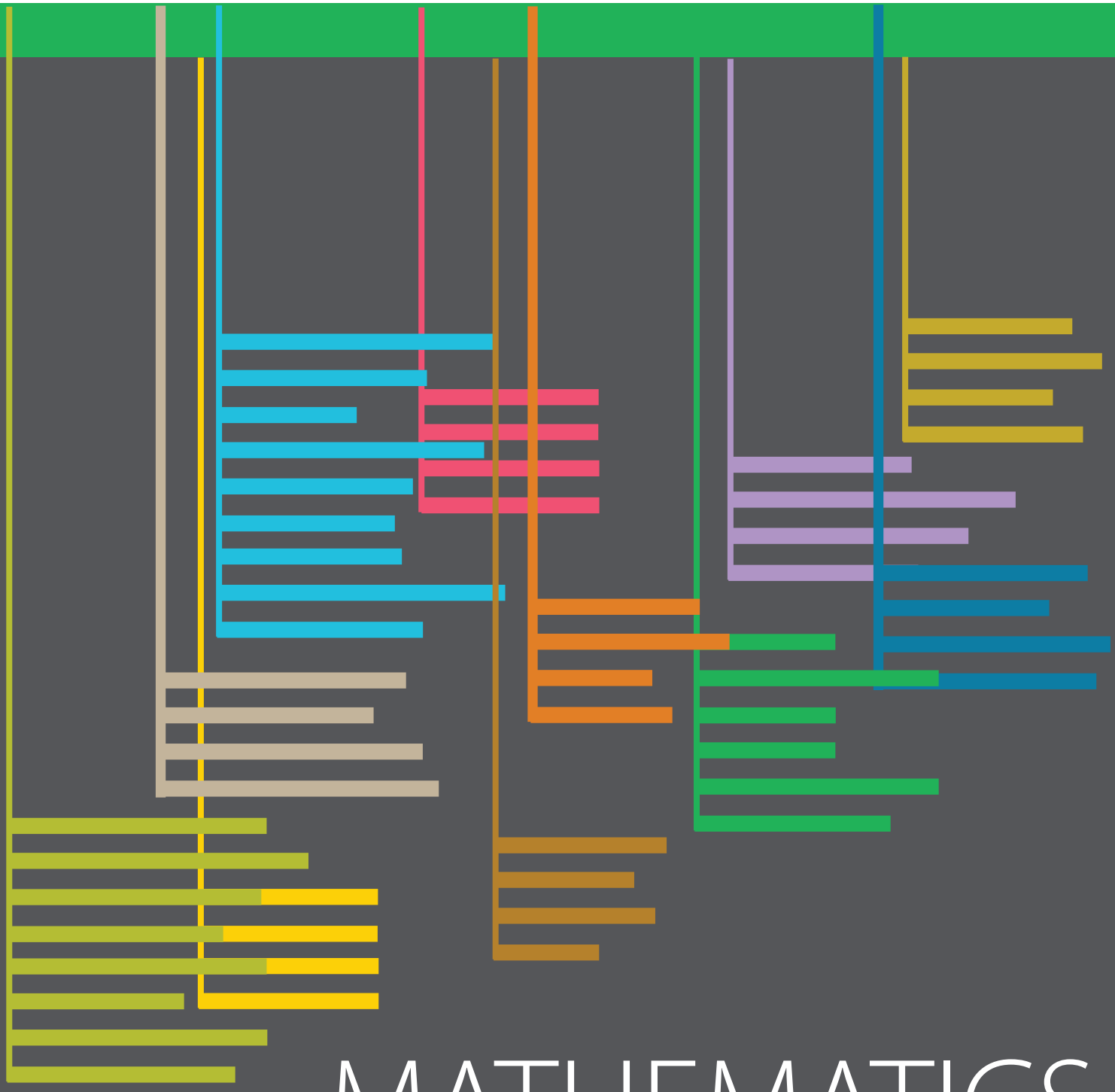
UTAH CORE STANDARDS



MATHEMATICS
High School (9–12)

UTAH STATE OFFICE OF EDUCATION 250 EAST 500 SOUTH P.O. BOX 144200 SALT LAKE CITY, UTAH 84114-4200 SYDNEE DICKSON, INTERIM STATE SUPERINTENDENT OF PUBLIC INSTRUCTION

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Utah STATE OFFICE of Education

UTAH CORE STATE STANDARDS *for* MATHEMATICS

HIGH SCHOOL (9–12)

Adopted August 2010

by the

Utah State Board of Education

Revised January 2016

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The Utah State Board of Education, in January of 1984, established policy requiring the identification of specific core standards to be met by all K–12 students in order to graduate from Utah's secondary schools. The Utah State Board of Education regularly updates the Utah Core Standards, while parents, teachers, and local school boards continue to control the curriculum choices that reflect local values.

The Utah Core Standards are aligned to scientifically based content standards. They drive high quality instruction through statewide comprehensive expectations for all students. The standards outline essential knowledge, concepts, and skills to be mastered at each grade level or within a critical content area. The standards provide a foundation for ensuring learning within the classroom.

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INTRODUCTION

Organization of the Standards

The Utah Core Standards are organized into **strands**, which represent significant areas of learning within content areas. Depending on the core area, these strands may be designated by time periods, thematic principles, modes of practice, or other organizing principles.

Within each strand are **standards**. A standard is an articulation of the demonstrated proficiency to be obtained. A standard represents an essential element of the learning that is expected. While some standards within a strand may be more comprehensive than others, all standards are essential for mastery.

Understanding Mathematics

These standards define what students should understand and be able to do in their study of mathematics. Asking a student to understand something means asking a teacher to assess whether the student has understood it. But what does mathematical understanding look like? One hallmark of mathematical understanding is the ability to justify, in a way appropriate to the student's mathematical maturity, why a particular mathematical statement is true or where a mathematical rule comes from. Mathematical understanding and procedural skill are equally important, and both are assessable using mathematical tasks of sufficient richness.

The standards set grade-specific standards but do not dictate curriculum or teaching methods, nor do they define the intervention methods or materials necessary to support students who are well below or well above grade-level expectations. It is also beyond the scope of the Standards to define the full range of supports appropriate for English language learners and for students with special needs. At the same time, all students must have the opportunity to learn and meet the same high standards if they are to access the knowledge and skills necessary in their post-school lives. The standards should be read as allowing for the widest possible range of students to participate fully from the outset, along with appropriate accommodations to ensure maximum participation of students with special education needs. No set of grade-specific standards can fully reflect the great variety in abilities, needs, learning rates, and achievement levels of students in any given classroom. However, the standards do provide clear signposts along the way to the goal of college and career readiness for all students.

What students can learn at any particular grade level depends upon what they have learned before. Ideally then, each standard in this document might have been phrased in the form, "Students who already know… should next come to learn ..." Grade placements for specific topics have been made on the basis of state and international comparisons and the collective experience and collective professional judgment of educators, researchers and mathematicians. Learning opportunities will continue to vary across schools and school systems, and educators should make every effort to meet the needs of individual students based on their current understanding.

Utah STATE OFFICE of Education

UTAH CORE STATE STANDARDS *for* MATHEMATICS HIGH SCHOOL COURSES

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SECONDARY MATHEMATICS I

THE FUNDAMENTAL PURPOSE OF SECONDARY MATHEMATICS I is to formalize and extend the mathematics that students learned in the middle grades. The critical areas, organized into units, deepen and extend understanding of linear relationships, in part by contrasting them with exponential phenomena, and in part by applying linear models to data that exhibit a linear trend. Secondary Mathematics I uses properties and theorems involving congruent figures to deepen and extend understanding of geometric knowledge from prior grades. The final unit in the course ties together the algebraic and geometric ideas studied. The Mathematical Practice Standards apply throughout each course and, together with the content standards, prescribe that students experience mathematics as a coherent, useful, and logical subject that makes use of their ability to make sense of problem situations.

CRITICAL AREA 1: By the end of eighth grade, students have had a variety of experiences working with expressions and creating equations. Students continue this work by using quantities to model and analyze situations, to interpret expressions, and by creating equations to describe situations.

CRITICAL AREA 2: In earlier grades, students define, evaluate, and compare functions, and use them to model relationships between quantities. Students will learn function notation and develop the concepts of domain and range. They move beyond viewing functions as processes that take inputs and yield outputs, and start viewing functions as objects in their own right. They explore many examples of functions, including sequences; they interpret functions given graphically, numerically, symbolically, and verbally, translate between representations, and understand the limitations of various representations. They work with functions given by graphs and tables, keeping in mind that, depending upon the context, these representations are likely to be approximate and incomplete. Their work includes functions that can be described or approximated by formulas, as well as those that cannot. When functions describe relationships between quantities arising from a context, students reason with the units in which those quantities are measured. Students build on and informally extend their understanding of integer exponents to consider exponential functions. They compare and contrast linear and exponential functions, distinguishing between additive and multiplicative change. They interpret arithmetic sequences as linear functions and geometric sequences as exponential functions.

CRITICAL AREA 3: By the end of eighth grade, students have learned to solve linear equations in one variable and have applied graphical and algebraic methods to analyze and solve systems of linear equations in two variables. This area builds on these earlier experiences by asking students to analyze and explain the process of solving an equation and to justify the process used in solving a system of equations. Students develop fluency writing, interpreting, and translating between various forms of linear equations and inequalities, and using them to solve problems. They master the solution of linear equations and apply related solution techniques and the laws of exponents to the creation and solution of simple exponential equations. Students explore

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systems of equations and inequalities, and they find and interpret their solutions. All of this work is grounded on understanding quantities and on relationships between them.

CRITICAL AREA 4: This area builds upon students' prior experiences with data, providing students with more formal means of assessing how a model fits data. Students use regression techniques to describe approximately linear relationships between quantities. They use graphical representations and knowledge of the context to make judgments about the appropriateness of linear models. With linear models, they look at residuals to analyze the goodness of fit.

CRITICAL AREA 5: In previous grades, students were asked to draw triangles based on given measurements. They also have prior experience with rigid motions (translations, reflections, and rotations) and have used these to develop notions about what it means for two objects to be congruent. In this unit, students establish triangle congruence criteria, based on analyses of rigid motions and formal constructions. They solve problems about triangles, quadrilaterals, and other polygons. They apply reasoning to complete geometric constructions and explain why they work.

CRITICAL AREA 6: Building on their work with the Pythagorean Theorem in eighth grade to find distances, students use a rectangular coordinate system to verify geometric relationships, including properties of special triangles and quadrilaterals and slopes of parallel and perpendicular lines.

Strand: MATHEMATICAL PRACTICES (MP)

The Standards for Mathematical Practice in Secondary Mathematics I describe mathematical habits of mind that teachers should seek to develop in their students. Students become mathematically proficient in engaging with mathematical content and concepts as they learn, experience, and apply these skills and attitudes **(Standards MP.1–8).**

- **Standard SI.MP.1 Make sense of problems and persevere in solving them.** Explain the meaning of a problem and look for entry points to its solution. Analyze givens, constraints, relationships, and goals. Make conjectures about the form and meaning of the solution, plan a solution pathway, and continually monitor progress asking, "Does this make sense?" Consider analogous problems, make connections between multiple representations, identify the correspondence between different approaches, look for trends, and transform algebraic expressions to highlight meaningful mathematics. Check answers to problems using a different method.

- **Standard SI.MP.2 Reason abstractly and quantitatively.** Make sense of the quantities and their relationships in problem situations. Translate between context and algebraic representations by contextualizing and decontextualizing quantitative relationships. This includes the ability to decontextualize a given situation, representing it algebraically and manipulating symbols fluently as well as the ability to contextualize algebraic representations to make sense of the problem.

- **Standard SI.MP.3 Construct viable arguments and critique the reasoning of others.** Understand and use stated assumptions, definitions, and previously established results in constructing arguments. Make conjectures and build a logical progression of statements to explore the truth of their conjectures. Justify conclusions and communicate them to others. Respond to the arguments of others by listening, asking clarifying questions, and critiquing the reasoning of others.

- **Standard SI.MP.4 Model with mathematics.** Apply mathematics to solve problems arising in everyday life, society, and the workplace. Make assumptions and approximations, identifying important quantities to construct a mathematical model. Routinely interpret mathematical results in the context of the situation and reflect on whether the results make sense, possibly improving the model if it has not served its purpose.

- **Standard SI.MP.5 Use appropriate tools strategically.** Consider the available tools and be sufficiently familiar with them to make sound decisions about when each tool might be helpful, recognizing both the insight to be gained as well as the limitations. Identify relevant external mathematical resources and use them to pose or solve problems. Use tools to explore and deepen their understanding of concepts.

- **Standard SI.MP.6 Attend to precision.** Communicate precisely to others. Use explicit definitions in discussion with others and in their own reasoning. They state the meaning of the symbols they choose. Specify units of measure and label axes to clarify the correspondence with quantities in a problem. Calculate accurately and efficiently, express numerical answers with a degree of precision appropriate for the problem context.

- **Standard SI.MP.7 Look for and make use of structure.** Look closely at mathematical relationships to identify the underlying structure by recognizing a simple structure within a more complicated structure. See complicated things, such as some algebraic expressions, as single objects or as being composed of several objects. *For example, see* $5 - 3(x - y)^2$ *as 5 minus a positive number times a square and use that to realize that its value cannot be more than 5 for any real numbers* x *and* y.

- **Standard SI.MP.8 Look for and express regularity in repeated reasoning.** Notice if reasoning is repeated, and look for both generalizations and shortcuts. Evaluate the reasonableness of intermediate results by maintaining oversight of the process while attending to the details.

Strand: NUMBER AND QUANTITY—Quantities (N.Q)

Reason quantitatively and use units to solve problems. Working with quantities and the relationships between them provides grounding for work with expressions, equations, and functions **(Standards N.Q.1–3).**

- **Standard N.Q.1** Use units as a way to understand problems and to guide the solution of multi-step problems; choose and interpret units consistently in formulas; choose and interpret the scale and the origin in graphs and data displays.

- **Standard N.Q.2** Define appropriate quantities for the purpose of descriptive modeling.

- **Standard N.Q.3** Choose a level of accuracy appropriate to limitations on measurement when reporting quantities.

Strand: ALGEBRA—Seeing Structure in Expressions (A.SSE)

Interpret the structure of expressions **(Standard A.SSE.1).**

- **Standard A.SSE.1** Interpret linear expressions and exponential expressions with integer exponents that represent a quantity in terms of its context.★

  a. Interpret parts of an expression, such as terms, factors, and coefficients.

  b. Interpret complicated expressions by viewing one or more of their parts as a single entity. *For example, interpret* $P(1+r)^n$ *as the product of* P *and a factor not depending on* P.

Strand: ALGEBRA—Creating Equations (A.CED)

Create equations that describe numbers or relationships. Limit these to linear equations and inequalities, and exponential equations. In the case of exponential equations, limit to situations requiring evaluation of exponential functions at integer inputs **(Standards A.CED.1–4).**

- **Standard A.CED.1** Create equations and inequalities in one variable and use them to solve problems. Include equations arising from linear and simple exponential functions.

- **Standard A.CED.2** Create equations in two or more variables to represent relationships between quantities; graph equations on coordinate axes with labels and scales.
- **Standard A.CED.3** Represent constraints by equations or inequalities and by systems of equations and/or inequalities, and interpret solutions as viable or non-viable options in a modeling context. *For example, represent inequalities describing nutritional and cost constraints on combinations of different foods.*
- **Standard A.CED.4** Rearrange formulas to highlight a quantity of interest, using the same reasoning as in solving equations. *For example, rearrange Ohm's Law* V = IR *to highlight resistance* R.

Strand: ALGEBRA—Reasoning With Equations and Inequalities (A.REI)

Understand solving equations as a process of reasoning and explain the reasoning **(Standard A.REI.1).** Solve equations and inequalities in one variable **(Standard A.REI.3).** Solve systems of equations. Build on student experiences graphing and solving systems of linear equations from middle school. Include cases where the two equations describe the same line—yielding infinitely many solutions—and cases where two equations describe parallel lines—yielding no solution; connect to GPE.5, which requires students to prove the slope criteria for parallel lines **(Standards A.REI.5–6).** Represent and solve equations and inequalities graphically **(Standards A.REI.10–12).**

- **Standard A.REI.1** Explain each step in solving a linear equation as following from the equality of numbers asserted at the previous step, starting from the assumption that the original equation has a solution. Construct a viable argument to justify a solution method. Students will solve exponential equations with logarithms in Secondary Mathematics III.
- **Standard A.REI.3** Solve equations and inequalities in one variable.
  - **a.** Solve one-variable equations and literal equations to highlight a variable of interest.
  - **b.** Solve compound inequalities in one variable, including absolute value inequalities.
  - **c.** Solve simple exponential equations that rely only on application of the laws of exponents (limit solving exponential equations to those that can be solved without logarithms). *For example,* $5^x = 125$ *or* $2^x = 1/16$.
- **Standard A.REI.5** Prove that, given a system of two equations in two variables, replacing one equation by the sum of that equation and a multiple of the other produces a system with the same solutions.
- **Standard A.REI.6** Solve systems of linear equations exactly and approximately (numerically, algebraically, graphically), focusing on pairs of linear equations in two variables.
- **Standard A.REI.10** Understand that the graph of an equation in two variables is the set of all its solutions plotted in the coordinate plane, often forming a curve (which could be a line).

- **Standard A.REI.11** Explain why the *x*-coordinates of the points where the graphs of the equations $y = f(x)$ and $y = g(x)$ intersect are the solutions of the equation $f(x) = g(x)$; find the solutions approximately; e.g., using technology to graph the functions, make tables of values, or find successive approximations. Include cases where $f(x)$ and/or $g(x)$ are linear and exponential functions.★

- **Standard A.REI.12** Graph the solutions to a linear inequality in two variables as a half-plane (excluding the boundary in the case of a strict inequality), and graph the solution set to a system of linear inequalities in two variables as the intersection of the corresponding half-planes.

Strand: FUNCTIONS—Interpreting Linear and Exponential Functions (F.IF)

Understand the concept of a linear or exponential function and use function notation. Recognize arithmetic and geometric sequences as examples of linear and exponential functions **(Standards F.IF.1–3).** Interpret linear or exponential functions that arise in applications in terms of a context **(Standards F.IF.4–6).** Analyze linear or exponential functions using different representations **(Standards F.IF.7,9).**

- **Standard F.IF.1** Understand that a function from one set (called the domain) to another set (called the range) assigns to each element of the domain exactly one element of the range. If *f* is a function and *x* is an element of its domain, then $f(x)$ denotes the output of *f* corresponding to the input *x*. The graph of *f* is the graph of the equation $y = f(x)$.

- **Standard F.IF.2** Use function notation, evaluate functions for inputs in their domains, and interpret statements that use function notation in terms of a context.

- **Standard F.IF.3** Recognize that sequences are functions, sometimes defined recursively, whose domain is a subset of the integers. Emphasize arithmetic and geometric sequences as examples of linear and exponential functions. *For example, the Fibonacci sequence is defined recursively by* $f(0) = f(1) = 1$, $f(n+1) = f(n) + f(n-1)$ *for* $n \geq 1$.

- **Standard F.IF.4** For a function that models a relationship between two quantities, interpret key features of graphs and tables in terms of the quantities, and sketch graphs showing key features given a verbal description of the relationship. *Key features include intercepts; intervals where the function is increasing, decreasing, positive, or negative; relative maximums and minimums; symmetries; and end behavior.*★

- **Standard F.IF.5** Relate the domain of a function to its graph and, where applicable, to the quantitative relationship it describes. *For example, if the function* h(n) *gives the number of person-hours it takes to assemble n engines in a factory, then the positive integers would be an appropriate domain for the function.*★

- **Standard F.IF.6** Calculate and interpret the average rate of change of a function (presented symbolically or as a table) over a specified interval. Estimate the rate of change from a graph.★

- **Standard F.IF.7** Graph functions expressed symbolically and show key features of the

graph, by hand in simple cases and using technology for more complicated cases.★

a. Graph linear functions and show intercepts.

e. Graph exponential functions, showing intercepts and end behavior.

■ **Standard F.IF.9** Compare properties of two functions, each represented in a different way (algebraically, graphically, numerically in tables, or by verbal descriptions). *For example, compare the growth of two linear functions, or two exponential functions such as* $y=3^n$ *and* $y=100 \cdot 2^n$.

Strand: FUNCTIONS—Building Linear or Exponential Functions (F.BF)

Build a linear or exponential function that models a relationship between two quantities **(Standards F.BF.1–2).** Build new functions from existing functions **(Standard F.BF.3).**

■ **Standard F.BF.1** Write a function that describes a relationship between two quantities.★

a. Determine an explicit expression, a recursive process, or steps for calculation from a context.

b. Combine standard function types using arithmetic operations. *For example, build a function that models the temperature of a cooling body by adding a constant function to a decaying exponential, and relate these functions to the model.*

■ **Standard F.BF.2** Write arithmetic and geometric sequences both recursively and with an explicit formula, use them to model situations, and translate between the two forms. Limit to linear and exponential functions. Connect arithmetic sequences to linear functions and geometric sequences to exponential functions. ★

■ **Standard F.BF.3** Identify the effect on the graph of replacing $f(x)$ by $f(x) + k$, for specific values of k (both positive and negative); find the value of k given the graphs. Relate the vertical translation of a linear function to its y-intercept. Experiment with cases and illustrate an explanation of the effects on the graph using technology.

Strand: FUNCTIONS—Linear and Exponential (F.LE)

Construct and compare linear and exponential models and solve problems **(Standards F.LE.1–3).** Interpret expressions for functions in terms of the situation they model. **(Standard F.LE.5).**

■ **Standard F.LE.1** Distinguish between situations that can be modeled with linear functions and with exponential functions.

a. Prove that linear functions grow by equal differences over equal intervals; exponential functions grow by equal factors over equal intervals.

b. Recognize situations in which one quantity changes at a constant rate per unit interval relative to another.

c. Recognize situations in which a quantity grows or decays by a constant percent rate per unit interval relative to another.

- **Standard F.LE.2** Construct linear and exponential functions, including arithmetic and geometric sequences, given a graph, a description of a relationship, or two input-output pairs (include reading these from a table).
- **Standard F.LE.3** Observe using graphs and tables that a quantity increasing exponentially eventually exceeds a quantity increasing linearly.
- **Standard F.LE.5** Interpret the parameters in a linear or exponential function in terms of a context. Limit exponential functions to those of the form $f(x) = b^x + k$.

Strand: GEOMETRY—Congruence (G.CO)

Experiment with transformations in the plane. Build on student experience with rigid motions from earlier grades **(Standards G.CO.1–5).** Understand congruence in terms of rigid motions. Rigid motions are at the foundation of the definition of congruence. Reason from the basic properties of rigid motions (that they preserve distance and angle), which are assumed without proof. Rigid motions and their assumed properties can be used to establish the usual triangle congruence criteria, which can then be used to prove other theorems **(Standards G.CO.6–8).** Make geometric constructions **(Standards G.CO.12–13).**

- **Standard G.CO.1** Know precise definitions of angle, circle, perpendicular line, parallel line, and line segment, based on the undefined notions of point, line, distance along a line, and distance around a circular arc.
- **Standard G.CO.2** Represent transformations in the plane using, for example, transparencies and geometry software; describe transformations as functions that take points in the plane as inputs and give other points as outputs. Compare transformations that preserve distance and angle to those that do not (e.g., translation versus horizontal stretch).
- **Standard G.CO.3** Given a rectangle, parallelogram, trapezoid, or regular polygon, describe the rotations and reflections that carry it onto itself.
- **Standard G.CO.4** Develop definitions of rotations, reflections, and translations in terms of angles, circles, perpendicular lines, parallel lines, and line segments.
- **Standard G.CO.5** Given a geometric figure and a rotation, reflection, or translation, draw the transformed figure using, for example, graph paper, tracing paper, or geometry software. Specify a sequence of transformations that will carry a given figure onto another. Point out the basis of rigid motions in geometric concepts, for example, translations move points a specified distance along a line parallel to a specified line; rotations move objects along a circular arc with a specified center through a specified angle.
- **Standard G.CO.6** Use geometric descriptions of rigid motions to transform figures and to predict the effect of a given rigid motion on a given figure; given two figures, use

the definition of congruence in terms of rigid motions to decide whether they are congruent.

- **Standard G.CO.7** Use the definition of congruence in terms of rigid motions to show that two triangles are congruent if and only if corresponding pairs of sides and corresponding pairs of angles are congruent.
- **Standard G.CO.8** Explain how the criteria for triangle congruence (ASA, SAS, and SSS) follow from the definition of congruence in terms of rigid motions.
- **Standard G.CO.12** Make formal geometric constructions with a variety of tools and methods (compass and straightedge, string, reflective devices, paper folding, dynamic geometric software, etc.). Emphasize the ability to formalize and defend how these constructions result in the desired objects. *For example, copying a segment; copying an angle; bisecting a segment; bisecting an angle; constructing perpendicular lines, including the perpendicular bisector of a line segment; and constructing a line parallel to a given line through a point not on the line.*
- **Standard G.CO.13** Construct an equilateral triangle, a square, and a regular hexagon inscribed in a circle. Emphasize the ability to formalize and defend how these constructions result in the desired objects.

Strand: GEOMETRY—Expressing Geometric Properties With Equations (G.GPE)

Use coordinates to prove simple geometric theorems algebraically **(Standards G.GPE.4–5, 7).**

- **Standard G.GPE.4** Use coordinates to prove simple geometric theorems algebraically. *For example, prove or disprove that a figure defined by four given points in the coordinate plane is a rectangle; prove or disprove that the point (1, √3) lies on the circle centered at the origin and containing the point (0, 2).*
- **Standard G.GPE.5** Prove the slope criteria for parallel and perpendicular lines; use them to solve geometric problems (e.g., find the equation of a line parallel or perpendicular to a given line that passes through a given point).
- **Standard G.GPE.7** Use coordinates to compute perimeters of polygons and areas of triangles and rectangles; connect with The Pythagorean Theorem and the distance formula.★

Strand: STATISTICS AND PROBABILITY— Interpreting Categorical and Quantitative Data (S.ID)

Summarize, represent, and interpret data on a single count or measurement variable **(Standards S.ID.1–3).** Summarize, represent, and interpret data on two categorical and quantitative variables **(Standard S.ID.6).** Interpret linear models building on students' work with linear relationships, and introduce the correlation coefficient **(Standards S.ID.7–9).**

- **Standard S.ID.1** Represent data with plots on the real number line (dot plots, histograms, and box plots).
- **Standard S.ID.2** Use statistics appropriate to the shape of the data distribution to compare center (median, mean) and spread (interquartile range, standard deviation) of two or more different data sets.
- **Standard S.ID.3** Interpret differences in shape, center, and spread in the context of the data sets, accounting for possible effects of extreme data points (outliers). Calculate the weighted average of a distribution and interpret it as a measure of center.
- **Standard S.ID.6** Represent data on two quantitative variables on a scatter plot, and describe how the variables are related.
    - **a.** Fit a linear function to the data; use functions fitted to data to solve problems in the context of the data. Use given functions, or choose a function suggested by the context. Emphasize linear and exponential models.
    - **b.** Informally assess the fit of a function by plotting and analyzing residuals. Focus on situations for which linear models are appropriate.
    - **c.** Fit a linear function for scatter plots that suggest a linear association.
- **Standard S.ID.7** Interpret the slope (rate of change) and the intercept (constant term) of a linear model in the context of the data.
- **Standard S.ID.8** Compute (using technology) and interpret the correlation coefficient of a linear fit.
- **Standard S.ID.9** Distinguish between correlation and causation.

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SECONDARY MATHEMATICS I—HONORS STANDARDS

Strand: NUMBER AND QUANTITY: VECTOR AND MATRIX QUANTITIES (N.VM)

Represent and model with vector quantities **(Standards N.VM.1–3).** Perform operations on vectors **(Standards N.VM.4–5).** Perform operations on matrices and use matrices in applications **(Standards N.VM.6–13).**

- **Standard N.VM.1** Recognize vector quantities as having both magnitude and direction. Represent vector quantities by directed line segments, and use appropriate symbols for vectors and their magnitudes (e.g., v, $|v|$, $||v||$, v).
- **Standard N.VM.2** Find the components of a vector by subtracting the coordinates of an initial point from the coordinates of a terminal point.
- **Standard N.VM.3** Solve problems involving velocity and other quantities that can be represented by vectors.
- **Standard N.VM.4** Add and subtract vectors.
  - **a.** Add vectors end-to-end, component-wise, and by the parallelogram rule. Understand that the magnitude of a sum of two vectors is typically not the sum of the magnitudes.
  - **b.** Given two vectors in magnitude and direction form, determine the magnitude and direction of their sum.
  - **c.** Understand vector subtraction $v - w$ as $v + (-w)$, where $-w$ is the additive inverse of w, with the same magnitude as w and pointing in the opposite direction. Represent vector subtraction graphically by connecting the tips in the appropriate order, and perform vector subtraction component-wise.
- **Standard N.VM.5** Multiply a vector by a scalar.
  - **a.** Represent scalar multiplication graphically by scaling vectors and possibly reversing their direction; perform scalar multiplication component-wise, e.g., as $c(v_x, v_y) = (cv_x, cv_y)$.
  - **b.** Compute the magnitude of a scalar multiple cv using $||cv|| = |c|v$. Compute the direction of cv knowing that when $|c|v \neq 0$, the direction of cv is either along v (for $c > 0$) or against vs (for $c < 0$).
- **Standard N.VM.6** Use matrices to represent and manipulate data, e.g., to represent pay-offs or incidence relationships in a network.
- **Standard N.VM.7** Multiply matrices by scalars to produce new matrices, e.g., as when all of the pay-offs in a game are doubled.

- **Standard N.VM.8** Add, subtract, and multiply matrices of appropriate dimensions.
- **Standard N.VM.9** Understand that, unlike multiplication of numbers, matrix multiplication for square matrices is not a commutative operation, but still satisfies the associative and distributive properties.
- **Standard N.VM.10** Understand that the zero and identity matrices play a role in matrix addition and multiplication similar to the role of 0 and 1 in the real numbers. The determinant of a square matrix is nonzero if and only if the matrix has a multiplicative inverse.
- **Standard N.VM.11** Multiply a vector (regarded as a matrix with one column) by a matrix of suitable dimensions to produce another vector. Work with matrices as transformations of vectors.
- **Standard N.VM.12** Work with 2×2 matrices as transformations of the plane, and interpret the absolute value of the determinant in terms of area.
- **Standard N.VM.13** Solve systems of linear equations up to three variables using matrix row reduction.

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SECONDARY MATHEMATICS II

THE FOCUS OF SECONDARY MATHEMATICS II is on quadratic expressions, equations, and functions and on comparing their characteristics and behavior to those of linear and exponential relationships from Secondary Mathematics I as organized into six critical areas, or units. The need for extending the set of rational numbers arises, and real and complex numbers are introduced so that all quadratic equations can be solved. The link between probability and data is explored through conditional probability and counting methods, including their use in making and evaluating decisions. The study of similarity leads to an understanding of right triangle trigonometry and connects to quadratics through Pythagorean relationships. Circles, with their quadratic algebraic representations, round out the course. The Mathematical Practice Standards apply throughout each course and, together with the content standards, prescribe that students experience mathematics as a coherent, useful, and logical subject that makes use of their ability to make sense of problem situations.

CRITICAL AREA 1: Students extend the laws of exponents to rational exponents and explore distinctions between rational and irrational numbers by considering their decimal representations. Students learn that when quadratic equations do not have real solutions the number system must be extended so that solutions exist, analogous to the way in which extending the whole numbers to the negative numbers allows $x+1 = 0$ to have a solution. Students explore relationships between number systems: whole numbers, integers, rational numbers, real numbers, and complex numbers. The guiding principle is that equations with no solutions in one number system may have solutions in a larger number system.

CRITICAL AREA 2: Students consider quadratic functions, comparing the key characteristics of quadratic functions to those of linear and exponential functions. They select from among these functions to model phenomena. Students learn to anticipate the graph of a quadratic function by interpreting various forms of quadratic expressions. In particular, they identify the real solutions of a quadratic equation as the zeros of a related quadratic function. When quadratic equations do not have real solutions, students learn that the graph of the related quadratic function does not cross the horizontal axis. They expand their experience with functions to include more specialized functions—absolute value, step, and those that are piecewise-defined.

CRITICAL AREA 3: Students begin this unit by focusing on the structure of expressions, rewriting expressions to clarify and reveal aspects of the relationship they represent. They create and solve equations, inequalities, and systems of equations involving exponential and quadratic expressions.

CRITICAL AREA 4: Building on probability concepts that began in the middle grades, students use the languages of set theory to expand their ability to compute and interpret theoretical and experimental probabilities for compound events, attending to mutually exclusive events, independent events, and conditional probability. Students should make use of geometric probability models wherever possible. They use probability to make informed decisions.

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CRITICAL AREA 5: Students apply their earlier experience with dilations and proportional reasoning to build a formal understanding of similarity. They identify criteria for similarity of triangles, use similarity to solve problems, and apply similarity in right triangles to understand right triangle trigonometry, with particular attention to special right triangles and the Pythagorean Theorem. It is in this unit that students develop facility with geometric proof. They use what they know about congruence and similarity to prove theorems involving lines, angles, triangles, and other polygons. They explore a variety of formats for writing proofs.

CRITICAL AREA 6: Students prove basic theorems about circles, such as a tangent line is perpendicular to a radius, inscribed angle theorem, and theorems about chords, secants, and tangents dealing with segment lengths and angle measures. In the Cartesian coordinate system, students use the distance formula to write the equation of a circle when given the radius and the coordinates of its center, and the equation of a parabola with vertical axis when given an equation of its directrix and the coordinates of its focus. Given an equation of a circle, they draw the graph in the coordinate plane, and apply techniques for solving quadratic equations to determine intersections between lines and circles or a parabola and between two circles. Students develop informal arguments justifying common formulas for circumference, area, and volume of geometric objects, especially those related to circles.

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Strand: MATHEMATICAL PRACTICES (MP)

The Standards for Mathematical Practice in Secondary Mathematics II describe mathematical habits of mind that teachers should seek to develop in their students. Students become mathematically proficient in engaging with mathematical content and concepts as they learn, experience, and apply these skills and attitudes **(Standards MP.1–8).**

- **Standard SII.MP.1 Make sense of problems and persevere in solving them.** Explain the meaning of a problem and look for entry points to its solution. Analyze givens, constraints, relationships, and goals. Make conjectures about the form and meaning of the solution, plan a solution pathway, and continually monitor progress asking, "Does this make sense?" Consider analogous problems, make connections between multiple representations, identify the correspondence between different approaches, look for trends, and transform algebraic expressions to highlight meaningful mathematics. Check answers to problems using a different method.

- **Standard SII.MP.2 Reason abstractly and quantitatively.** Make sense of the quantities and their relationships in problem situations. Translate between context and algebraic representations by contextualizing and decontextualizing quantitative relationships. This includes the ability to decontextualize a given situation, representing it algebraically and manipulating symbols fluently as well as the ability to contextualize algebraic representations to make sense of the problem.

- **Standard SII.MP.3 Construct viable arguments and critique the reasoning of others.** Understand and use stated assumptions, definitions, and previously established results in constructing arguments. Make conjectures and build a logical progression of statements to explore the truth of their conjectures. Justify conclusions and communicate them to others. Respond to the arguments of others by listening, asking clarifying questions, and critiquing the reasoning of others.

- **Standard SII.MP.4 Model with mathematics.** Apply mathematics to solve problems arising in everyday life, society, and the workplace. Make assumptions and approximations, identifying important quantities to construct a mathematical model. Routinely interpret mathematical results in the context of the situation and reflect on whether the results make sense, possibly improving the model if it has not served its purpose.

- **Standard SII.MP.5 Use appropriate tools strategically.** Consider the available tools and be sufficiently familiar with them to make sound decisions about when each tool might be helpful, recognizing both the insight to be gained as well as the limitations. Identify relevant external mathematical resources and use them to pose or solve problems. Use tools to explore and deepen their understanding of concepts.

- **Standard SII.MP.6 Attend to precision.** Communicate precisely to others. Use explicit definitions in discussion with others and in their own reasoning. They state the meaning of the symbols they choose. Specify units of measure and label axes to clarify the correspondence with quantities in a problem. Calculate accurately and efficiently, express numerical answers with a degree of precision appropriate for the problem context.

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- **Standard SII.MP.7 Look for and make use of structure.** Look closely at mathematical relationships to identify the underlying structure by recognizing a simple structure within a more complicated structure. See complicated things, such as some algebraic expressions, as single objects or as being composed of several objects. *For example, see* $5 - 3(x - y)^2$ *as 5 minus a positive number times a square and use that to realize that its value cannot be more than 5 for any real numbers* x *and* y.

- **Standard SII.MP.8 Look for and express regularity in repeated reasoning.** Notice if reasoning is repeated, and look for both generalizations and shortcuts. Evaluate the reasonableness of intermediate results by maintaining oversight of the process while attending to the details.

Strand: NUMBER AND QUANTITY—The Real Number System (N.RN)

Extend the properties of exponents to rational exponents **(Standards N.RN.1–2).** Use properties of rational and irrational numbers **(Standard N.RN. 3).**

- **Standard N.RN.1** Explain how the definition of the meaning of rational exponents follows from extending the properties of integer exponents to those values, allowing for a notation for radicals in terms of rational exponents. *For example, we define* $5^{1/3}$ *to be the cube root of 5 because we want* $(5^{1/3})^3 = 5^{(1/3)3}$ *to hold, so* $(5^{1/3})^3$ *must equal 5.*

- **Standard N.RN.2** Rewrite expressions involving radicals and rational exponents using the properties of exponents.

- **Standard N.RN.3** Explain why sums and products of rational numbers are rational, that the sum of a rational number and an irrational number is irrational, and that the product of a nonzero rational number and an irrational number is irrational. Connect to physical situations (e.g., finding the perimeter of a square of area 2).

Strand: NUMBER AND QUANTITY—The Complex Number System (N.CN)

Perform arithmetic operations with complex numbers **(Standards N.CN.1–2).** Use complex numbers in polynomial identities and equations **(Standards N.CN.7–9).**

- **Standard N.CN.1** Know there is a complex number i such that $i^2 = -1$, and every complex number has the form $a + bi$ with a and b real.

- **Standard N.CN.2** Use the relation $i^2 = -1$ and the commutative, associative, and distributive properties to add, subtract, and multiply complex numbers. Limit to multiplications that involve i^2 as the highest power of i.

- **Standard N.CN.7** Solve quadratic equations with real coefficients that have complex solutions.

- **Standard N.CN.8** Extend polynomial identities to the complex numbers. Limit to quadratics with real coefficients. *For example, rewrite* $x^2 + 4$ *as* $(x + 2i)(x - 2i)$.

■ **Standard N.CN.9** Know the Fundamental Theorem of Algebra; show that it is true for quadratic polynomials.

Strand: ALGEBRA—Seeing Structure in Expression (A.SSE)

Interpret the structure of expressions **(Standards A.SSE.1–2).** Write expressions in equivalent forms to solve problems, balancing conceptual understanding and procedural fluency in work with equivalent expressions **(Standard A.SSE.3).**

■ **Standard A.SSE.1** Interpret quadratic and exponential expressions that represent a quantity in terms of its context.★

a. Interpret parts of an expression, such as terms, factors, and coefficients.

b. Interpret increasingly more complex expressions by viewing one or more of their parts as a single entity. Exponents are extended from the integer exponents to rational exponents focusing on those that represent square or cube roots.

■ **Standard A.SSE.2** Use the structure of an expression to identify ways to rewrite it. For example, see $x^4 - y^4$ as $(x^2)^2 - (y^2)^2$, thus recognizing it as a difference of squares that can be factored as $(x^2 - y^2)(x^2 + y^2)$.

■ **Standard A.SSE.3** Choose and produce an equivalent form of an expression to reveal and explain properties of the quantity represented by the expression. *For example, development of skill in factoring and completing the square goes hand in hand with understanding what different forms of a quadratic expression reveal.* ★

a. Factor a quadratic expression to reveal the zeros of the function it defines.

b. Complete the square in a quadratic expression to reveal the maximum or minimum value of the function it defines.

c. Use the properties of exponents to transform expressions for exponential functions. *For example, the expression* 1.15^t *can be rewritten as* $(1.15^{1/12})^{12t} \approx 1.012^{12t}$ *to reveal the approximate equivalent monthly interest rate if the annual rate is 15%.*

Strand: ALGEBRA—Arithmetic With Polynomials and Rational Expressions (A.APR)

Perform arithmetic operations on polynomials. Focus on polynomial expressions that simplify to forms that are linear or quadratic in a positive integer power of x **(Standard A.APR.1).**

■ **Standard A.APR.1** Understand that polynomials form a system analogous to the integers, namely, they are closed under the operations of addition, subtraction, and multiplication; add, subtract, and multiply polynomials.

Strand: ALGEBRA—Creating Equations (A.CED)

Create equations that describe numbers or relationships. Extend work on linear and exponential equations to quadratic equations **(Standards A.CED.1–2, 4).**

- **Standard A.CED.1** Create equations and inequalities in one variable and use them to solve problems. Include equations arising from linear and quadratic functions, and simple rational and exponential functions.
- **Standard A.CED.2** Create equations in two or more variables to represent relationships between quantities; graph equations on coordinate axes with labels and scales.
- **Standard A.CED.4** Rearrange formulas to highlight a quantity of interest, using the same reasoning as in solving equations; extend to formulas involving squared variables. *For example, rearrange the formula for the volume of a cylinder* $V = \pi r^2 h$.

Strand: ALGEBRA—Reasoning With Equations and Inequalities (A.REI)

Solve equations and inequalities in one variable **(Standard A.REI.4).** Solve systems of equations. Extend the work of systems to include solving systems consisting of one linear and one nonlinear equation **(Standard A.REI.7).**

- **Standard A.REI.4** Solve quadratic equations in one variable.
  - **a.** Use the method of completing the square to transform any quadratic equation in x into an equation of the form $(x – p)^2 = q$ that has the same solutions. Derive the quadratic formula from this form.
  - **b.** Solve quadratic equations by inspection (e.g., for $x^2 = 49$), taking square roots, completing the square, the quadratic formula and factoring, as appropriate to the initial form of the equation. Recognize when the quadratic formula gives complex solutions and write them as $a \pm bi$ for real numbers a and b.
- **Standard A.REI.7** Solve a simple system consisting of a linear equation and a quadratic equation in two variables algebraically and graphically. *For example, find the points of intersection between the line* $y = –3x$ *and the circle* $x^2 + y^2 = 3$.

Strand: FUNCTIONS—Interpret Functions (F.IF)

Interpret quadratic functions that arise in applications in terms of a context **(Standards F.IF.4–6).** Analyze functions using different representations **(Standards F.IF.7–9).**

- **Standard F.IF.4** For a function that models a relationship between two quantities, interpret key features of graphs and tables in terms of the quantities, and sketch graphs showing key features given a verbal description of the relationship. *Key features include intercepts; intervals where the function is increasing, decreasing, positive, or negative; relative maximums and minimums; symmetries; and end behavior.*★

■ **Standard F.IF.5** Relate the domain of a function to its graph and, where applicable, to the quantitative relationship it describes. Focus on quadratic functions; compare with linear and exponential functions. *For example, if the function* h(n) *gives the number of person-hours it takes to assemble n engines in a factory, then the positive integers would be an appropriate domain for the function.*★

■ **Standard F.IF.6** Calculate and interpret the average rate of change of a function (presented symbolically or as a table) over a specified interval. Estimate the rate of change from a graph.★

■ **Standard F.IF.7** Graph functions expressed symbolically and show key features of the graph, by hand in simple cases and using technology for more complicated cases.★

a. Graph linear and quadratic functions and show intercepts, maxima, and minima.

b. Graph piecewise-defined functions and absolute value functions. Compare and contrast absolute value and piecewise-defined functions with linear, quadratic, and exponential functions. Highlight issues of domain, range, and usefulness when examining piecewise-defined functions.

■ **Standard F.IF.8** Write a function defined by an expression in different but equivalent forms to reveal and explain different properties of the function.

a. Use the process of factoring and completing the square in a quadratic function to show zeros, extreme values, and symmetry of the graph, and interpret these in terms of a context.

b. Use the properties of exponents to interpret expressions for exponential functions. *For example, identify percent rate of change in functions such as* $y = (1.02)^{t}$, $y = (0.97)^{t}$, $y = (1.01)^{12t}$, $y = (1.2)^{t/10}$, *and classify them as representing exponential growth or decay.*

■ **Standard F.IF.9** Compare properties of two functions, each represented in a different way (algebraically, graphically, numerically in tables, or by verbal descriptions). Extend work with quadratics to include the relationship between coefficients and roots, and that once roots are known, a quadratic equation can be factored. *For example, given a graph of one quadratic function and an algebraic expression for another, say which has the larger maximum.*

Strand: FUNCTIONS—Building Functions (F.BF)

Build a function that models a relationship between two quantities **(Standard F.BF.1).** Build new functions from existing functions **(Standard F.BF.3).**

■ **Standard F.BF.1** Write a quadratic or exponential function that describes a relationship between two quantities.★

a. Determine an explicit expression, a recursive process, or steps for calculation from a context.

b. Combine standard function types using arithmetic operations. *For example, build a function that models the temperature of a cooling body by adding a constant function to a decaying exponential, and relate these functions to the model.*

■ **Standard F.BF.3** Identify the effect on the graph of replacing *f(x) by f(x) + k, k f(x), f(kx), and f(x + k)* for specific values of *k* (both positive and negative); find the value of *k* given the graphs. Focus on quadratic functions and consider including absolute value functions. Experiment with cases and illustrate an explanation of the effects on the graph using technology. Include recognizing even and odd functions from their graphs and algebraic expressions for them.

Strand: FUNCTIONS—Linear, Quadratic, and Exponential Models (F.LE)

Construct and compare linear, quadratic, and exponential models and solve problems **(Standard F.LE.3).**

■ **Standard F.LE.3** Observe using graphs and tables that a quantity increasing exponentially eventually exceeds a quantity increasing linearly, quadratically, or (more generally) as a polynomial function. Compare linear and exponential growth to quadratic growth.

Strand: FUNCTIONS—Trigonometric Functions (F.TF)

Prove and apply trigonometric identities. Limit θ to angles between 0 and 90 degrees. Connect with the Pythagorean Theorem and the distance formula **(Standard F.TF.8).**

■ **Standard F.TF.8** Prove the Pythagorean identity $\sin^2(\theta) + \cos^2(\theta) = 1$ and use it to find sin (θ), cos (θ), or tan (θ), given sin (θ), cos (θ), or tan (θ), and the quadrant of the angle.

Strand: GEOMETRY—Congruence (G.CO)

Prove geometric theorems. Encourage multiple ways of writing proofs, such as narrative paragraphs, flow diagrams, two-column format, and diagrams without words. Focus on the validity of the underlying reasoning while exploring a variety of formats for expressing that reasoning **(Standards G.CO.9–11).**

■ **Standard G.CO.9** Prove theorems about lines and angles. *Theorems include: vertical angles are congruent; when a transversal crosses parallel lines, alternate interior angles are congruent and corresponding angles are congruent; points on a perpendicular bisector of a line segment are exactly those equidistant from the segment's endpoints.*

■ **Standard G.CO.10** Prove theorems about triangles. *Theorems include: measures of interior angles of a triangle sum to 180°; base angles of isosceles triangles are congruent; the segment joining midpoints of two sides of a triangle is parallel to the third side and half the length; the medians of a triangle meet at a point.*

■ **Standard G.CO.11** Prove theorems about parallelograms. *Theorems include: opposite sides are congruent, opposite angles are congruent, the diagonals of a parallelogram bisect each other, and conversely, rectangles are parallelograms with congruent diagonals.*

Strand: GEOMETRY—Similarity, Right Triangles, and Trigonometry (G.SRT)

Understand similarity in terms of similarity transformations **(Standards G.SRT.1–3).** Prove theorems involving similarity **(Standards G.SRT.4–5).** Define trigonometric ratios and solve problems involving right triangles **(Standards G.SRT.6–8).**

- **Standard G.SRT.1** Verify experimentally the properties of dilations given by a center and a scale factor.

  a. A dilation takes a line not passing through the center of the dilation to a parallel line, and leaves a line passing through the center unchanged.

  b. The dilation of a line segment is longer or shorter in the ratio given by the scale factor.

- **Standard G.SRT.2** Given two figures, use the definition of similarity in terms of similarity transformations to decide whether they are similar; explain using similarity transformations the meaning of similarity for triangles as the equality of all corresponding pairs of angles and the proportionality of all corresponding pairs of sides.

- **Standard G.SRT.3** Use the properties of similarity transformations to establish the AA criterion for two triangles to be similar.

- **Standard G.SRT.4** Prove theorems about triangles. *Theorems include: a line parallel to one side of a triangle divides the other two proportionally and conversely; the Pythagorean Theorem (proved using triangle similarity).*

- **Standard G.SRT.5** Use congruence and similarity criteria for triangles to solve problems and to prove relationships in geometric figures.

- **Standard G.SRT.6** Understand that by similarity, side ratios in right triangles are properties of the angles in the triangle, leading to definitions of trigonometric ratios for acute angles.

- **Standard G.SRT.7** Explain and use the relationship between the sine and cosine of complementary angles.

- **Standard G.SRT.8** Use trigonometric ratios and the Pythagorean Theorem to solve right triangles in applied problems.

Strand: GEOMETRY—Circles (G.C)

Understand and apply theorems about circles **(Standard G.C.1–4).** Find arc lengths and areas of sectors of circles. Use this as a basis for introducing the radian as a unit of measure. It is not intended that it be applied to the development of circular trigonometry in this course **(Standard G.C.5).**

- **Standard G.C.1** Prove that all circles are similar.

- **Standard G.C.2** Identify and describe relationships among inscribed angles, radii, and chords. *Relationships include the relationship between central, inscribed, and circumscribed angles; inscribed angles on a diameter are right angles; the radius of a circle is perpendicular to the tangent where the radius intersects the circle.*
- **Standard G.C.3** Construct the inscribed and circumscribed circles of a triangle, and prove properties of angles for a quadrilateral inscribed in a circle.
- **Standard G.C.4** Construct a tangent line from a point outside a given circle to the circle.
- **Standard G.C.5** Derive, using similarity, the fact that the length of the arc intercepted by an angle is proportional to the radius, and define the radian measure of the angle as the constant of proportionality; derive the formula for the area of a sector.

Strand: GEOMETRY—Expressing Geometric Properties With Equations (G.GPE)

Translate between the geometric description and the equation for a conic section **(Standard G.GPE.1).** Use coordinates to prove simple geometric theorems algebraically. Include simple proofs involving circles **(Standard G.GPE.4).** Use coordinates to prove simple geometric theorems algebraically **(Standard G.GPE.6).**

- **Standard G.GPE.1** Derive the equation of a circle of given center and radius using the Pythagorean Theorem; complete the square to find the center and radius of a circle given by an equation.
- **Standard G.GPE.4** Use coordinates to prove simple geometric theorems algebraically. *For example, prove or disprove that a figure defined by four given points in the coordinate plane is a rectangle; prove or disprove that the point (1, √3) lies on the circle centered at the origin and containing the point (0, 2).*
- **Standard G.GPE.6** Find the point on a directed line segment between two given points that partitions the segment in a given ratio.

Strand: GEOMETRY—Geometric Measurement and Dimension (G.GMD)

Explain volume formulas and use them to solve problems **(Standards G.GMD.1, 3).**

- **Standard G.GMD.1** Give an informal argument for the formulas for the circumference of a circle, area of a circle, volume of a cylinder, pyramid, and cone. Informal arguments for area formulas can make use of the way in which area scale under similarity transformations: when one figure in the plane results from another by applying a similarity transformation with scale factor k, its area is k^2 times the area of the first. *Use dissection arguments, Cavalieri's principle, and informal limit arguments.*
- **Standard G.GMD.3** Use volume formulas for cylinders, pyramids, cones, and spheres to solve problems. Informal arguments for volume formulas can make use of the way in which volume scale under similarity transformations: when one figure results from

another by applying a similarity transformation, volumes of solid figures scale by k^3 under a similarity transformation with scale factor k. ★

Strand: STATISTICS—Interpreting Categorical and Quantitative Data (S.ID)

Summarize, represent, and interpret data on two categorical or quantitative variables **(Standard S.ID.5).**

- **Standard S.ID.5** Summarize categorical data for two categories in two-way frequency tables. Interpret relative frequencies in the context of the data (including joint, marginal, and condition reltive frequencies). Recognize possible associations and trends in the date.

Strand: STATISTICS—Conditional Probability and the Rules of Probability (S.CP)

Understand independence and conditional probability and use them to interpret data **(Standards S.CP.1, 4–5).** Use the rules of probability to compute probabilities of compound events in a uniform probability model **(Standard S.CP.6).**

- **Standard S.CP.1** Describe events as subsets of a sample space (the set of outcomes) using characteristics (or categories) of the outcomes, or as unions, intersections, or complements of other events ("or," "and," "not").

- **Standard S.CP.4** Construct and interpret two-way frequency tables of data when two categories are associated with each object being classified. Use the two-way table as a sample space to decide if events are independent and to approximate conditional probabilities. *For example, collect data from a random sample of students in your school on their favorite subject among math, science, and English. Estimate the probability that a randomly selected student from your school will favor science given that the student is in tenth grade. Do the same for other subjects and compare the results.*

- **Standard S.CP.5** Recognize and explain the concepts of conditional probability and independence in everyday language and everyday situations. *For example, compare the chance of having lung cancer if you are are a smoker with the chance of being a smoker if you have lung cancer.*

- **Standard S.CP.6** Find the conditional probability of A given B as the fraction of B's outcomes that also belong to A, and interpret the answer in terms of the model.

SECONDARY MATHEMATICS II—HONORS STANDARDS

Strand: NUMBER AND QUANTITY—Complex Number System (N.CN)

Perform arithmetic operations with complex numbers **(Standard N.CN.3).** Represent complex numbers and their operations on the complex plane **(Standards N.CN.4–5).**

- **Standard N.CN.3** Find the conjugate of a complex number; use conjugates to find moduli and quotients of complex numbers.
- **Standard N.CN.4** Represent complex numbers on the complex plane in rectangular form, and explain why the rectangular form of a given complex number represents the same number.
- **Standard N.CN.5** Represent addition, subtraction, and multiplication geometrically on the complex plane; use properties of this representation for computation. *For example, $(-1 + \sqrt{3}\,i)3 = 8$ because $(-1 + \sqrt{3}\,i)$ has modulus 2 and argument 120°.*

Strand: ALGEBRA—Reasoning With Equations and Inequalities (A.REI)

Solve systems of equations **(Standards A.REI.8–9).**

- **Standard A.REI.8** Represent a system of linear equations as a single-matrix equation in a vector variable.
- **Standard A.REI.9** Find the inverse of a matrix if it exists, and use it to solve systems of linear equations (using technology for matrices of dimension 3 x 3 or greater).

Strand: FUNCTIONS—Interpreting Functions (F.IF)

Analyze functions using different representations **(Standards F.IF.10–11).**

- **Standard F.IF.10** Use sigma notation to represent the sum of a finite arithmetic or geometric series.
- **Standard F.IF.11** Represent series algebraically, graphically, and numerically.

Strand: GEOMETRY—Expressing Geometric Properties With Equations (G-GPE)

Translate between the geometric description and the equation for a conic section **(Standards G.GPE.2–3).**

- **Standard G.GPE.2** Derive the equation of a parabola given a focus and directrix.
- **Standard G.GPE.3** Derive the equations of ellipses and hyperbolas given the foci, using the fact that the sum or difference of distances from the foci is constant.

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Strand: STATISTICS AND PROBABILITY—Conditional Probability and the Rules of Probability (S.CP)

Understand independence and conditional probability and use them to interpret data **(Standards S.CP.2–3).** Use the rules of probability to compute probabilities of compound events in a uniform probability model **(Standards S.CP.7–8).**

- **Standard S.CP.2** Understand that two events A and B are independent if the probability of A and B occurring together is the product of their probabilities, and use this characterization to determine if they are independent.
- **Standard S.CP.3** Understand the conditional probability of *A* given *B* as *P(A and B)/P(B)*, and interpret independence of A and *B* as saying that the conditional probability of *B* given *A* is the same as the probability of *B*.
- **Standard S.CP.7** Apply the Addition Rule, $P(A \text{ or } B) = P(A) + P(B) - P(A \text{ and } B)$, and interpret the answer in terms of the model.
- **Standard S.CP.8** Apply the general Multiplication Rule in a uniform probability model, $P(A \text{ and } B) = P(A)P(B|A) = P(B)P(A|B)$, and interpret the answer in terms of the model.

SECONDARY MATHEMATICS III

IN SECONDARY MATHEMATICS III students pull together and apply the accumulation of learning that they have from their previous courses, with content grouped into four critical areas, organized into units. They apply methods from probability and statistics to draw inferences and conclusions from data. Students expand their repertoire of functions to include polynomial, rational, and radical functions. They expand their study of right triangle trigonometry to include general triangles. And, finally, students bring together all of their experience with functions and geometry to create models and solve contextual problems. The Mathematical Practice Standards apply throughout each course and, together with the content standards, prescribe that students experience mathematics as a coherent, useful, and logical subject that makes use of their ability to make sense of problem situations.

CRITICAL AREA 1: Students see how the visual displays and summary statistics they learned in earlier grades relate to different types of data and to probability distributions. They identify different ways of collecting data—including sample surveys, experiments, and simulations—and the role that randomness and careful design play in the conclusions that can be drawn.

CRITICAL AREA 2: This area develops the structural similarities between the system of polynomials and the system of integers. Students draw on analogies between polynomial arithmetic and base-ten computation, focusing on properties of operations, particularly the distributive property. Students connect multiplication of polynomials with multiplication of multi-digit integers, and division of polynomials with long division of integers. Students identify zeros of polynomials and make connections between zeros of polynomials and solutions of polynomial equations. The unit culminates with the fundamental theorem of algebra. Rational numbers extend the arithmetic of integers by allowing division by all numbers except 0. Similarly, rational expressions extend the arithmetic of polynomials by allowing division by all polynomials except the zero polynomial. A central theme of this unit is that the arithmetic of rational expressions is governed by the same rules as the arithmetic of rational numbers.

CRITICAL AREA 3: Students develop the Laws of Sines and Cosines in order to find missing measures of general (not necessarily right) triangles. They are able to distinguish whether three given measures (angles or sides) define 0, 1, 2, or infinitely many triangles. This discussion of general triangles opens up the idea of trigonometry applied beyond the right triangle—that is, at least to obtuse angles. Students build on this idea to develop the notion of radian measure for angles and extend the domain of the trigonometric functions to all real numbers. They apply this knowledge to model simple periodic phenomena.

CRITICAL AREA 4: Students synthesize and generalize what they have learned about a variety of function families. They extend their work with exponential functions to include solving exponential equations with logarithms. They explore the effects of transformations on graphs of diverse functions, including functions arising in an application, in order to abstract the general principle that transformations on a graph

always have the same effect regardless of the type of the underlying functions. They identify appropriate types of functions to model a situation, adjust parameters to improve the model, and compare models by analyzing the appropriateness of the fit and making judgments about the domain over which a model is a good fit. The description of modeling as "the process of choosing and using mathematics and statistics to analyze empirical situations, to understand them better, and to make decisions" is at the heart of this area. The narrative discussion and diagram of the modeling cycle should be considered when knowledge of functions, statistics, and geometry is applied in a modeling context.

Strand: MATHEMATICAL PRACTICES (MP)

The Standards for Mathematical Practice in Secondary Mathematics III describe mathematical habits of mind that teachers should seek to develop in their students. Students become mathematically proficient in engaging with mathematical content and concepts as they learn, experience, and apply these skills and attitudes **(Standards MP.1–8).**

- **Standard SIII.MP.1 Make sense of problems and persevere in solving them.** Explain the meaning of a problem and look for entry points to its solution. Analyze givens, constraints, relationships, and goals. Make conjectures about the form and meaning of the solution, plan a solution pathway, and continually monitor progress asking, "Does this make sense?" Consider analogous problems, make connections between multiple representations, identify the correspondence between different approaches, look for trends, and transform algebraic expressions to highlight meaningful mathematics. Check answers to problems using a different method.

- **Standard SIII.MP.2 Reason abstractly and quantitatively.** Make sense of the quantities and their relationships in problem situations. Translate between context and algebraic representations by contextualizing and decontextualizing quantitative relationships. This includes the ability to decontextualize a given situation, representing it algebraically and manipulating symbols fluently as well as the ability to contextualize algebraic representations to make sense of the problem.

- **Standard SIII.MP.3 Construct viable arguments and critique the reasoning of others.** Understand and use stated assumptions, definitions, and previously established results in constructing arguments. Make conjectures and build a logical progression of statements to explore the truth of their conjectures. Justify conclusions and communicate them to others. Respond to the arguments of others by listening, asking clarifying questions, and critiquing the reasoning of others.

- **Standard SIII.MP.4 Model with mathematics.** Apply mathematics to solve problems arising in everyday life, society, and the workplace. Make assumptions and approximations, identifying important quantities to construct a mathematical model. Routinely interpret mathematical results in the context of the situation and reflect on whether the results make sense, possibly improving the model if it has not served its purpose.

- **Standard SIII.MP.5 Use appropriate tools strategically.** Consider the available tools and be sufficiently familiar with them to make sound decisions about when each tool might be helpful, recognizing both the insight to be gained as well as the limitations. Identify relevant external mathematical resources and use them to pose or solve problems. Use tools to explore and deepen their understanding of concepts.

- **Standard SIII.MP.6 Attend to precision.** Communicate precisely to others. Use explicit definitions in discussion with others and in their own reasoning. They state the meaning of the symbols they choose. Specify units of measure and label axes to clarify the correspondence with quantities in a problem. Calculate accurately and efficiently, express numerical answers with a degree of precision appropriate for the problem context.

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- **Standard SIII.MP.7 Look for and make use of structure.** Look closely at mathematical relationships to identify the underlying structure by recognizing a simple structure within a more complicated structure. See complicated things, such as some algebraic expressions, as single objects or as being composed of several objects. *For example, see* $5 - 3(x - y)^2$ *as 5 minus a positive number times a square and use that to realize that its value cannot be more than 5 for any real numbers* x *and* y.

- **Standard SIII.MP.8 Look for and express regularity in repeated reasoning.** Notice if reasoning is repeated, and look for both generalizations and shortcuts. Evaluate the reasonableness of intermediate results by maintaining oversight of the process while attending to the details.

Strand: NUMBER AND QUANTITY—The Complex Number System (N.CN)

Use complex numbers in polynomial identities and equations. Build on work with quadratic equations in Secondary Mathematics II **(Standards N.CN.8–9).**

- **Standard N.CN.8** Extend polynomial identities to the complex numbers. *For example, rewrite* $x^2 + 4$ *as* $(x + 2i)(x - 2i)$.

- **Standard N.CN.9** Know the Fundamental Theorem of Algebra. Limit to polynomials with real coefficients.

Strand: ALGEBRA—Seeing Structures in Expressions (A.SSE)

Interpret the structure of expressions. Extend to polynomial and rational expressions **(Standards A.SSE.1–2).** Write expressions in equivalent forms to solve problems **(Standard A.SSE.4).**

- **Standard A.SSE.1** Interpret polynomial and rational expressions that represent a quantity in terms of its context.★
  - **a.** Interpret parts of an expression, such as terms, factors, and coefficients.
  - **b.** Interpret complex expressions by viewing one or more of their parts as a single entity. *For example, examine the behavior of* $P(1+r/n)^{nt}$ *as* n *becomes large.*

- **Standard A.SSE.2** Use the structure of an expression to identify ways to rewrite it. *For example, see* $x^4 - y^4$ *as* $(x^2)^2 - (y^2)^2$*, thus recognizing it as a difference of squares that can be factored as* $(x^2 - y^2)(x^2 + y^2)$.

- **Standard A.SSE.4** Understand the formula for the sum of a series and use the formula to solve problems.
  - **a.** Derive the formula for the sum of an arithmetic series.
  - **b.** Derive the formula for the sum of a geometric series, and use the formula to solve problems. Extend to infinite geometric series. *For example, calculate mortgage payments.*★

Strand: ALGEBRA—Arithmetic With Polynomials and Rational Expressions (A.APR)

Perform arithmetic operations on polynomials, extending beyond the quadratic polynomials **(Standard A.APR.1).** Understand the relationship between zeros and factors of polynomials **(Standards A.APR.2–3).** Use polynomial identities to solve problems **(Standards A.APR.4–5).** Rewrite rational expressions **(Standards A.APR.6–7).**

- **Standard A.APR.1** Understand that all polynomials form a system analogous to the integers, namely, they are closed under the operations of addition, subtraction, and multiplication; add, subtract, and multiply polynomials.
- **Standard A.APR.2** Know and apply the Remainder Theorem: For a polynomial $p(x)$ and a number a, the remainder on division by $x – a$ is $p(a)$, so $p(a) = 0$ if and only if $(x – a)$ is a factor of $p(x)$.
- **Standard A.APR.3** Identify zeros of polynomials when suitable factorizations are available, and use the zeros to construct a rough graph of the function defined by the polynomial.
- **Standard A.APR.4** Prove polynomial identities and use them to describe numerical relationships. *For example, the polynomial identity* $(x^2 + y^2)^2 = (x^2 – y^2)^2 + (2xy)^2$ *can be used to generate Pythagorean triples.*
- **Standard A.APR.5** Know and apply the Binomial Theorem for the expansion of $(x + y)^n$ in powers of x and y for a positive integer n, where x and y are any numbers. *For example, with coefficients determined by Pascal's Triangle.*
- **Standard A.APR.6** Rewrite simple rational expressions in different forms; write $a(x)/b(x)$ in the form $q(x) + r(x)/b(x)$, where $a(x)$, $b(x)$, $q(x)$, and $r(x)$ are polynomials with the degree of $r(x)$ less than the degree of $b(x)$, using inspection, long division or, for the more complicated examples, a computer algebra system.
- **Standard A.APR.7** Understand that rational expressions form a system analogous to the rational numbers, closed under addition, subtraction, multiplication, and division by a nonzero rational expression; add, subtract, multiply, and divide rational expressions.

Strand: ALGEBRA: CREATING EQUATIONS (A.CED)

Create equations that describe numbers or relationships, using all available types of functions to create such equations **(Standards A.CED.1–4).**

- **Standard A.CED.1** Create equations and inequalities in one variable and use them to solve problems. Include equations arising from linear and quadratic functions, and simple rational and exponential functions.
- **Standard A.CED.2** Create equations in two or more variables to represent relationships between quantities; graph equations on coordinate axes with labels and scales.
- **Standard A.CED.3** Represent constraints by equations or inequalities, and by systems of equations and/or inequalities, and interpret solutions as viable or non-viable options in

a modeling context. *For example, maximizing the volume of a box for a given surface area while drawing attention to the practical domain.*

■ **Standard A.CED.4** Rearrange formulas to highlight a quantity of interest, using the same reasoning as in solving equations. *For example, rearrange the compound interest formula to solve for* t: $A = P(1+r/n)^{nt}$

Strand: ALGEBRA: REASONING WITH EQUATIONS AND INEQUALITIES (A.REI)

Understand solving equations as a process of reasoning and explain the reasoning **(Standard A.REI.2).** Represent and solve equations and inequalities graphically **(Standard A.REI.11).**

■ **Standard A.REI.2** Solve simple rational and radical equations in one variable, and give examples showing how extraneous solutions may arise.

■ **Standard A.REI.11** Explain why the *x*-coordinates of the points where the graphs of the equations $y = f(x)$ *and* $y = g(x)$ intersect are the solutions of the equation $f(x) = g(x)$; find the solutions approximately, for example, using technology to graph the functions, make tables of values, or find successive approximations. Include cases where $f(x)$ and/or $g(x)$ are linear, polynomial, rational, absolute value, exponential, and logarithmic functions.★

Strand: FUNCTIONS—Interpreting Functions (F.IF)

Interpret functions that arise in applications in terms of a context **(Standards F.IF.4–6).** Analyze functions using different representations **(Standards F.IF.7–9).**

■ **Standard F.IF.4** For a function that models a relationship between two quantities, interpret key features of graphs and tables in terms of the quantities, and sketch graphs showing key features given a verbal description of the relationship. *Key features include intercepts; intervals where the function is increasing, decreasing, positive, or negative; relative maximums and minimums; symmetries; end behavior; and periodicity.*★

■ **Standard F.IF.5** Relate the domain of a function to its graph and, where applicable, to the quantitative relationship it describes. *For example, if the function* h(n) *gives the number of person-hours it takes to assemble* n *engines in a factory, then the positive integers would be an appropriate domain for the function.*★

■ **Standard F.IF.6** Calculate and interpret the average rate of change of a function (presented symbolically or as a table) over a specified interval. Estimate the rate of change from a graph.★

■ **Standard F.IF.7** Graph functions expressed symbolically and show key features of the graph, by hand in simple cases and using technology for more complicated cases.★

b. Graph square root, cube root, and piecewise-defined functions, including step functions and absolute value functions. Compare and contrast square root, cubed root, and step functions with all other functions.

c. Graph polynomial functions, identifying zeros when suitable factorizations are available, and showing end behavior.

d. Graph rational functions, identifying zeros and asymptotes when suitable factorizations are available, and showing end behavior.

e. Graph exponential and logarithmic functions, showing intercepts and end behavior; and trigonometric functions, showing period, midline, and amplitude.

■ **Standard F.IF.8** Write a function defined by an expression in different but equivalent forms to reveal and explain different properties of the function.

■ **Standard F.IF.9** Compare properties of two functions each represented in a different way (algebraically, graphically, numerically in tables, or by verbal descriptions). *For example, given a graph of one quadratic function and an algebraic expression for another, say which has the larger maximum.*

Strand: FUNCTIONS—Building Functions (F.BF)

Build a function that models a relationship between two quantities. Develop models for more complex or sophisticated situations **(Standards F.BF.1).** Build new functions from existing functions **(Standards F.BF.3–4).**

■ **Standard F.BF.1** Write a function that describes a relationship between two quantities.★

b. Combine standard function types using arithmetic operations. *For example, build a function that models the temperature of a cooling body by adding a constant function to a decaying exponential, and relate these functions to the model.*

■ **Standard F.BF.3** Identify the effect on the graph of replacing *f(x) by f(x) + k, k f(x), f(kx), and f(x + k)* for specific values of *k* (both positive and negative); find the value of *k* given the graphs. Note the effect of multiple transformations on a single function and the common effect of each transformation across function types. Include functions defined only by a graph. Experiment with cases and illustrate an explanation of the effects on the graph using technology. Include recognizing even and odd functions from their graphs and algebraic expressions for them.

■ **Standard F.BF.4** Find inverse functions.

a. Solve an equation of the form *f(x) = c* for a simple function *f* that has an inverse and write an expression for the inverse. Include linear, quadratic, exponential, logarithmic, rational, square root, and cube root functions. *For example,* $f(x) = 2x^3$ *or* $f(x) = (x+1)/(x-1)$ *for* $x \neq 1$.

Strand: FUNCTIONS—Linear, Quadratic, and Exponential Models (F.LE)

Construct and compare linear, quadratic, and exponential models and solve problems **(Standards F.LE.3–4).** Interpret expressions for functions in terms of the situation it models.

Introduce $f(x) = e^x$ as a model for continuous growth **(Standard F.LE.5).**

- **Standard F.LE.3** Observe using graphs and tables that a quantity increasing exponentially eventually exceeds a quanitity increasing linearly, quadratically, or (more generally) as a polynomial function.
- **Standard F.LE.4** For exponential models, express as a logarithm the solution to $ab^{ct} = d$ where *a, c*, and *d* are numbers and the base *b* is 2, 10, or *e*; evaluate the logarithm using technology. Include the relationship between properties of logarithms and properties of exponents, such as the connection between the properties of exponents and the basic logarithm property that log $xy = log\ x + log\ y$.
- **Standard F.LE.5** Interpret the parameters in a linear, quadratic, or exponential function in terms of a context.

Strand: FUNCTIONS—Trigonometric Functions (F.TF)

Extend the domain of trigonometric functions using the unit circle **(Standards F.TF.1–3).** Model periodic phenomena with trigonometric functions **(Standards F.TF.5–7).**

- **Standard F.TF.1** Understand radian measure of an angle as the length of the arc on the unit circle subtended by the angle.
- **Standard F.TF.2** Explain how the unit circle in the coordinate plane enables the extension of trigonometric functions to all real numbers, interpreted as radian measures of angles traversed counterclockwise around the unit circle.
- **Standard F.TF.3** Use special triangles to determine geometrically the values of sine, cosine, tangent for $\pi/3$, $\pi/4$ and $\pi/6$, and use the unit circle to express the values of sine, cosine, and tangent for $\pi - x$, $\pi + x$, and $2\pi - x$ in terms of their values for *x,* where *x* is any real number.
- **Standard F.TF.5** Choose trigonometric functions to model periodic phenomena with specified amplitude, frequency, and midline.★
- **Standard F.TF.7** Use inverse functions to solve trignometric equations that arise in modeling context; evaluate the solutions using technology and interpret them in terms of context. Limit solutions to a given interval. ★

Strand: GEOMETRY—Similarity, Right Triangles, and Trigonometry (G.SRT)

Apply trigonometry to general triangles. With respect to the general case of the Laws of Sines and Cosines, the definitions of sine and cosine must be extended to obtuse angles **(Standards G.SRT.9–11).**

- **Standard G.SRT.9** Derive the formula $A = ½\ ab\ sin(C)$ for the area of a triangle by drawing an auxiliary line from a vertex perpendicular to the opposite side.
- **Standard G.SRT.10** Prove the Laws of Sines and Cosines and use them to solve problems.

- **Standard G.SRT.11** Understand and apply the Law of Sines and the Law of Cosines to find unknown measurements in right and non-right triangles (e.g., surveying problems, resultant forces).

Strand: GEOMETRY—Geometric Measurement and Dimension (G.GMD)

Visualize relationships between two-dimensional and three-dimensional objects **(Standards G.MD.4).**

- **Standard G.GMD.4** Identify the shapes of two-dimensional cross-sections of three-dimensional objects, and identify three-dimensional objects generated by rotations of two-dimensional objects.

Strand: GEOMETRY—Modeling With Geometry (G.MG)

Apply geometric concepts in modeling situations **(Standards G.MG.1–3).**

- **Standard G.MG.1** Use geometric shapes, their measures, and their properties to describe objects (e.g., modeling a tree trunk or a human torso as a cylinder).★
- **Standard G.MG.2** Apply concepts of density based on area and volume in modeling situations (e.g., persons per square mile, BTUs per cubic foot).★
- **Standard G.MG.3** Apply geometric methods to solve design problems (e.g., designing an object or structure to satisfy physical constraints or minimize cost; working with typographic grid systems based on ratios).★

Strand: STATISTICS—Interpreting Categorical and Quantitative Data (S.ID)

Summarize, represent, and interpret data on a single count or measurement variable. While students may have heard of the normal distribution, it is unlikely that they will have prior experience using it to make specific estimates. Build on students' understanding of data distributions to help them see how the normal distribution uses area to make estimates of frequencies (which can be expressed as probabilities). Emphasize that only some data are well described by a normal distribution **(Standard S.ID.4).**

- **Standard S.ID.4** Use the mean and standard deviation of a data set to fit it to a normal distribution and to estimate population percentages. Recognize that there are data sets for which such a procedure is not appropriate. Use calculators, spreadsheets, and tables to estimate areas under the normal curve.

Strand: STATISTICS—Making Inferences and Justifying Conclusions (S.IC)

Understand and evaluate random processes underlying statistical experiments **(Standard S.IC.1).** Draw and justify conclusions from sample surveys, experiments, and observational studies. In earlier grades, students are introduced to different ways of collecting data and use

graphical displays and summary statistics to make comparisons. These ideas are revisited with a focus on how the way in which data is collected determines the scope and nature of the conclusions that can be drawn from that data. The concept of statistical significance is developed informally through simulation as meaning a result that is unlikely to have occurred solely as a result of random selection in sampling or random assignment in an experiment. For S.IC.4, focus on the variability of results from experiments—that is, focus on statistics as a way of dealing with, not eliminating, inherent randomness **(Standards S.IC.3–4, 6).**

- **Standard S.IC.1** Understand that statistics allow inferences to be made about population parameters based on a random sample from that population.
- **Standard S.IC.3** Recognize the purposes of and differences among sample surveys, experiments, and observational studies; explain how randomization relates to each.
- **Standard S.IC.4** Use data from a sample survey to estimate a population mean or proportion; develop a margin of error through the use of simulation models for random sampling.
- **Standard S.IC.6** Evaluate reports based on data.

SECONDARY MATHEMATICS III—HONORS STANDARDS

Strand: NUMBER AND QUANTITY—Complex Number System (N.CN)

Perform arithmetic operations with complex numbers **(Standard N.CN.3).** Represent complex numbers and their operations on the complex plane **(Standard N.CN.4–6).** Use complex numbers in polynomial identities and equations **(Standard N.CN.10).**

- **Standard N.CN.3** Find the conjugate of a complex number; use conjugates to find moduli and quotients of complex numbers.
- **Standard N.CN.4** Represent complex numbers on the complex plane in rectangular form and polar form (including real and imaginary numbers), and explain why the rectangular form of a given complex number represents the same number.
- **Standard N.CN.5** Represent addition, subtraction, multiplication, and conjugation of complex numbers geometrically on the complex plane; use properties of this representation for computation. *For example, (-1 + √3 i)3 = 8 because (-1 + √3 i) has modulus 2 and argument 120°.*
- **Standard N.CN.6** Calculate the distance between numbers in the complex plane as the modulus of the difference, and the midpoint of a segment as the average of the numbers at its endpoints.
- **Standard N.CN.10** Multiply complex numbers in polar form and use DeMoivre's Theorem to find roots of complex numbers.

Strand: FUNCTIONS—Interpreting Functions (F.IF)

Analyze functions using different representations **(Standard F.IF.7, d and f).**

- **Standard F.IF.7** Graph functions expressed symbolically, and show key features of the graph, by hand in simple cases and using technology for more complicated cases.

  d. Graph rational functions, identifying zeros, asymptotes, and point discontinuities when suitable factorizations are available, and showing end behavior.

  f. Define a curve parametrically and draw its graph.

Strand: FUNCTIONS—Building Functions (F.BF).

Build a function that models a relationship between two quantities **(Standard F.BF.1.c).** Build new functions from existing functions **(Standards F.BF.4, b,c,d–5).**

- **Standard F.BF.1** Write a function that describes a relationship between two quantities.

  c. Compose functions. *For example, if* T(y) *is the temperature in the atmosphere as a function of height, and* h(t) *is the height of a weather balloon as a function of time, then* T(h(t)) *is the temperature at the location of the weather balloon as a function of time.*

- **Standard F.BF.4** Find inverse functions.
  - **b.** Verify by composition that one function is the inverse of another.
  - **c.** Read values of an inverse function from a graph or a table, given that the function has an inverse.
  - **d.** Produce an invertible function from a non-invertible function by restricting the domain.
- **Standard F.BF.5** Understand the inverse relationship between exponents and logarithms, and use this relationship to solve problems involving logarithms and exponents.

Strand: FUNCTIONS—Trigonometric Functions (F.TF)

Extend the domain of trigonometric functions using the unit circle **(Standard F.TF.4).** Model periodic phenomena with trigonometric functions **(Standards F.TF.6–7).** Prove and apply trigonometric identities **(Standard F.TF.9).**

- **Standard F.TF.4** Use the unit circle to explain symmetry (odd and even) and periodicity of trigonometric functions.
- **Standard F.TF.6** Understand that restricting a trigonometric function to a domain on which it is always increasing or always decreasing allows its inverse to be constructed.
- **Standard F.TF.7** Use the inverse functions to solve trigonometric equations that arise in the modeling contexts; evaluate the solutions using technology, and interpret them in terms of the context.
- **Standard F.TF.9** Prove the addition and subtraction formulas for sine, cosine, and tangent, and use them to solve problems.

Strand: GEOMETRY—Geometric Measurement and Dimension (G.GMD)

Explain volume formulas and use them to solve problems **(Standard G.GMD.2).**

- **Standard G.GMD.2** Give an informal argument using Cavalieri's principle for the formulas for the volume of a sphere and other solid figures.

Strand: STATISTICS AND PROBABILITY—Conditional Probability and the Rules of Probability (S.CP)

Use the rules of probability to compute probabilities of compound events in a uniform probability model **(Standard S.CP.9).**

- **Standard S.CP.9** Use permutations and combinations to compute probabilities of compound events and solve problems.

PRECALCULUS

Number and Quantity

- **Vector and Matrix Quantities**
- **Complex Number System**

Algebra

- **Reasoning With Equations and Inequalities**

Functions

- **Interpreting Functions**
- **Building Functions**
- **Trigonometric Functions**

Geometry

- **Geometric Measurement and Dimension**
- **Expressing Geometric Properties with Equations**

Statistics

- **Conditional Probability and the Rules of Probability**

MATHEMATICAL PRACTICES

1. Make sense of problems and persevere in solving them.
2. Reason abstractly and quantitatively.
3. Construct viable arguments and critique the reasoning of others.
4. Model with mathematics.
5. Use appropriate tools strategically.
6. Attend to precision.
7. Look for and make use of structure.
8. Look for and express regularity in repeated reasoning.

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Strand: MATHEMATICAL PRACTICES (MP)

The Standards for Mathematical Practice in PreCalculus describe mathematical habits of mind that teachers should seek to develop in their students. Students become mathematically proficient in engaging with mathematical content and concepts as they learn, experience, and apply these skills and attitudes **(Standards MP.1–8).**

- **Standard P.MP.1 Make sense of problems and persevere in solving them.** Explain the meaning of a problem and look for entry points to its solution. Analyze givens, constraints, relationships, and goals. Make conjectures about the form and meaning of the solution, plan a solution pathway, and continually monitor progress asking, "Does this make sense?" Consider analogous problems, make connections between multiple representations, identify the correspondence between different approaches, look for trends, and transform algebraic expressions to highlight meaningful mathematics. Check answers to problems using a different method.

- **Standard P.MP.2 Reason abstractly and quantitatively.** Make sense of the quantities and their relationships in problem situations. Translate between context and algebraic representations by contextualizing and decontextualizing quantitative relationships. This includes the ability to decontextualize a given situation, representing it algebraically and manipulating symbols fluently as well as the ability to contextualize algebraic representations to make sense of the problem.

- **Standard P.MP.3 Construct viable arguments and critique the reasoning of others.** Understand and use stated assumptions, definitions, and previously established results in constructing arguments. Make conjectures and build a logical progression of statements to explore the truth of their conjectures. Justify conclusions and communicate them to others. Respond to the arguments of others by listening, asking clarifying questions, and critiquing the reasoning of others.

- **Standard P.MP.4 Model with mathematics.** Apply mathematics to solve problems arising in everyday life, society, and the workplace. Make assumptions and approximations, identifying important quantities to construct a mathematical model. Routinely interpret mathematical results in the context of the situation and reflect on whether the results make sense, possibly improving the model if it has not served its purpose.

- **Standard P.MP.5 Use appropriate tools strategically.** Consider the available tools and be sufficiently familiar with them to make sound decisions about when each tool might be helpful, recognizing both the insight to be gained as well as the limitations. Identify relevant external mathematical resources and use them to pose or solve problems. Use tools to explore and deepen their understanding of concepts.

- **Standard P.MP.6 Attend to precision.** Communicate precisely to others. Use explicit definitions in discussion with others and in their own reasoning. They state the meaning of the symbols they choose. Specify units of measure and label axes to clarify the correspondence with quantities in a problem. Calculate accurately and efficiently, express numerical answers with a degree of precision appropriate for the problem context.

- **Standard P.MP.7 Look for and make use of structure.** Look closely at mathematical relationships to identify the underlying structure by recognizing a simple structure within a more complicated structure. See complicated things, such as some algebraic expressions, as single objects or as being composed of several objects. *For example, see* $5 - 3(x - y)^2$ *as 5 minus a positive number times a square and use that to realize that its value cannot be more than 5 for any real numbers* x *and* y.

- **Standard P.MP.8 Look for and express regularity in repeated reasoning.** Notice if reasoning is repeated, and look for both generalizations and shortcuts. Evaluate the reasonableness of intermediate results by maintaining oversight of the process while attending to the details.

Strand: NUMBER AND QUANTITY—Vector and Matrix Quantities (N.VM)

Represent and model with vector quantities **(Standards 1–3).** Perform operations on vectors **(Standards 4–5).** Perform operations on matrices and use matrices in applications **(Standards 6–13).**

- **Standard N.VM.1** Recognize vector quantities as having both magnitude and direction. Represent vector quantities by directed line segments, and use appropriate symbols for vectors and their magnitudes (e.g., *v,* $|v|$, $||v||$, *v).*

- **Standard N.VM.2** Find the components of a vector by subtracting the coordinates of an initial point from the coordinates of a terminal point.

- **Standard N.VM.3** Solve problems involving velocity and other quantities that can be represented by vectors.

- **Standard N.VM. 4** Add and subtract vectors.

  a. Add vectors end to end, component-wise, and by the parallelogram rule. Understand that the magnitude of a sum of two vectors is typically not the sum of the magnitudes.

  b. Given two vectors in magnitude and direction form, determine the magnitude and direction of their sum.

  c. Understand vector subtraction $v - w$ *as* $v + (-w)$, where $-w$ is the additive inverse of w, with the same magnitude as w and pointing in the opposite direction. Represent vector subtraction graphically by connecting the tips in the appropriate order, and perform vector subtraction component-wise.

- **Standard N.VM.5** Multiply a vector by a scalar.

  a. Represent scalar multiplication graphically by scaling vectors and possibly reversing their direction; perform scalar multiplication component-wise, e.g., *as* $c(vx, vy) = (cvx, cvy)$.

b. Compute the magnitude of a scalar multiple cv using $||cv|| = |c|v$. Compute the direction of cv knowing that when $|c|v \neq 0$, the direction of cv is either along v (for $c > 0$) or against vs (for $c < 0$).

- **Standard N.VM.6** Use matrices to represent and manipulate data, e.g., to represent payoffs or incidence relationships in a network.
- **Standard N.VM.7** Multiply matrices by scalars to produce new matrices, e.g., as when all of the payoffs in a game are doubled.
- **Standard N.VM.8** Add, subtract, and multiply matrices of appropriate dimensions.
- **Standard N.VM.9** Understand that, unlike multiplication of numbers, matrix multiplication for square matrices is not a commutative operation, but still satisfies the associative and distributive properties.
- **Standard N.VM.10** Understand that the zero and identity matrices play a role in matrix addition and multiplication similar to the role of 0 and 1 in the real numbers. The determinant of a square matrix is nonzero if and only if the matrix has a multiplicative inverse.
- **Standard N.VM.11** Multiply a vector (regarded as a matrix with one column) by a matrix of suitable dimensions to produce another vector. Work with matrices as transformations of vectors.
- **Standard N.VM.12** Work with 2×2 matrices as transformations of the plane, and interpret the absolute value of the determinant in terms of area.
- **Standard N.VM.13** Solve systems of linear equations up to three variables using matrix row reduction.

Strand: NUMBER AND QUANTITY—Complex Number Systems (N.CN)

Perform arithmetic operations with complex numbers **(Standard 3).** Represent complex numbers and their operations on the complex plane **(Standards 4–6).** Use complex numbers in polynomial identities and equations **(Standard 10).**

- **Standard N.CN.3** Find the conjugate of a complex number; use conjugates to find moduli and quotients of complex numbers.
- **Standard N.CN.4** Represent complex numbers on the complex plane in rectangular and polar form (including real and imaginary numbers), and explain why the rectangular and polar forms of a given complex number represent the same number.
- **Standard N.CN.5** Represent addition, subtraction, multiplication, and conjugation of complex numbers geometrically on the complex plane; use properties of this representation for computation. *For example, $(-1 + \sqrt{3}\,i)^3 = 8$, because $(-1 + \sqrt{3}\,i)$ has modulus 2 and argument 120°.*

- **Standard N.CN.6** Calculate the distance between numbers in the complex plane as the modulus of the difference, and the midpoint of a segment as the average of the numbers at its endpoints.
- **Standard N.CN.10** Multiply complex numbers in polar form and use DeMoivre's Theorem to find roots of complex numbers.

Strand: ALGEBRA: REASONING WITH EQUATIONS AND INEQUALITIES (A.REI)

Solve systems of equations **(Standards 8–9).**

- **Standard A.REI.8** Represent a system of linear equations as a single matrix equation in a vector variable.
- **Standard A.REI.9** Find the inverse of a matrix, if it exists, and use it to solve systems of linear equations (using technology for matrices of dimension 3 x 3 or greater).

Strand: FUNCTIONS—Interpreting Functions (F.IF)

Analyze functions using different representations **(Standard 7, 10–11).**

- **Standard F.IF.7** Graph functions expressed symbolically, and show key features of the graph, by hand in simple cases and using technology for more complicated cases.
  - **d.** Graph rational functions, identifying zeros, asymptotes, and point discontinuities when suitable factorizations are available, and showing end behavior.
  - **f.** Define a curve parametrically and draw its graph.
- **Standard F.IF.10** Use sigma notation to represent the sum of a finite arithmetic or geometric series.
- **Standard F.IF.11** Represent series algebraically, graphically, and numerically.

Strand: FUNCTIONS—Building Functions (F.BF)

Build a function that models a relationship between two quantities **(Standard 1).** Build new functions from existing functions **(Standard 4–5).**

- **Standard F.BF.1** Write a function that describes a relationship between two quantities.
  - **c.** Compose functions. *For example, if* T(y) *is the temperature in the atmosphere as a function of height, and* h(t) *is the height of a weather balloon as a function of time, then* T(h(t)) *is the temperature at the location of the weather balloon as a function of time.*
- **Standard F.BF.4** Find inverse functions.
  - **b.** Verify by composition that one function is the inverse of another.

c. Read values of an inverse function from a graph or a table, given that the function has an inverse.

d. Produce an invertible function from a non-invertible function by restricting the domain.

■ **Standard F.BF.5** Understand the inverse relationship between exponents and logarithms and use this relationship to solve problems involving logarithms and exponents.

Strand: FUNCTIONS—Trigonometric Functions (F.TF)

Extend the domain of trigonometric functions using the unit circle **(Standard 4).** Model periodic phenomena with trigonometric functions **(Standard 6–7).** Prove and apply trigonometric identities **(Standard 9).**

■ **Standard F.TF.4** Use the unit circle to explain symmetry (odd and even) and periodicity of trigonometric functions.

■ **Standard F.TF.6** Understand that restricting a trigonometric function to a domain on which it is always increasing or always decreasing allows its inverse to be constructed.

■ **Standard F.TF.7** Use inverse functions to solve trigonometric equations that arise in modeling contexts; evaluate the solutions using technology, and interpret them in terms of the context

■ **Standard F.TF.9** Prove the addition and subtraction formulas for sine, cosine, and tangent, and use them to solve problems.

Strand: GEOMETRY—Geometric Measurement and Dimension (G.GMD)

Explain volume formulas and use them to solve problems **(Standard 2).**

■ **Standard G.GMD.2** Give an informal argument using Cavalieri's principle for the formulas for the volume of a sphere and other solid figures.

Strand: GEOMETRY—Expressing Geometric Properties With Equations (G.GPE)

Translate between the geometric description and the equation for a conic section **(Standards 2–3).**

■ **Standard G.GPE.2** Derive the equation of a parabola given a focus and a directrix.

■ **Standard G.GPE.3** Derive the equations of ellipses and hyperbolas given the foci, using the fact that the sum or difference of distances from the foci is constant.

Strand: STATISTICS—Conditional Probability and the Rules of Probability (S.CP)

Understand independence and conditional probability and use them to interpret data **(Standards 2–3).** Use the rules of probability to compute probabilities of compound events in a uniform probability model **(Standards 7–9).**

- **Standard S.CP.2** Understand that two events *A* and *B* are independent if the probability of *A* and *B* occurring together is the product of their probabilities, and use this characterization to determine if they are independent.
- **Standard S.CP.3** Understand the conditional probability of *A* given *B* as *P(A and B)/P(B)*, and interpret independence of *A* and *B* as saying that the conditional probability of *B* given *A* is the same as the probability of *B*.
- **Standard S.CP.7** Apply the Addition Rule, $P(A \text{ or } B) = P(A) + P(B) - P(A \text{ and } B)$, and interpret the answer in terms of the model.
- **Standard S.CP.8** Apply the general Multiplication Rule in a uniform probability model, $P(A \text{ and } B) = P(A)P(B|A) = P(B)P(A|B)$, and interpret the answer in terms of the model.
- **Standard S.CP.9** Use permutations and combinations to compute probabilities of compound events and solve problems.

ADA Compliant May 2021

APPENDICES

Utah Secondary
Supplemental Standards
for
MATHEMATICS
High School (9–12)

Adopted May 2021
by the
Utah State Board of Education

ADA Compliant May 2021

The Utah State Board of Education, in January of 1984, established policy requiring the identification of specific core standards to be met by all K–12 students in order to graduate from Utah's secondary schools. The Utah State Board of Education regularly updates the Utah Core Standards, while parents, teachers, and local school boards continue to control the curriculum choices that reflect local values.

The Utah Core Standards are aligned to scientifically based content standards. They drive high quality instruction through statewide comprehensive expectations for all students. The standards outline essential knowledge, concepts, and skills to be mastered at each grade level or within a critical content area. The standards provide a foundation for ensuring learning within the classroom.

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District	*Name*	*City*
District 1	**Jennie L. Earl**	Morgan, UT
District 2	**Scott L. Hansen**	Liberty, UT
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District 4	**Brent J. Strate**	South Ogden, UT
District 5	**Laura Belnap**	Bountiful, UT
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District 14	**Mark Huntsman**	Fillmore, UT
District 15	**Kristan Norton**	St. George, UT
	Sydnee Dickson	State Superintendent of Public Instruction
	Lorraine Austin	Secretary to the Board

5/2021

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SECONDARY SUPPLEMENTAL MATHEMATICS STANDARDS

COLLEGE PREP MATHEMATICS

Prerequisite: Secondary III

College Prep Math formalizes and reinforces concepts from the Secondary Mathematics series to provide students with the foundational skills and understanding prerequisite to College Algebra (1050). Students will reason abstractly and quantitatively while solving linear and quadratic equations and linear inequalities. They will efficiently use polynomial and rational expressions and functions, radicals and complex numbers, and exponential and logarithmic expressions and functions to model and solve mathematical problems. They will explore conic sections and represent parabolic data. Throughout this course, students will make sense of problems and persevere in solving them, use tools strategically, and attend to precision.

Standard I: Students will develop fluency with the language and operations of algebra to evaluate, analyze, and solve problems.

Objective 1: Perform operations and simplify expressions with rational, irrational, and complex numbers.

a. Rewrite expressions involving radicals and rational exponents using the properties of exponents.
b. Add, subtract, multiply, and divide radical expressions.
c. Simplify expressions involving complex numbers and express them in standard form, $a + bi$.

Objective 2: Solve systems of equations and inequalities.

a. Solve systems of linear equations in two variables algebraically and graphically.
b. Solve a simple system consisting of a linear equation and a quadratic equation in two variables algebraically and graphically.
c. Use matrices to solve systems of linear equations.*
d. Model and solve problems arising from authentic situations using systems of equations and inequalities.

Objective 3: Solve and graph quadratic equations.

a. Use the method of completing the square to transform any quadratic equation into an equation of the form $(x - p)^2 = q$, and use this to derive the quadratic formula.
b. Solve quadratic equations by inspection, taking square roots, factoring, completing the square, and using the quadratic formula.
c. Model and solve real-world situations using quadratic relationships in two variables.

Objective 4: Simplify rational and radical expressions and solve and graph rational and radical equations.

a. Simplify and rewrite simple rational expressions in different forms using algebraic techniques, including long division.
b. Solve rational equations.
c. Solve radical equations in one variable, including those with extraneous solutions.
d. Graph rational functions, identifying domain, range, zeros and asymptotes, and showing end behavior.

Mathematical Language and Symbols Students Should Use:

asymptote	completing the square	complex number	
compound inequality	extraneous root	quadratic formula	radical
rational	irrational	system of equations	minimum
maximum	intercept		

Standard II: Students will understand and represent functions and analyze function behavior.

Objective 1: Understand the concept of a function and use function notation.

a. Understand that a function from one set (called the domain) to another set (called the range) assigns to each element of the domain exactly one element of the range.
b. Determine when a relation is a function.
c. Use function notation, evaluate functions for inputs in their domains, and interpret statements that use function notation in terms of a context.

Objective 2: Analyze functions using different representations.

a. Graph linear, quadratic, square root, cube root, piecewise, and polynomial functions expressed symbolically, and show key features of the graphs—by hand in simple cases, and using technology for more complicated cases.
b. Identify key features of functions from either graphs or equations.
c. Write a function in different but equivalent forms to reveal and explain different properties of the function.
d. Compare properties of two functions, each represented in a different way (algebraically, graphically, numerically in tables, or by verbal description).

Objective 3: Build new functions from existing functions.

a. Combine standard function types using arithmetic operations.
b. Compose functions.
c. Find the inverse of a function when it exists.
d. Produce an invertible function from a non-invertible function by restricting the domain.
e. Graph functions using transformations of parent functions.

Objective 4: Construct exponential models and use them to solve problems.

a. Understand that exponential functions grow by equal factors over equal intervals.
b. Recognize and model situations in which a quantity grows or decays by a constant percent rate per unit interval relative to another.
c. Construct exponential functions, including geometric sequences, when given a graph, a description of a relationship, or two input-output pairs.
d. Graph exponential functions.
e. Interpret the parameters in an exponential function in terms of a context.

Objective 5: Understand the inverse relationship between exponents and logarithms and use this relationship to solve problems.

a. Simplify, expand, or condense simple logarithmic expressions.
b. Connect the manipulation of logarithmic expressions with the laws of exponents.
c. Convert logarithms between bases.
d. Graph logarithmic functions.
e. Model and solve authentic problem situations using logarithms.

Mathematical Language and Symbols Students Should Use:

domain	exponential function	function	f(x)
interval notation	inverse	logarithm	parameter
parent function	piecewise function	range	relation
transformation	intercept	maximum	minimum

Standard III: Students will apply geometric concepts in modeling situations.

Objective 1: Use coordinate algebra to represent and analyze geometric situations.

a. Prove the slope criteria for parallel and perpendicular lines and solve related problems.

b. Use coordinates to compute perimeters of polygons and areas of triangles and rectangles using the distance formula.

c. Use coordinates to model geometric problems involving distances and relationships and apply those models to contextual situations.*

Objective 2: Translate between the geometric descriptions and the equations for conic sections.

a. Derive the equation of a circle, given center and radius, using the Pythagorean Theorem.

b. Complete the square to find the center and radius of a circle when given the equation.

c. Write an equation of a parabola in the form $y = a(x\text{-}h)^2 + k$ when given the graph or equation, or when given the focus and directrix.

d. Identify the vertices, foci, and intercepts of ellipses and hyperbolas with centers at the origin.*

e. Derive the equations of ellipses and hyperbolas given the foci, using the fact that the sum or difference of distances from the foci is constant.*

Mathematical Language and Symbols Students Should Use:

eccentricity	ellipse	foci	hyperbola	parabola
vertex	parallel	perpendicular		
Pythagorean Theorem	directrix			

* May be eliminated in a semester course.

Standard IV: Students will apply statistical methods to make informed decisions.*

Objective 1: Formulate questions and answer these questions by organizing, summarizing, and analyzing data.

a. Collect data, and classify the data as univariate or bivariate and categorical or quantitative.
b. Summarize distributions using measures of center and variability, and communicate findings coherently using graphical representations and data summaries.
c. Use the mean and standard deviation of normally distributed data to estimate population percentages.
d. Fit a function to bivariate data.

Objective 2: Make inferences and justify conclusions using data.

a. Understand statistics as a process for making inferences about population parameters based on a random sample from that population.
b. Make inferences and justify conclusions based on data collection.
c. Use data from a sample survey to estimate a population mean or proportion.

Objective 3: Use the rules of probability to compute probabilities, and use probabilities to interpret data.

a. Understand and calculate probabilities of independent events.
b. Understand and calculate conditional probabilities.
c. Use permutations and combinations to compute probabilities of compound events and solve problems.
d. Calculate expected values and use them to solve problems.

Mathematical Language and Symbols Students Should Use:

bivariate	categorical	cumulative frequency plot	
interquartile range	measures of center	measures of variability (spread)	
normal distribution	quantitative data	univariate	regression

INTRODUCTORY CALCULUS

Prerequisite: Pre-calculus or Math 1050

Introductory Calculus is a year-long course where students will develop a conceptual understanding of limits, derivatives, and integrals through discovery and applications. Students will study polynomial, radical, exponential, logarithmic, and rational functions with an emphasis on graphical analysis preparatory to the study of limits. Students will compare the concept of slope with the formal definition of the derivative as well as develop and apply different techniques of integration to model and solve a variety of problems. Students will survey the basic topics of calculus and demonstrate their application in a wide variety of fields. Students will explore calculus concepts through inquiry using technology and develop explicit methods to use calculus in applications.

Standard I: Students will develop a conceptual understanding of limits and the characteristics of functions.

Objective 1: Analyze and perform applications using characteristics of functions, including polynomial, radical, rational, piece-wise, power, exponential, and logarithmic functions.

a. Sketch graphs showing key features of functions.
b. Perform transformations of functions graphically and algebraically by changing one or more parameters in $y = af(x)$, $y = f(ax)$, $y = f(x) + a$, and $y = f(x + a)$ for both positive and negative a.
c. Describe and analyze local and global behavior of functions, including increasing, decreasing, asymptotic and end behavior.
d. Recognize even and odd functions from their graphs and algebraic expressions for them.
e. Identify the domain, range, and other attributes of families of functions.

Objective 2: Demonstrate an understanding of the nature of limits.

a. Estimate the limit of a function at a point using graphs and tables.
b. Calculate limits using algebra.
c. Calculate limits involving infinity.
d. Use the asymptotic behavior of a function to identify limits.

Objective 3: Understand the concept of continuity as a property of functions.

a. Understand the definition of continuity as it applies to functions.
b. Understand the role of limits in continuity.
c. Demonstrate an understanding of the implications of continuity on the graphs of functions, including use of the Intermediate Value Theorem and Extreme Value Theorem.

Mathematical Language and Symbols Students Should Use:

polynomial	radical	rational	piece-wise	power
exponential	logarithmic	$f(x)$		$\lim_{x \to n} f(x)$
asymptote	even function	odd function	limit	
continuity	Intermediate Value Theorem		Extreme Value Theorem	

Standard II: Students will understand the derivative as an instantaneous rate of change and will use it to analyze curves, optimize values, and model rates of change in applied contexts.

Objective 1: Develop an understanding of derivatives graphically, numerically, and analytically.

a. Interpret derivatives as a limit of average rates or as an instantaneous rate of change in various contexts.
b. Define derivatives as the limit of the difference quotient.
c. Estimate, calculate, and interpret the derivative of a function.
d. Approximate a slope using local linearity, secant lines and tangent lines.
e. Discover and understand the implications of continuity for differentiability.

Objective 2: Manipulate and simplify derivatives using properties.

a. Find derivatives of polynomial functions using the power rule.
b. Understand and use derivative rules for sums, products, and quotients of functions.
c. Calculate the derivative of a composite function using the chain rule.
d. Find derivatives by implicit differentiation with respect to a specified variable.

Objective 3: Solve application problems involving polynomial, exponential, and logarithmic models, including applications in business, economics, and physics.

a. Construct a framework for the application of derivative by solving various real-life problems.
b. Use optimization techniques to maximize and/or minimize functions.
c. Graph functions to illustrate curves.
d. Explain the meaning of the second derivative.
e. Model rates of change, including related rates problems.

Mathematical Language and Symbols Students Should Use:

rate of change	derivative	differentiable	power rule
product rule	quotient rule	composite function	chain rule
local linearity	implicit differentiation	$\frac{\Delta Y}{\Delta X}, \frac{DY}{DX}$	

Standard III: Students will develop and apply different techniques of integration to model and solve a variety of problems.

Objective 1: Use various numerical methods to approximate definite integrals of functions represented as equations, graphs, and tables.

a. Understand the definite integral of a function as the area under the graph of that function between two points.
b. Approximate the area under the curve by using areas of familiar geometric shapes.
c. Approximate the area under the curve by using left, right, and midpoint Riemann sums.
d. Approximate the area under the curve by using trapezoidal sums.
e. Recognize the definite integral as a limit of Riemann sums.

Objective 2: Use the Fundamental Theorem of Calculus to find definite integrals and to solve differential equations.

a. Understand antidifferentiation and the indefinite integral in terms of reversing the operation of differentiation.
b. Understand that the antiderivative is unique up to a constant.
c. Relate antiderivatives and definite integrals by the Fundamental Theorem of Calculus.
d. Use an initial condition to find a specific antiderivative.
e. Create slope fields from differential equations and use them to draw possible antiderivatives.
f. Understand properties of integrals.

Objective 3: Model, solve, and interpret applications of antiderivatives.

a. Approximate and evaluate change in various contexts numerically, algebraically, and graphically.
b. Use integration techniques to solve geometric problems, including finding the area between two curves.
c. Use integration techniques to solve problems in physics.

Mathematical Language and Symbols Students Should Use:

average value definite integral $\int_b^a f(x)$ Riemann sum antiderivative Fundamental Theorem of Calculus

INTRODUCTORY STATISTICS

Prerequisite: Secondary Math II

Statistics is a branch of mathematics that explores concrete connections with everyday living. Students will develop critical thinking skills with life-long application. Students will gather, graph, examine, compare and interpret data using technology, including graphing calculators or computer statistics software. They will describe data and make informed decisions and predictions based on data.

*Note: The course may be taught as a one-semester or two semester course. Content marked with * may be reduced in depth or eliminated for a one-semester course.*

Standard I: Students will understand, use, and evaluate random processes underlying statistical analysis.

Objective 1: Use sample survey data collected through random samples to draw conclusions about populations.

a. Recognize sources of bias in surveys, and discuss how surveys may be intentionally biased to support certain agendas.
b. Explain the importance of randomness in good survey design.
c. Pose a question, choose an appropriate method of random selection, conduct a survey, and summarize the results in graphical displays.
d. Distinguish between different survey designs such as SRS, cluster sampling, stratified sampling, and systematic sampling.*

Objective 2: Describe and use the features of good experimental design, such as random assignment of treatments, controls, placebos, blinding, and blocking.

a. Distinguish between an observational study and an experiment, and be able to select which method is appropriate to collect desired information.
b. Recognize possible sources of bias in various experiments, and describe how the features of good experimental design will reduce bias.
c. Pose a question, conduct one or more simple experiments using appropriate features of experimental design for the data that is being collected, and summarize the results in graphical displays.
d. Explain the importance of experimental ethics, and debate historical violations of experimental ethics.*

Objective 3: Discuss and interpret surveys, experiments, and observations using information from government data, current events, medical experiments, polls, and news media.

a. Consider the reasonableness of claims of data from various sources, using examples to illustrate the uses and misuses of statistics that appear in the media.
b. Distinguish between causality and correlation, and be able to recognize unwarranted conclusions.
c. Recognize when data is misrepresented by graphical manipulation, such as modified axes or use of incorrect visual proportions.
d. Discuss the role of government reports such as the consumer price index for making comparisons in data.*
e. Calculate percent change and perform simple calculations for price changes over the years due to inflation.*

Mathematical Language and Symbols Students Should Use: data,

survey	SRS (simple random sampling)		cluster sampling
stratified sampling	systematic sampling	experiment	observation
treatment	control	placebo	blinding
blocking	census	percent change	
consumer price index	causality	correlation	

Standard II: Students will summarize and interpret data.

Objective 1: Interpret and display data by selecting appropriate graphical methods.

a. Distinguish between quantitative and categorical data.
b. Use quantitative data to create dot plots, stem plots, histograms, box plots, and scatter plots and use them to make sense of the data.
c. Use categorical data to create circle graphs, bar graphs and frequency tables and use them to make sense of the data.

Objective 2: Summarize data and be able to use technology such as calculators or computer software to assist in calculations.

a. Calculate measures of center, and estimate center from data presented in a variety of forms, such as charts, tables, and graphs.
b. Select and interpret appropriate measures of spread.
c. Describe the distribution of data considering shape, skewness, modality, and outliers.

Objective 3: Use data summaries to interpret and compare data.

a. Describe and compare individual performances in terms of quartiles, percentiles and standard deviations.
b. Interpret differences in shape, center, and spread in the context of the data sets, accounting for possible effects of outliers.
c. Use statistics appropriate to the shape of the data distribution to compare center and spread of two or more different data sets.

Objective 4: Describe the characteristics of the normal distribution, and create an understanding of the standard deviation as a measure of spread.

a. Examine data sets that approximate the normal distribution, and recognize the characteristics of data that are normally distributed.
b. Compare individual measurements using the mean and standard deviation to find standardized scores and identify unusual data points.
c. Use the 68%–95%–99.7% rule to determine the probability of events.
d. Use the 68%–95%–99.7% rule to create and explain confidence intervals.*

Mathematical Language and Symbols Students Should Use:

quantitative data	categorical data	dot plot	stem plot	bar graph
frequency table	mean	median	mode	range
IQR (inter-quartile range)		standard deviation		
standardized scores	skewed	outlier	quartile	percentile
normal distribution	mean	standard	deviation	
68%–95%–99.7% rule				

Standard III: Students will make inferences and justify conclusions based on data.

Objective 1: Summarize, represent, and interpret bivariate data.

a. Create and use graphs of bivariate data to visually assess trends and recognize patterns.
b. Calculate regression lines and correlation coefficients for linear data using technology such as calculators or computer software.
c. Use regression equations to make appropriate predictions.
d. Interpret the slope (rate of change) and the intercept (constant term) of a linear model in the context of the data.
e. Make predictions based on patterns and trends of non-linear data, such as seasonal data, tidal tables, sunspots, and population changes.*

Objective 2: Display and compare data to make predictions and formulate conclusions.

a. Describe the effect of outliers on predictions.
b. Recognize and discuss the pitfalls of extrapolation in predictions.
c. Compare actual data measurements with predicted values, and discuss the reasonableness of predictions.

Objective 3: Make inferences and justify conclusions from sample surveys, experiments, and observational studies.

a. Understand statistics as a process for making inferences about population parameters based on a random sample from that population.
b. Understand and interpret confidence intervals generated from data.*
c. Use the results of hypothesis testing to interpret sample data and draw conclusions.*

Mathematical Language and Symbols Students Should Use:

bivariate data	linear and non-linear data	slope	rate of change
regression	correlation	coefficient	scatter plot
interpolation	extrapolation	prediction	hypothesis test
confidence interval			

Standard IV: Students will understand and use probability rules.

Objective 1: Use the rules of probability to calculate independent and conditional probabilities in real contexts.

a. Distinguish between subjective, experimental, and theoretical probability.
b. Calculate probabilities using addition and multiplication rules, tree diagrams, and twoway tables using correct probability notation.

Objective 1: Use the rules of probability to calculate independent and conditional probabilities in real contexts.

a. Distinguish between subjective, experimental, and theoretical probability.
b. Calculate probabilities using addition and multiplication rules, tree diagrams, and twoway tables using correct probability notation.
c. Calculate conditional probabilities of compound events using two-way tables and Venn diagrams.
d. Use permutations and combinations to find probabilities.*

Objective 2: Adapt probability models to solve real-world problems.

a. Perform simulations to estimate probability outcomes using technology and objects such as coins, spinners, cards, and dice.
b. Identify and explain common misconceptions regarding probability, including long-run vs. short-run behavior.
c. Discuss probability applications in decision making, using terms such as "odds" and "risk," including applications in insurance, medical treatments, and extreme sports.*

Objective 3: Use probability to make decisions and analyze outcomes.*

a. Calculate expected values and use them to solve problems.*
b. Develop a probability distribution for a random variable and find the expected value.*
c. Weigh the possible outcomes of a decision by assigning probabilities to payoff values and finding expected values.*
d. Use probabilities to make fair decisions.*

Mathematical Language and Symbols Students Should Use:

theoretical	subjective and experimental probability		independence
conditionalprobability	two-way table	permutation	combination
simulation	tree diagram	addition and multiplication laws	
P(A), P(A|B), ${}_nC_r$, ${}_nP_r$			

MATHEMATICAL DECISION MAKING FOR LIFE

Prerequisite: Secondary Mathematics II

Mathematical Decision Making is a four-quarter course for seniors. The course includes mathematical decision making in finance, modeling, probability and statistics, and making choices. The four quarters of instruction are independent of each other, allowing students to enter and exit the course quarterly. Students will make sense of authentic problems and persevere in solving them. They will reason abstractly and quantitatively while communicating mathematics to others. Students will use appropriate tools, including technology, to model mathematics. Students will use structure and regularity of reasoning to describe mathematical situations and solve problems.

QUARTER A—MATHEMATICAL DECISION MAKING: FINANCE

Standard I: Students will use mathematical analysis to manage personal resources and make financially sound decisions.

Objective 1: Determine, represent and analyze mathematical models for various types of income calculations.

a. Compute and compare hourly wages, given commissions or salaries and hours worked.

b. Compute gross earnings based on commissions, salaries, hourly wages, or piece-work.

c. Compute net earnings after common payroll deductions.

d. Research and compare annual earnings for various employment opportunities.

Objective 2: Create, represent, and justify personal budgets.

a. Create spreadsheets, tables, and charts that represent personal income and expenses.

b. Calculate the total costs of owning a car, including monthly payments, insurance, maintenance, and fuel.

c. Analyze and model periodic monthly expenditures, including those that change during the year such, as heating and cooling costs.

Objective 3: Analyze mathematical models related to investing and borrowing money.

a. Compute and compare the anticipated earnings for investments and savings plans.

b. Interpret stock market data charts.

c. Research and predict retirement income from savings, Social Security benefits, pensions, and investments.

d. Compute the costs of loans for monthly payments.
e. Compare time and costs required to borrow money compared to saving for purchase of an item.
f. Analyze various types of loans to determine the best loan for a given situation.

Objective 4: Analyze numerical data to make quantitative and qualitative decisions.

a. Research, compare, and contrast published ratios, rates, ratings, averages, weighted averages, and indices to make informed decisions.
b. Use spreadsheets to manage large quantities of data.
c. Understand and analyze situations involving large numbers, such as national debt or national budgets.

Mathematical Language and Symbols Students Should Use:

APR (fixed and variable)	adjustments	annual fees	bi-weekly
capital gain/loss	compound interest	co-payment	deductible
deductions	FICA	future value	graduated
gross	index	interest	net
percent change	premium	principal	semi-monthly
stocks	weighted average	yield	

QUARTER B—MATHEMATICAL DECISION MAKING: MODELING

Standard II: Students will use mathematical models to organize, communicate, and solve problems.

Objective 1: Use matrices to represent and analyze mathematical situations.

a. Use matrices to represent and manipulate data.
b. Multiply matrices by scalars to produce new matrices.
c. Add, subtract, and multiply matrices of appropriate dimensions.
d. Use matrices to represent geometric transformations.
e. Use matrices to solve applied problems.

Objective 2: Model mathematical problems with geometric tools.

a. Use geometric methods to solve design problems.
b. Calculate measures of perimeter, surface area, area, and volume, and apply those measures to relevant situations.

Objective 3: Use mathematics to model and solve problems involving change.

a. Analyze and solve problems involving models for linear, exponential, and logistic growth and decay.
b. Identify, model, and solve problems involving cyclical change that can be represented using trigonometric functions.
c. Identify, model, and solve problems involving change that can be represented with a piecewise function.
d. Model and solve problems involving recursion or iteration.

Mathematical Language and Symbols Students Should Use:

amplitude	area	cyclical	exponential	iteration
linear	logistic	matrix	piecewise	perimeter
period	recursion	scalar	surface area	volume

QUARTER C—MATHEMATICAL DECISION MAKING: PROBABILITY AND STATISTICS

Standard III: Students will use statistics and probability to make decisions.

Objective 1: Understand and communicate statistical information.

a. Report results of statistical studies in both oral and written form, including graphical representations.
b. Describe strengths and weaknesses of sampling techniques, data and graphical displays, and interpretations of summary statistics.
c. Identify uses and misuses of statistical analyses.

Objective 2: Develop and evaluate inferences and predictions that are based on data.

a. Understand and evaluate random processes underlying statistical experiments.
b. Determine possible sources of statistical bias and describe the potential impact of such bias on a study.
c. Make inferences and justify conclusions from sample surveys, experiments, and observational studies.
d. Use data from a sample survey to estimate a population mean or proportion.

Objective 3: Apply statistical methods to design and conduct a survey or an experiment.

a. Formulate a question that can be analyzed using statistical methods.
b. Determine possible sources of variability of data, including both those that can and cannot be controlled.

c. Identify the population of interest, select an appropriate sampling technique, and collect data.
d. Create graphical displays of data.
e. Calculate and compare measures of central tendency, spread, and unusual features in data.

Objective 4: Use the rules of probability to calculate independent and conditional probabilities in real contexts.

a. Distinguish between subjective, experimental, and theoretical probability.
b. Calculate probabilities using addition and multiplication rules, tree diagrams, and two-way tables using correct probability notation.
c. Calculate conditional probabilities of compound events using two-way tables and Venn diagrams.
d. Use permutations and combinations to find probabilities.

Objective 5: Analyze risk and return in the context of everyday situations.

a. Construct and analyze tree diagrams, Venn diagrams, and area models to make decisions in problem situations.
b. Construct and interpret two-way frequency tables of data.
c. Weigh the possible outcomes of a decision by assigning probabilities to payoff values and finding expected values.
d. Use probabilities to make fair decisions.
e. Analyze decisions and strategies using probability concepts.

Mathematical Language and Symbols Students Should Use:

- bias
- combination
- conditional
- expected value
- experiment
- experimental probability
- fair decision
- frequency table
- independent
- sample
- survey
- observational study
- permutation
- randomization
- theoretical probability
- tree diagram
- variability
- Venn diagram
- measures of central tendency (mean, median, mode)
- measures of spread (range, standard deviation)

QUARTER D—MATHEMATICAL DECISION MAKING: USING MODELS TO MAKE CHOICES

Standard IV: Students will use mathematical models to analyze situations and make choices.

Objective 1: Construct viable arguments and critique the reasoning of others.

a. Use stated assumptions, definitions, and previously established results to construct an argument.
b. Make conjectures and build a logical progression of statements to explore the truth of conjectures.
c. Recognize and use counterexamples.
d. Justify and communicate conclusions, and respond to the arguments of others.
e. Compare two plausible arguments and make decisions based on correct logic.

Objective 2: Analyze and evaluate the mathematics behind various ranking and selection methods.

a. Analyze and apply various ranking algorithms to determine an appropriate method for a given situation.
b. Evaluate various voting and selection processes to determine an appropriate method for a given situation.
c. Analyze and apply various algorithms for making fair divisions.

Objective 3: Construct, analyze, and interpret flow charts.

a. Construct flowcharts to describe processes or problem-solving procedures.
b. Analyze flowcharts and follow procedures to solve problems.
c. Evaluate efficiency of control processes.
d. List requirements and restrictions needed for a suggested algorithm.

Objective 4: Use a variety of graphical models to represent network and scheduling problems.

a. Solve scheduling problems using mathematical models.
b. Explore shortest route and fastest route situations.
c. Solve precedence or critical paths problems to facilitate "what if" scenarios.

Mathematical Language and Symbols Students Should Use:

- algorithm
- counterexample
- critical paths
- Euler path
- flow chart
- logic
- minimal spanning trees
- truth table
- vertex-edge graph

MATHEMATICS OF PERSONAL FINANCE

Prerequisite: Currently enrolled in or successful completion of Secondary Math II

Mathematics of Personal Finance is designed for students in their junior or senior year of high school. The course represents content from mathematics and personal finance that are essential for students who will assume roles as consumers, money managers and members of a global workforce. Successful completion of all four quarters of this course will fulfill 1 credit of supplemental mathematics requirements.

Standard I: Students will use number sense, perform operations, solve problems and make decisions using rational and irrational numbers to set and implement financial goals.

Objective 1: Determine, represent and analyze mathematical models and formulas for various types of financial calculations.

a. Use financial formulas that require operations with real numbers.
b. Interpret the meaning of integers in financial situations.
c. Use the irrational number e to evaluate continuously compounded interest.
d. Calculate net and gross income.

Objective 2: Analyze financial plans and calculated costs of personal finance.

a. Describe the value and use of savings in financial planning.
b. Explain how government regulations protect savers.
c. Compare the risk, return, liquidity, and costs for savings and investments.
d. Describe the value of investing and types of investments in the financial planning process.
e. Explain the effects of inflation on savings and investments.
f. Analyze the relationship between risk and return.
g. Describe and select appropriate financial products for different financial goals.

Objective 3: Complete calculations associated with personal finance and financial planning using technology.

a. Calculate future value of investments and present value of investments.
b. Calculate stock market transaction fees, stock splits and dividend income.
c. Compute monthly payments for loans of different types.

d. Determine the required time to pay off loans.
e. Compute the anticipated earnings for investments and savings plans.
f. Compute average daily balances on loans and investments.

Mathematical and Financial Language and Symbols Students Should Use

- decimals
- rate
- irrational numbers
- deductions
- APR (fixed vs. variable)
- percent change
- bi-weekly
- capital gain/loss
- mutual funds
- co-payments
- investments
- percent
- rational numbers
- graduated commission
- FICA
- weekly
- simple moving average
- semi-monthly
- CD
- premium
- withholdings
- estimate
- integers
- gross
- adjustments
- 52 week high and low
- arithmetic average
- monthly
- bonds
- future value
- expenditures
- exponent
- net
- exempt
- annual fees
- commission
- stocks
- deductibles
- balances

Exploratory Concepts and Skills

- NASDAQ
- NYSE
- S & P
- Dow Jones
- Tax forms

Standard II: Students will use graphs, charts and tables for financial decision-making and planning.

Objective 1: Summarize and interpret information from graphs, tables and charts.

a. Create and interpret scatter plots, line graphs, bar graphs, circle graphs, histograms, and pictographs associated with finances.
b. Use regression to interpolate and extrapolate data associated with finances.
c. Use charts, tables and graphs to identify and track fixed and variable expenses and assets.
d. Use spreadsheets to evaluate data.
e. Interpret stock market data charts.

Objective 2: Solve problems using graphical representations

a. Represent the value of an employee benefit package graphically.
b. Compare different employee benefit packages in the same job.
c. Compare different employee benefit packages in different jobs.
d. Analyze graphs displaying various interest rates on returns of investment.

e. Graph the costs of various aspects of renting a home vs. purchasing a home.
f. Graph the growth of different long term investments and income earnings.
g. Graph linear and exponential functions associated with finance.

Mathematical and Financial Language and Symbols Students Should Use

plots	interpolate	extrapolate	data
information	charts	graphs	regression
benefit packages	amortization charts	bankruptcy	tax tables

Exploratory Concepts and Skills

- Creating plots
- Regressions
- Evaluation of when to use different types of graphs

Standard III: Students will use algebra associated with personal finances.

Objective 1: Analyze monthly, annual, and life-long financial plans.

a. Compare time and costs required to pay off a loan compared to saving for a purchase.
b. Compute monthly net and gross earnings based on various earning possibilities.
c. Compute annual net and gross earnings based on various earning possibilities.
d. Calculate present and future investment values.
e. Use algebraic functions to calculate average daily balances on loans and investments.

Objective 2: Work with functions associated with finance and financial planning.

a. Interpret the rate of change of functions associated with financial planning.
b. Represent financial scenarios with linear and non-linear functions.
c. Use functions to make predictions for future financial values.
d. Distinguish between financial contexts that result in linear and non-linear functions.

Objective 3: Determine, represent and analyze relationships for various types of growth and decay models.

a. Compute depreciation problems that involve exponential decay.
b. Compute inflation problems that involve exponential growth.
c. Identify growth and decay situations from real world problems.

d. Calculate purchasing power.

Mathematical and Financial Language and Symbols Students Should Use

function	linear	non-linear	extrapolate
rate	average	stock splits	dividend income
earnings	exponential growth	exponential decay	

Exploratory Concepts and Skills

- Explore national debt projections
- Calculate income and taxes for a vocation in which they are interested

Standard IV: Students will use rational decision making for financial planning.

Objective 1: Create, represent and justify personal monthly and yearly budgets.

a. Create spreadsheets, tables, or charts that represent personal income and expenses.
b. Design a plan to reach a specific financial goal.
c. List advantages of designing and following a personal financial plan.
d. Analyze the role of cultural, social, and emotional influences on financial behavior.

Objective 2: Relate financial decisions to personal and societal consequences.

a. Recognize that individuals are responsible for their finances.
b. Describe the social and economic consequences of bankruptcy.
c. Describe the consequences of excessive debt including increased consumer costs, inflation, and family instability.
d. Evaluate the role of emotions when making financial decisions.

Objective 3: Research annual earnings for various employment opportunities in the job market to make mathematical decisions for personal income.

a. Create budgets for various earning opportunities including fluctuations in income.
b. Follow and explain the earnings or losses associated with a given investment in the market.
c. Collect, organize and interpret data associated with utilities, cash flow, housing, food, entertainment, medical, and transportation expenses.
d. Compare various vehicles for attaining short-term and long-term financial goals.

Mathematical and Financial Language and Symbols Students Should Use

decimals	percent	exponent	rate	fluctuations
compound interest	interpolate	extrapolate	mean	median
mode	estimate	variable rate of change		
constant rate of change	total	simple interest	units	
gains	losses	variance	consumer costs	
inflation				

Exploratory Concepts and Skills

- Creating spreadsheets and data tables.
- Extrapolating data, interpolating data.
- Writing programs for predicting future values.
- Writing and evaluating formulas.

Standard V: Students will understand and explain the relationship between income sources and career preparation.

Objective 1: Identify various forms of income and analyze factors that affect income.

a. Identify sources of income.
b. Compare common employee benefits.
c. Compare income to the cost-of-living in various geographical areas.
d. Analyze how economic conditions affect income.

Objective 2: Identify and understand required income withholdings.

a. List the reasons for taxation and uses of tax revenues.
b. Describe the purposes of Social Security and Medicare.
c. Identify wages and withholding on an employee payment record.
d. Demonstrate how to complete personal state and federal income tax forms.

Objective 3: Analyze criteria for selecting a career and the impact of career choices on income and financial stability.

a. Describe the correlation between income and a worker's skills, education, the value of the work to society, condition of the economy, and the supply and demand for workers.
b. Develop career plans that include educational requirements, skill development, and income potential.
c. Analyze the costs and benefits of obtaining additional education or developing new skills for the workplace.
d. Identify the risks and rewards of entrepreneurship/self-employment.

Mathematical and Financial Language and Symbols Students Should Use

wages	investments	self-employment	leave
retirement	cost-of-living	economic effect	entrepreneur

Exploratory Concepts and Skills

- Creating spreadsheets.
- Compare early retirement benefits to full retirement benefits of Social Security
- Create case studies of income earnings for various careers

Standard VI: Students will use principles of money management.

Objective 1: Describe the role of planning and maintaining a balanced budget.

a. Develop, monitor, and evaluate a personal budget.
b. Discuss opportunity costs and trade-offs on budget implementation.
c. Identify and discuss the social and personal consequences of not following a budget.
d. Compare and evaluate various tools available for keeping track of budgets.
e. Demonstrate knowledge of financial transactions, checking and savings accounts associated financial services.
f. Demonstrate how to manage a checking account.
g. Evaluate the impact of major purchases on budgeting.

Objective 2: Understand credit uses and costs.

a. Discuss the history and role of credit.
b. List basic types of credit.
c. Describe the risks and responsibilities associated with using credit.
d. Identify methods of establishing and maintaining a good credit rating.
e. Explain the purpose of co-signers and collateral when applying for a loan.
f. Identify warning signs of credit abuse and ways to correct credit problems.
g. Calculate and compare costs associated with the use of credit.
h. Calculate how long it takes to repay debt and the total costs when a borrower makes minimum payments.

Objective 3: Describe the impact of credit on money management.

a. Compare the advantages and disadvantages of different payment methods.
b. Compare the services of various types of financial institutions and identify advantages of comparison shopping before selecting financial services.
c. Describe the relationship between a credit rating and the cost of credit and factors that affect credit worthiness.
d. Explain the value of credit reports and scores to borrowers and lenders.

Objective 4: Describe the rights and responsibilities of buyers and sellers under consumer protection laws.

a. Explain the purposes and features of consumer protection laws, agencies and sources of assistances.
b. Describe ways to avoid identity theft and fraud.
c. Explain the importance of understanding financial contracts.
d. List possible actions a consumer can take in response to excessive debt and collection practices.
e. Describe ways to avoid financial scams and schemes designed to defraud consumers.

Objective 5: Discuss the purposes for insurance and risk management.

a. Identify common types of insurance and their terminology.
b. Describe how insurance and other risk-management strategies protect against financial loss.
c. Discuss insurance needs at various life stages.
d. Identify the importance of estate planning.
e. Discuss the consequences of being under-insured.

Mathematical and Financial Language and Symbols Students Should Use

insurance	auto insurance	health insurance	disability
renter's insurance	homeowners insurance	term	long term
whole life	deductible	premium	grace period
wills	trusts		

Exploratory Concepts and Skills

- Estate planning
- Comparison shopping for insurance
- Exploring risk management sources

MODERN MATHEMATICS

Prerequisite: Secondary Math II

This course introduces students to topics in modern mathematics as they apply to real-world contexts. The course extends students' understanding of the mathematics developed in Algebra 1 and Geometry. The course is intended to help students develop an understanding of how mathematics describes and explains the world in which they live. Students will extend their mathematical literacy, problem-solving skills, and enthusiasm for the power and beauty of mathematics as a tool for quantifying their world.

Teachers will select a minimum of five objectives per semester to explore, and may modify indicators to meet those objectives. Teachers are encouraged to select topics which are of particular interest to their students. Because the topics within the course are not intended to build on one another, students may enter or exit the class throughout the academic year.

Standard 1: Students will expand number sense to understand the language and operations of number systems.

Objective 1: Use concepts of number theory and information systems to effectively manage large amounts of data.

a. Expand understanding of the decimal system by exploring other number-base systems.
b. Use various methods to write and decipher codes.
c. Determine validity of ISBN, UPC and credit card numbers using modular arithmetic.
d. Compute using modular arithmetic.

Objective 2: Use matrices to model, organize, and solve problems involving multiple variables.

a. Use matrices as a way to organize information.
b. Perform basic matrix calculations to solve problems in context.
c. Use matrices and technology to solve systems of linear equations.

Objective 3: Recognize sequences as mathematical patterns and use them to model authentic situations.

a. Find the n^{th} term in arithmetic or geometric sequences.
b. Represent arithmetic and geometric sequences explicitly and recursively.
c. Explore sequences to model authentic situations.

Mathematical Language and Symbols Students Should Use:

binary	hexadecimal	matrix/matrices	
inductive reasoning	deductive reasonin	ISBN	UPC
exponential growth	Fibonacci sequence	Pascal's triangle	modular
arithmetic			

Standard II: Students will use functions to model and solve problems.

Objective 1: Use linear systems of equations and inequalities to model and solve problems.

a. Use algebra and technology to solve systems of linear equations.
b. Use linear programming to maximize or minimize an objective function in context.

Objective 2: Use exponential functions to model and solve problems.

a. Graph and evaluate exponential functions.
b. Use an exponential function to generate values and make predictions in problems that involve exponential growth and decay.
c. Understand the inverse relationship between exponents and logarithms and use this relationship to solve problems.

Mathematical Language and Symbols Students Should Use:

exponential functions	logarithm	maximize	minimize
Linear programming	constraints	base	exponent
objective function			

Standard III: Students will solve problems using symmetry, transformations, graphs, and measurement.

Objective 1: Use concepts of Chaos Theory to describe the behavior of dynamic systems that are highly sensitive to initial conditions.

a. Distinguish between dynamic and non-dynamic systems.
b. Create fractals with and without technology.
c. Use fractals to describe dynamic systems.
d. Investigate the effects of different initial conditions on dynamic systems using technology.

Objective 2: Use concepts from Graph Theory to model and solve problems.

a. Use graphs to represent the relations of various structures and networks.
b. Identify characteristics of a given graph.

c. Classify graphs according to structure.

d. Solve problems of practical interest using graphs.

Objective 3: Extend geometry ideas to analyze art, architecture, music, and nature.

a. Describe artistic and natural structures and phenomena in terms of transformations, symmetry and fractals.

b. Identify projections used in art, architecture, and music.

c. Analyze the use of proportion in art, architecture, and music.

Mathematical Language and Symbols Students Should Use:

Chaos Theory	strange attractors	scaling	bifurcation
dynamic system	connectedness	completeness	regularity
Golden ratio	Graph Theory	initial sensitivity	butterfly effect
fractal			

Standard IV: Students will understand and apply concepts from probability and statistics to solve problems.

Objective 1: Use the rules of probability to calculate independent and conditional probabilities in real contexts.

a. Distinguish between subjective, experimental, and theoretical probability.

b. Calculate probabilities using addition and multiplication rules, tree diagrams, and twoway tables using correct probability notation.

c. Calculate conditional probabilities of compound events using two-way tables and Venn diagrams.

d. Use permutations and combinations to find probabilities.

Objective 2: Use graphs and numerical summaries to describe univariate data.

a. Display numerical univariate data using stemplots, line plots and histograms.

b. Describe the center and spread of a distribution using mean and standard deviation or median and quartiles.

c. Analyze distributions, and be able to explain what may have caused the distribution to be normal, symmetrical, skewed left, skewed right, or bimodal.

d. Use the empirical rule (68–95–99.7) to determine the proportion of a normally distributed population that falls within a given range of values.

Objective 3: Use graphs and numerical summaries to describe and analyze bivariate data.

a. Use technology to graph bivariate data and calculate the regression line of the scatterplot.
b. Calculate the correlation coefficient of bivariate data using technology and use correlation to determine the direction and strength of the regression line.
c. Use a regression line to make predictions and analyze characteristics of data.

Objective 4: Use appropriate sampling techniques to describe a population.

a. Compare and contrast survey, observation, and experimental methods for obtaining sample data.
b. Select a simple random sample from a given population.
c. Design an experiment using randomization.
d. Identify improper sampling techniques when taking sample surveys.
e. Generalize results of a survey or experiment.

Mathematical Language and Symbols Students Should Use:

conditional	unconditional	multiplication rule	
probability tree	stem plot	line plot	
histogram	mean	median	quartile
standard deviation	univariate	bivariate	
correlation coefficient	regression	survey	
observational study	experiment	control group	placebo
Venn diagram	two-way table	permutation	combination

Standard V: Students will think logically and solve problems.

Objective 1: Solve standard and non-standard problems.

a. Use a variety of problem-solving strategies such as drawing a picture, making a systematic list, eliminating possibilities, looking for patterns, guessing and checking, identifying sub-problems, analyzing units, solving a related problem, working backwards, or using Venn diagrams to solve contextual problems.
b. Connect problem-solving strategies to traditional algorithms.
c. Describe how various problem-solving strategies are related.

Objective 2: Use logical reasoning to create convincing arguments and develop patterns of successful decision making.

a. Use logic notation to create propositions or logic statements.

b. Use truth tables to determine the veracity of compound and conditional logic statements.
c. Identify equivalent logic statements.

Objective 3: Represent and compare finite and infinite groups using sets.

a. Use set notation to represent groups.
b. Use Venn diagrams to represent and solve problems involving combinations of sets.
c. Use Venn diagrams and set notation to explore the real number system.
d. Use Venn diagrams and set notation to explore logic.

Mathematical Language and Symbols Students Should Use:

Venn diagram	truth table	logic notation
compound statement	conditional statement	equivalent statement
intersection	union	subset
empty set	complement	cardinality
$\varnothing, \in, \subset, \cap, \cup,$	bounded	unbounded
	countable	uncountable

■ **inside back cover**

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Superintendent of Public Instruction

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