Core Content

Cluster Title: Represent and model with vector quantities.
Standard: (+) N.VM.1: Recognize vector quantities as having both magnitude and direction. Represent vector quantities by directed line segments, and use appropriate symbols for vectors and their magnitudes (e.g., $\boldsymbol{v}$, $\|\boldsymbol{v}\|$,$\|\|\boldsymbol{v}\|\|$, v).
Concepts and Skills to Master
• Recognize vector quantities as having both magnitude and direction. • Represent vector quantities by directed line segments and use appropriate symbols for vectors (***v)*** and their magnitudes (e.g., $\|\boldsymbol{v}\|$, $\|\|\boldsymbol{v}\|\|$, v). • Find the magnitude of a vector.

Supports for Teachers

Critical Background Knowledge	
Pythagorean Theorem, distance formula	
Academic Vocabulary	
Vector, magnitude, displacement	
Suggested Instructional Strategies	**Resources**
• Relate vectors to bearings. • Relate vectors to velocity of planes when affected by crosswinds.	
Sample Formative Assessment Tasks	
Skill-based Task Find $\|\boldsymbol{v}\|$ if $\boldsymbol{v}$ = <7,-12>.	**Problem Task** A car has driven 125 km due west, then 60 km due south. Represent the displacement of the car with a vector. Find the magnitude of the vector to find the displacement of the car.

Core Content

Cluster Title: Represent and model with vector quantities.
Standard: (+) N.VM.2: Find the components of a vector by subtracting the coordinates of an initial point from the coordinates of a terminal point.
Concepts and Skills to Master
• Find the horizontal and vertical components of a vector by subtracting the coordinates of an initial point from the coordinates of a terminal point.

Supports for Teachers

Critical Background Knowledge	
Definition of vector, vector notation, coordinate plane, graphing points	
Academic Vocabulary	
Vector, components, initial point, terminal point	
Suggested Instructional Strategies	**Resources**
Explore vectors using contextual situations such as air or sea navigation.	
Sample Formative Assessment Tasks	
Skill-based Task Write the components of a vector whose initial point is (-7,2) and whose terminal point is (5,-3).	**Problem Task** Create pairs of initial and terminal points that represent the vector $\boldsymbol{v}$ = <-2,5>.

Core Content

Cluster Title: Represent and model with vector quantities.
Standard: (+) N.VM.3: Solve problems involving velocity and other quantities that can be represented by vectors.
Concepts and Skills to Master
• Represent real world contexts with geometric vector models. • Solve contextual problems involving velocity and other quantities that can be represented by vectors in a variety of disciplines (e.g. science, sports, medicine).

Supports for Teachers

Critical Background Knowledge	
• Find the direction and magnitude of a vector. • Graph vectors. • Use vector notation.	
Academic Vocabulary	
Vector, direction, magnitude, velocity, force	
Suggested Instructional Strategies	**Resources**
• Use contextual problems to explore applications of vectors. • Have students create contextual situations for given vectors. • Use tools (e.g. compass, ruler, cm. paper) to model vector situations geometrically.	
Sample Formative Assessment Tasks	
Skill-based Task You are going to swim across a 20 m. river with a current of 6 kph. Draw a scale model of the vector that represents the path of your swim and estimate how far down-stream you are when you reach the other side.	**Problem Task** A car is travelling north at 45 mph and collides into another car travelling east at 30 mph. Represent the collision graphically.

Core Content

Cluster Title: Perform operations on vectors.
Standard (Honors) N.VM.4: Add and subtract vectors. a. Add vectors end-to-end, component-wise, and by the parallelogram rule. Understand that the magnitude of a sum of two vectors is typically not the sum of the magnitudes. b. Given two vectors in magnitude and direction form, determine the magnitude and direction of their sum. c. Understand vector subtraction ***v*** – ***w*** as ***v*** + (–***w***), where –***w*** is the additive inverse of ***w***, with the same magnitude as ***w*** and pointing in the opposite direction. Represent vector subtraction graphically by connecting the tips in the appropriate order and perform vector subtraction component-wise.
Concepts and Skills to Master
• Draw vectors end-to-end to find the resultant sum of the vectors. • Add vectors using components. • Use the parallelogram rule to find the sum of two vectors. • Understand that the magnitude of a sum of two vectors is typically not the sum of the magnitudes. • Understand vector subtraction as the vector you would add to **w** to get **v**.

Supports for Teachers

Critical Background Knowledge	
• Vector notation, magnitude, and direction • Graph ordered pairs and vectors • Distance Formula	
Academic Vocabulary	
vector, end-to-end, component-wise, parallelogram rule, magnitude, resultant vector	
Suggested Instructional Strategies	**Resources**
• Use contextual situations from a variety of disciplines to model vector addition. • Use real-life examples to justify why vectors cannot be added by summing magnitudes.	Precalculus textbook

Sample Formative Assessment Tasks	
Skill-based Task Given the vectors <4, 7> and <-1, 2> select a method to find their sum. What is the magnitude of the sum?	**Problem Task** Under what conditions is the sum of the magnitudes of two vectors be equal to the magnitude of the sum?

Core Content

Cluster Title: Perform operations on vectors.
Standard (Honors) N.VM.5: Multiply a vector by a scalar. a. Represent scalar multiplication graphically by scaling vectors and possibly reversing their direction; perform scalar multiplication component-wise, e.g., as $c(v_x, v_y) = (cv_x, cv_y)$. b. Compute the magnitude of a scalar multiple $c\boldsymbol{v}$ using $\|\|c\boldsymbol{v}\|\| = |c|v$. Compute the direction of $c\boldsymbol{v}$ knowing that when $|c|v \neq 0$, the direction of $c\boldsymbol{v}$ is either along $\boldsymbol{v}$ (for $c > 0$) or against $\boldsymbol{v}$ (for $c < 0$).
Concepts and Skills to Master
• Represent scalar multiplication graphically. • Compute the product of a scalar and a vector.

Supports for Teachers

Critical Background Knowledge	
Distributive property, draw a vector,	
Academic Vocabulary	
Scalar, vector, product	
Suggested Instructional Strategies	**Resources**
• Connect scalar multiplication to dilations and similarity. • Explore representations and magnitudes created by scalar multiples of vectors.	Precalculus textbook IB Maths SL textbooks
Sample Formative Assessment Tasks	
Skill-based Task Draw and find the magnitude of -3$\boldsymbol{v}$ where $\boldsymbol{v}$=<-2, 3>.	**Problem Task** Under what conditions is a scalar product of the sum of two vectors the same as the sum of the scalar products of the two vectors?

Core Content

Cluster Title: Perform operations on matrices and use matrices in applications.
Standard (Honors) N.VM.6: Use matrices to represent and manipulate data, e.g., to represent payoffs or incidence relationships in a network.
Concepts and Skills to Master
• Organize data in a matrix. • Identify and name matrix properties (e.g. dimensions) accurately. • Interpret data in a matrix. • Recognize and use matrix notation.

Supports for Teachers

Critical Background Knowledge	
Organize data in a table	
Academic Vocabulary	
Row, column, dimension, square matrix, row matrix, column matrix	
Suggested Instructional Strategies	**Resources**
• Use matrices to represent a logic problem. • Relate matrices to tables and spreadsheets. • Find examples in the media of data that can be represented in a matrix (e.g. sports, marketing, consumer data)	Newspapers, magazines
Sample Formative Assessment Tasks	
Skill-based Task At *Shop Here* oranges are $.32 each, plums are $.45 each and apples are $.52 each. At *Wonderful Foods* oranges are $.35 each, plums are $.58 each, and apples are $.48 each. Organize this information into a 2X3 matrix and into a 3X2 matrix.	**Problem Task** Organize data from the newspaper into a matrix.

Core Content

Cluster Title: Perform operations on matrices and use matrices in applications.
Standard (Honors) N.VM.7: Multiply matrices by scalars to produce new matrices, e.g., as when all of the payoffs in a game are doubled.
Concepts and Skills to Master
• Understand that scalar multiplication does not change the order of elements in a matrix. • Multiply a matrix by a scalar.

Supports for Teachers

Critical Background Knowledge	
Distributive Property	
Academic Vocabulary	
scalar	
Suggested Instructional Strategies	**Resources**
• Interpret scalar multiplication in real world contexts. • Multiply using a variety of scalars (e.g. fractions, integers) • Use scalar multiplication with a matrix representing a polygon to create a dilation. • Generalize scalar multiplication to include variables.	
Sample Formative Assessment Tasks	
Skill-based Task Multiply: $\frac{-a}{2}\begin{bmatrix} 5 & 0 \\ x & -1 \\ -3 & 2.5 \end{bmatrix}$	**Problem Task** Create a story context for: $1.5\begin{bmatrix} 3 & 9 & 11 \\ 11 & 6 & 8 \end{bmatrix}$

Core Content

Cluster Title: Perform operations on matrices and use matrices in applications.
Standard (Honors) N.VM.8: Add, subtract, and multiply matrices of appropriate dimensions.
Concepts and Skills to Master
• Recognize the necessary conditions for matrix operations. • Add and subtract matrices by hand and using technology. • Multiply matrices by hand and using technology. • Explain the meaning of the result of matrix operations in context.

Supports for Teachers

Critical Background Knowledge	
Dimensions of matrices, row, column, order of operations	
Academic Vocabulary	
Row, column, matrix	
Suggested Instructional Strategies	**Resources**
• Connect matrix operations to a context. • Use matrix operations to perform geometric transformations.	

Sample Formative Assessment Tasks

Skill-based Task

$$\begin{bmatrix} 3 & 0 & -3 \\ 4 & 1 & -5 \end{bmatrix} + \begin{bmatrix} 2 \\ -4 \end{bmatrix} \begin{bmatrix} 5 & -8 & 0 \end{bmatrix}$$

Problem Task

The elements of **A** represent the number of three different parts in production at two factories. The elements of **B** represent the labor hours required to produce each part at each of the two factories. What is the meaning of each element in **AB**? in **BA**?

$$A = \begin{bmatrix} 40 & 30 & 80 \\ 20 & 70 & 35 \end{bmatrix}, B = \begin{bmatrix} 4 & 3 \\ 2 & 5 \\ 6 & 2 \end{bmatrix}$$

Core Content

Cluster Title: Perform operations on matrices and use matrices in applications.
Standard (Honors) N.VM.9: Understand that, unlike multiplication of numbers, matrix multiplication for square matrices is not a commutative operation, but still satisfies the associative and distributive properties.
Concepts and Skills to Master
• Understand that multiplication of matrices is not commutative. • Understand that the associative and distributive properties hold for matrix multiplication.

Supports for Teachers

Critical Background Knowledge	
Matrix multiplication, properties of real numbers	
Academic Vocabulary	
Associative, commutative, distributive, square matrix	
Suggested Instructional Strategies	**Resources**
• Explore the result of a variety of matrix operations on square matrices using technology.	
Sample Formative Assessment Tasks	
Skill-based Task Show that multiplication of square matrices is not commutative.	**Problem Task** Create two square matrices such that **AB=BA.**

Core Content

Cluster Title: Perform operations on matrices and use matrices in applications.
Standard (Honors) N.VM.10: Understand that the zero and identity matrices play a role in matrix addition and multiplication similar to the role of 0 and 1 in the real numbers. The determinant of a square matrix is nonzero if and only if the matrix has a multiplicative inverse.
Concepts and Skills to Master
• Recognize and create matrices that are identity matrices. • Determine additive and multiplicative identities and inverses of a matrix when they exist. • Find the determinant of a matrix using technology. • Use the determinant to determine if a square matrix has an inverse.

Supports for Teachers

Critical Background Knowledge	
Multiplication of matrices, additive and multiplicative identities and additive and multiplicative inverses of real numbers, division by zero as undefined	
Academic Vocabulary	
Identity, inverse, determinant, square matrix, non-zero, variable matrix, singular matrix	
Suggested Instructional Strategies	**Resources**
• Solve matrix equations in the form AX+B=C, where A, B, and C are number matrices and X is a variable matrix. • Explore addition and multiplication of identity matrices. • Determine multiplicative inverses by hand for 2X2 matrices and using technology for larger matrices.	

Sample Formative Assessment Tasks	
Skill-based Task Find the inverse of the following matrix, if it exists: $\begin{bmatrix} 3 & -4 \\ -2 & 5 \end{bmatrix}$	**Problem Task** Compare and contrast the process of solving a linear equation with the process of solving a matrix equation using properties. If **AB=I**, what can you say about **BA**? Explain.

Core Content

Cluster Title: Perform operations on matrices and use matrices in applications.
Standard (Honors) N.VM.11: Multiply a vector (regarded as a matrix with one column) by a matrix of suitable dimensions to produce another vector. Work with matrices as transformations of vectors.
Concepts and Skills to Master
• Define and represent a vector as a matrix with one column. • Recognize that multiplication of a vector (**v**) by a matrix (**A**) is calculated as **Av**. • Understand that a matrix is a representation of a function where **v** is the input, and the product of **A** and **v** is the output. • Transform a vector using a matrix.

Supports for Teachers

Critical Background Knowledge	
Multiplication of matrices, vectors	
Academic Vocabulary	
Vector, matrix, transformation, column matrix	
Suggested Instructional Strategies	**Resources**
• Explore transformations by trying different values in a transformation matrix and observing the resultant vector. • Apply transformations of matrices to cryptology.	Illuminations, Computer Animation IB Maths SL Textbook Inspire calculator, *Geometer's Sketchpad, IMP* (Key Curriculum) *Year 4 "As the Cube Turns"*
Sample Formative Assessment Tasks	
Skill-based Task Transform the vector <2,1> using the transformation matrix $\begin{bmatrix} -2 & 0 \\ 0 & 2 \end{bmatrix}$ and describe the result.	**Problem Task** Find a transformation matrix that would halve the magnitude of a vector and rotate it 90 degrees.

Core Content

Cluster Title: Perform operations on matrices and use matrices in applications.
Standard (Honors) N.VM.12: Work with 2 × 2 matrices as transformations of the plane and interpret the absolute value of the determinant in terms of area.
Concepts and Skills to Master
• Recognize matrix transformations as a function. • Transform geometric figures using 2X2 matrices. • Find the area of a triangle using determinants.

Supports for Teachers

Critical Background Knowledge	
Area of a triangle, ordered pairs, definition of a function, determinant, matrix operations, absolute value	
Academic Vocabulary	
Matrix, determinant, transformation	
Suggested Instructional Strategies	**Resources**
• Find the area of a triangle using ½ the absolute value of the determinant of the square matrix representing the coordinates of the vertices of a polygon.	
Sample Formative Assessment Tasks	
Skill-based Task Use matrix arithmetic to translate the triangle with coordinates (2,4), (-1,3) and (0,-2) three units to the right and one unit down.	**Problem Task** Extend the process of finding the area of a triangle using determinants to other polygons.

Core Content

Cluster Title: Perform operations on matrices and use matrices in applications.
Standard (Honors): Solve systems of linear equations using matrices.
Concepts and Skills to Master
• Represent a system of linear equations using matrices. • Solve a system of two equations with two unknowns by hand using matrices. • Use technology to solve a system of three or more equations using matrices.

Supports for Teachers

Critical Background Knowledge
• Methods of solving systems of linear equations in two-variables • Identity matrix • Inverse matrix • Find a determinant
Academic Vocabulary
Matrices, row-echelon form, inverse, identity, determinant, dependent, inconsistent, singular matrix

Suggested Instructional Strategies	**Resources**
• Use row-echelon form to solve systems of equations. • Use matrix equations to solve systems. • Use contextual situations with multiple variables to explore the power of matrices. • Explore dependent and inconsistent systems of equations.	

Sample Formative Assessment Tasks	
Skill-based Task Solve using a matrix: $4x - 4y = 5$ $6x + 8y = -3$	**Problem Task** Create a system of equations such that the reduced row-echelon form on your calculator returns the matrix: $\begin{bmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}$. What is the graphical interpretation of this result?