pattern and write expressions.
    - Predict and calculate rates of change.
    - Definition of function.
    - Linear functions and their graphs.
    - Quadratic functions.
    - Transformation of functions.
    - Inverses – square root functions.

Pedagogy

The purpose of the Elementary Mathematics Endorsement courses is to ensure that practicing teachers gain the mathematical content knowledge needed to teach mathematical concepts to students in the elementary grades. Teachers must also, however, know how to transfer that content knowledge and the conceptual understandings inherent to the content to students. An understanding of sound pedagogical practice is essential to that transfer. Following are suggested pedagogical concepts and strategies that should be infused into the courses to aid teachers in student instruction. These concepts should never be taught in isolation, but should be modeled throughout the courses. It is not necessary nor intended that all the following concepts would be infused in every course, but they should be covered in their entirety in the series.

Knowledge of mathematics learners:

- Employ prior knowledge, skills, dispositions, beliefs, conceptions, misconceptions and confusions around particular topics;
- Employ knowledge of cognition, child development, and learning theory;
- Use knowledge of learners culturally and socially to know what motivates and inspires them.

Knowledge of effective mathematics teaching practices for student and teacher learning:

- Design, select and/or adapt worthwhile tasks and sequences of examples that support a particular learning goal, including tasks to be solved by teachers themselves, and others for which they try to anticipate children's solutions, and design responses.
- Develop sensitivity with the careful use of technical language, attending to both mathematical integrity, and usability by learners.
- Construct and evaluate multiple representations of a mathematical idea or process, and establish correspondences between representations.
- Use questions to effectively probe mathematical understanding and make productive use of responses.
- Develop learners' skills at clear and coherent public mathematical communication in a classroom setting, using a variety of resources and media.
- Model mathematical practices – questioning, representing, communicating, problem solving, inquiring and conjecturing, making connections, reasoning and proving, self-assessing and cultivate the learning of such practices.
- Use various instructional applications of technology and be able to make judicious, mathematically and pedagogically grounded use of them.
- Construct and mathematically evaluate contexts or story problems for various mathematical situations and processes.
- Analyze and evaluate student work, and design appropriate responses.
- Develop skillful and flexible use of different instructional formats – whole group, small group, partner, and individual – in support of learning goals.

- Know the resources for managing the diversities – cultural, linguistic, gender, socioeconomic, developmental – of the classroom, and ways to skillfully deploy them in support of the learning of all students.
- Understand and address diversity in planning and implementing instruction and in helping other teachers foster mathematics classroom environments that support the learning of all students.
- Understand and utilize various cultures' approaches to learning mathematics.

Knowledge of mathematics curriculum and assessment:

- Know common developmental paths (learning trajectories) related to foundational mathematical topics (e.g., place value, rational numbers and equivalence); understand these developmental paths, and use them to sequence activities to build learning environments that are developmentally appropriate and effective.
- Select, use, adapt and determine the suitability of mathematics curricula and teaching materials (e.g., textbooks, technology, manipulatives) for particular learning goals.
- Be cognizant of a formative assessment cycle (administering a formative assessment task, analyzing student work from the task, using that analysis to enhance teacher knowledge, and designing and teaching reengagement lessons) and resources.
- Provide appropriate interpretations of assessment results, and communicate results (in context) to specific individuals and groups (e.g., students, parents, caregivers, colleagues, administrators, policymakers, community members).

(Standards for Elementary Mathematics Specialists, Association of Mathematics Teacher Educators, 2009)

Possible Assignments:

- Mathematical problem solving tasks: Strategically selected mathematical tasks focused on developing problem solving strategies and exposing important ideas about algebraic reasoning.
- Clinical interview: Design an interview to determine student understanding of algebraic reasoning, including interview questions with extensions to press for student understanding. Videotape and conduct the interview. Analyze the video using a rubric that assesses both student understanding and the interview process.
- Differentiated lesson: Select one of the course objectives. Develop a lesson to meet the objective for the whole group and addition differentiation activities for at least three populations that could include gifted students, English Language Learners, Special Education, and/or other groups.
- Action research project: Action research is a particular approach to research that aims to improve practice by addressing real-world situations and needs through focused, reflective practice and the application of interventions or procedures that are designed for improving performance. Develop a classroom research project about teaching concepts of algebra for your own classroom. Consult the

mathematics education literature, citing at least three sources from peer-reviewed journals. Develop a research question which could be a new teaching, assessment, or curriculum strategy for rational numbers in your class. Collect data, and analyze the results.

- Case study: Write a case presenting the mathematical thinking of a student or group of students. Be sure to include details in your narrative, such as student dialogue, your questions, and what you were thinking as you listened to the students. Analyze the student thinking and discuss the questions that are raised for you in the students' mathematical thinking.
- Content exams, performance tasks, and homework for determining participants' mathematical knowledge for teaching on the topic of algebraic reasoning.
- Lesson study cycle: Work with a group to develop a task-based lesson including anticipated student responses, questions for classroom discourse, and formative assessment. One person in the group will teach the lesson while other members of the group observe student thinking. Group members refine the lesson in response to the observation data and then teach and analyze the lesson in their own classes.
- Reflections on your own mathematical understanding: Keep a reflections log that discusses: a mathematical idea from the session that is new or important for you, a question you have, and an application of the mathematical idea for your classroom.

Suggested Texts:

Van De Walle, John A. et. al., (2009). *Elementary and Middle School Mathematics, Teaching Developmentally (7th Edition).* Boston, MA: Allyn & Bacon.

Ministry of Education, Singapore. *Singapore New Elementary Mathematics Textbooks I & II.* Oregon City, OR: Marshall Cavendish Education.

Beckman, Sybilla. (2008). *Mathematics for Elementary Teachers* (*2nd Edition*). Reading, MA: Addison Wesley.

Gross, Ken I. et. al., *Algebra Notes (2009).* University of Vermont.

Supplementary Texts/Readings:

Carpenter, Thomas P. et. al., (2003). *Thinking Mathematically: Integrating Arithmetic and Algebra in Elementary School*. Chicago, IL: Heinemann.

Schifter, Deborah, et. al., (2007). *Reasoning Algebraically About Operations Casebook: A Collaborative Project by the Staff and Participants of Teaching to the Big Ideas (Developing Mathematical Ideas).* Palo Alto, CA: Dale Seymour Publications.

Schifter, Deborah, et. A., (2008). *Developing Mathematical Ideas Patterns, Functions, and Change Casebook.* Palo Alto, CA: Dale Seymour Publications.