

Utah's

3-Tier Model of Mathematics Instruction



Tier 3.

Provides intense, targeted intervention focused on essential knowledge and skills.

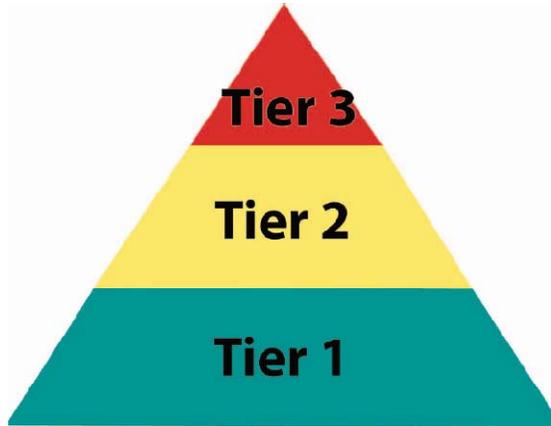
Tier 2.

Provides intervention remediating specific conceptual and skill deficits in one or more critical areas of mathematics.

Tier 1.

Guarantees access to the Utah Mathematics Core Curriculum for all students.





Utah's 3-Tier Model of Mathematics Instruction

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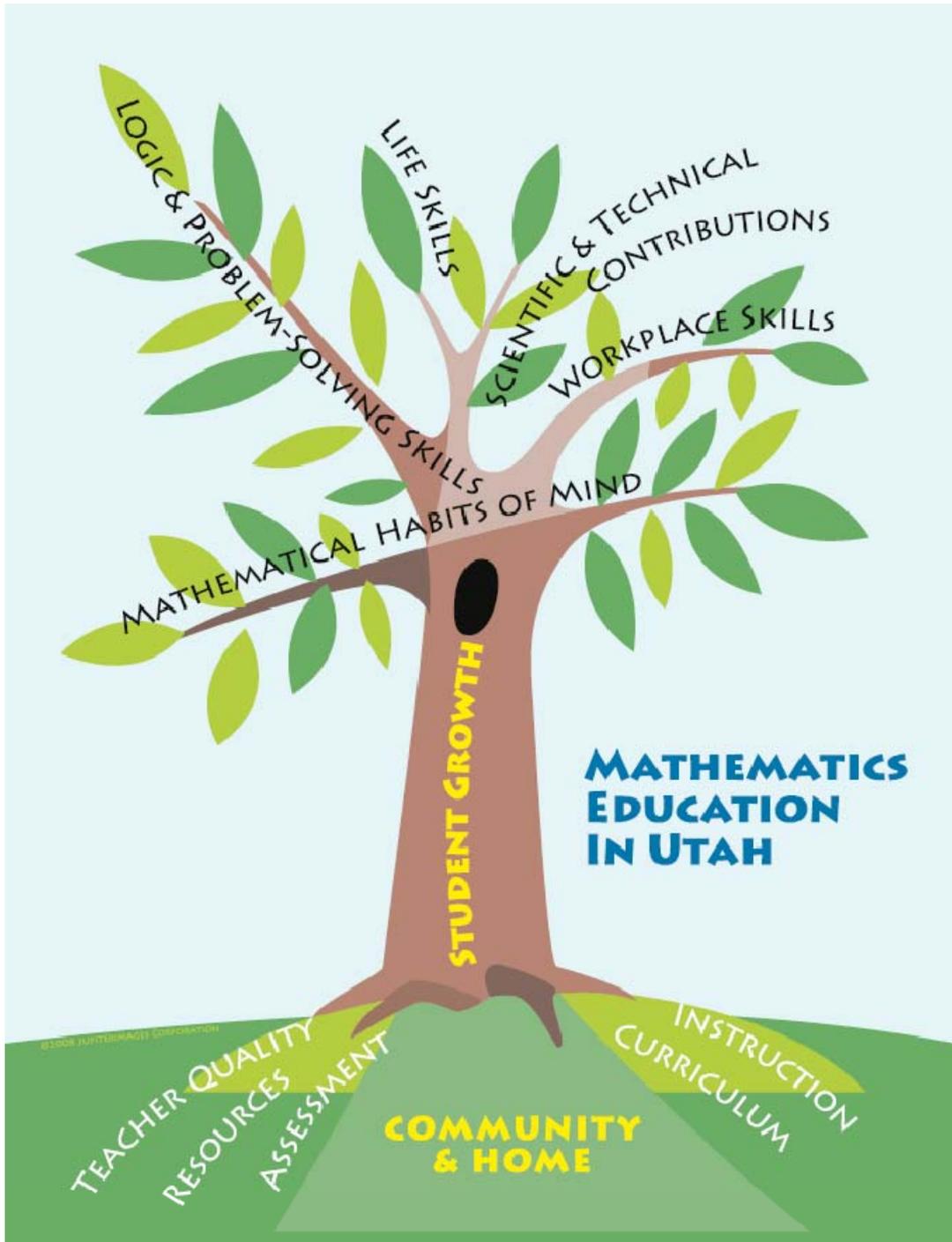
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Utah's Mathematics Model



The mission of mathematics education in Utah is to promote student growth and learning in mathematics in order to prepare students to thrive and contribute in the global economy of the 21st century.

Foreword

Proficiency in mathematics is critical for individual success and for the well-being of our nation. Mathematics supports the development of human minds and the growth and maintenance of our scientific and technical culture. Increasing involvement in a global economic network requires increased attention to developing critical thinking and problem-solving skills commonly developed in mathematics classes. Parents, educators, and business and political leaders recognize mathematics as integral to the success of our society. For these reasons, it is imperative that all students become mathematically proficient and have the opportunity to achieve their full mathematical potential.

A team of educators—including mathematics education professionals, special education professionals, and administrators—developed the 3-Tier Model of Mathematics Instruction as a guide for educators seeking to improve instruction and intervention in mathematics. This guide is based on current research and best-practices in mathematics education and supports the mission of mathematics education in Utah, which is to promote student growth and learning in mathematics in order to prepare students to thrive and contribute in the global economy of the 21st Century.

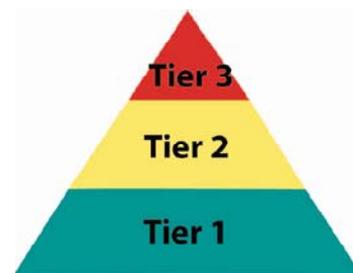
The Utah Mathematics Model on the previous page uses a living tree as a metaphor for student growth in mathematics. One of the foundations, or roots, of student achievement is instruction. The 3-Tier Model of Mathematics Instruction is one of the resources educators can use to support student growth in mathematics. Use of the 3-Tier Model will enable teachers to help students produce thinking skills and habits that will sustain them in life, in the workplace, and as they contribute to society.

Many people view mathematics as difficult and struggle with building meaning that will allow them to apply their skills to the world around them. Others lack the rudimentary mathematical skills that will support them as they attempt to build on previous learning to achieve greater knowledge. Proper instruction can alleviate such barriers to mathematics acquisition. Research clearly supports the fact that teachers with a high degree of content and pedagogical knowledge are essential to student success in mathematics classrooms. The 3-Tier Model of Mathematics Instruction provides support for teachers in developing mathematical competency in their students. This model also emphasizes the importance of local school administrators as instructional leaders, the use of data to guide instruction, appropriate intervention, parental involvement, and other research-based practices.

Utah's 3-Tier Model of Mathematics Instruction provides Utah educators with a process for delivering quality, research-based instruction using Utah's Core Curriculum. The model provides a foundation for reducing the prevalence of students struggling with mathematics by creating a seamless K-12 instructional system which aligns with state and federal legislative requirements. Utah's model is also a framework to guide data collection to determine student responsiveness to interventions and planning for appropriate instruction and remediation.

Introduction to Utah's Tiered Model of Mathematics Instruction

The Utah 3-Tier Model of Mathematics Instruction provides a model for delivering high-quality, comprehensive mathematics instruction for **all** students from kindergarten through high school. The model provides a framework for teachers and administrators as they seek to provide quality instruction and targeted interventions that lead to proficiency in and understanding of mathematics. The model is composed of three tiers, or levels, of instruction. The model includes guidance in curriculum, instruction, assessment, teacher quality, resources, scheduling, settings, and relationships with community and home.



Student Movement through the Tiers

Student movement through the tiers is a fluid process based on student assessment data and collaborative team decisions. At any time during this process, a student may be referred for consideration for a 504 plan and/or special education evaluation. Tutoring may be necessary in any of the tiers to provide extra practice and support to help students maintain progress.

Tier 1

Tier 1 instruction guarantees access to the Mathematics Core Curriculum for **all** students. Most students will demonstrate proficiency with effective Tier 1 instruction consisting of differentiated instructional methods and appropriate material selection. The majority of students will reach proficiency with effective Tier 1 instruction. Formative and summative assessments inform instructional decisions and measure student progress.

Tier 2

Tier 2 instruction provides intervention remediating the specific conceptual and skill deficits of students who fail to meet core objectives in one or more critical areas of mathematics. Tier 2 intervention supplements Tier 1 instruction, but does not replace it. Tier 2 interventions are systematic, explicit, and aligned with Tier 1 instruction, with targeted interventions based on the needs of individual students. The duration of this instruction varies based on frequent student assessments and is provided by or under the direction of the classroom teacher. The percentage of students who require Tier 2 intervention will vary, but is commonly less than 20 percent per core objective.

Tier 3

Tier 3 instruction provides intense, targeted intervention to students who have not responded to Tier 2 intervention. Tier 3 instruction is focused on essential pre-requisite knowledge and skills necessary to access Tier 1 Core Curriculum. Tier 3 replaces Tier 2 intervention and includes weekly progress monitoring. Tier 3 instruction may involve specialists beyond the regular classroom teacher. This small percentage of students usually has severe deficiencies in specific mathematics skills and requires instruction that is more intensive and specifically designed to meet their individual needs.

Student Movement through the Tiers

The purpose of tiered instruction is to facilitate success in core mathematics, and thus student participation within the tiers should remain fluid. When students struggle with Tier 1 mathematics instruction, the additional supports of Tier 2 or Tier 3 instruction must be made available to them. As students respond to instruction and intervention and are able to succeed in Tier 1 mathematics instruction, they no longer require the additional supports of Tier 2 or Tier 3 interventions. Students should not remain in Tier 2 or Tier 3 instruction for extended periods of time.

The 3-Tier Model of Mathematics Instruction does not describe students, but rather the instruction needed to help **all** students understand core mathematics ideas and demonstrate mathematics skills. Consequently, there is Tier 2 and Tier 3 instruction, but not Tier 2 or Tier 3 students.

Data collection and analysis must play a key role in determining appropriate instruction and intervention for students at all levels. Formative assessments such as frequent progress monitoring enable educators to make informed decisions regarding appropriate instructional methods for student progress.

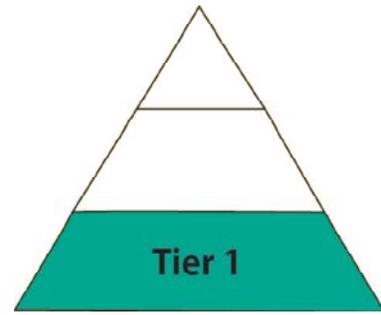
Collaborative teams may be established to evaluate student progress and propose effective interventions. These teams, composed of teachers, administrators and other educators, meet regularly to determine appropriate instruction and required remediation. Progress monitoring and data analysis inform the team as they evaluate student achievement and consider possible intervention strategies. These teams make decisions about student movement between tiers.

When a student does not respond to Tier 2 or Tier 3 interventions, the team may decide to recommend possible changes in course enrollment or evaluation for additional services. At any time during this process, a student may be referred for consideration for a 504 plan or special education evaluation.

**Implementation of
the
3-Tier Model of
Mathematics
Instruction in
Elementary
Schools**

K-6 Tier 1 Core Classroom Instruction for All Students

Tier 1 instruction guarantees access to the core mathematics curriculum for **all** students. Teachers use appropriate materials and effective practices to teach the Elementary Mathematics Core Curriculum. Differentiated instruction occurs using various instructional delivery methods for the whole class and in small groups. Frequently collected formative assessment data is used to monitor and maintain ongoing success in mathematics by informing instruction. **All** students take formative and summative assessments to show proficiency and to demonstrate student learning over time. Assessments measure student growth/gain over time and identify students who are experiencing difficulties or who need enrichment.

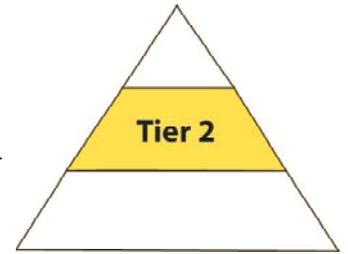


Tier 1 Essential Elements of Instruction

Component	Instruction
Curriculum	<ul style="list-style-type: none"> Utah Elementary Math Core Curriculum
Instruction	<ul style="list-style-type: none"> Whole-class conceptual development of core grade level mathematics ideas and skills Differentiated instruction for conceptual development, skill acquisition, and application, including re-teaching, enrichment, and/or additional practice Guided practice Use of physical, visual, and abstract representations Manipulatives and technology as instructional tools and for individual skill practice Applications of skills throughout the day across content areas
Assessment	<ul style="list-style-type: none"> Screening assessments Diagnostic assessments Formative assessments Summative assessments
Resources	<ul style="list-style-type: none"> Instructional materials aligned to the grade level core Content materials that support the Intended Learning Outcomes Content materials that support the Five Strands of Mathematical Proficiency Pacing guides Criterion Referenced Test (CRT) Reference Sheets (grades 4-6) Assessment Accommodation Manual Utah Test Item Pool (UTIPS) Manipulatives and technology Access to content support personnel Trained paraprofessionals and volunteers to provide support under the direction of the classroom teacher
Teacher Quality	<ul style="list-style-type: none"> Highly qualified elementary classroom teacher Ongoing professional development and collaboration in content and pedagogy
Scheduling	<ul style="list-style-type: none"> Recommended at least one hour of core mathematics instruction every day in grades 1-6 Majority of time in conceptual development and skill acquisition Recommended one-fourth of the kindergarten day engaged in learning core mathematics
Setting	<ul style="list-style-type: none"> General education classroom
Community and Home	<ul style="list-style-type: none"> Parent training and appropriate materials Home practice and support Communication of clear expectations and student progress Application of skills and strategies at home and in the community Encouragement of teacher/home, school/business, and elementary/secondary partnerships

K-6 Tier 2 Supplemental Targeted Instruction

The purpose of Tier 2 intervention is to provide supplemental instruction for students whose needs have not been met through Tier 1 instruction. Tier 2 is targeted instruction given in addition to Tier 1 instruction, by or under the direction of the general education teacher. This instruction remediates the specific conceptual and skill deficits of students who fail to meet Tier 1 core objectives in one or more critical areas of mathematics. Tier 2 instruction is systematic, explicit, and aligned with Tier 1 instruction. Instructional interventions are more intense and differentiated based on the needs of individual students as determined by assessment data.

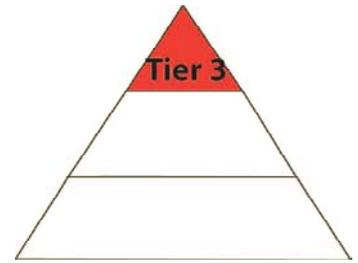


Tier 2 Essential Elements of Instruction

Component	Instruction
Curriculum	<ul style="list-style-type: none"> Emphasis on student needs within the Utah Mathematics Core Curriculum
Instruction	<ul style="list-style-type: none"> Targeted, systematic, explicit instruction Differentiated instruction in small groups Explicit connections between physical, visual, and abstract representations Additional conceptual development of core mathematics ideas and skills Guided practice to develop skills for independent practice Manipulatives and technology for instruction and individual skill practice
Assessment	<ul style="list-style-type: none"> Formative assessments Diagnostic assessments and progress monitoring every two to three weeks for making targeted instructional decisions Communication within the school regarding student progress Communication with parents regarding student progress and needs
Resources	<ul style="list-style-type: none"> Intervention and content materials that support Tier 1 instruction Utah Mathematics Core materials that target one or more student needs Content materials that support the Intended Learning Outcomes Content materials that support the Five Strands of Mathematical Proficiency Pacing guides CRT Reference Sheets (grades 4-6) Utah Test Item Pool (UTIPS) USOE Assessment Accommodation Manual Access to content support personnel Manipulatives and technology Highly trained paraprofessionals and trained volunteers
Teacher Quality	<ul style="list-style-type: none"> Highly qualified elementary classroom teacher with support from a 504 specialist, special education teacher, or English as a Second Language (ESL) endorsed teacher Ongoing professional development and collaboration in content and pedagogy
Scheduling	<ul style="list-style-type: none"> Recommended four to five times per week, for an additional 15-20 minutes beyond Tier 1 instruction Majority of time in targeted conceptual development and skill acquisition Application of skills and strategies in other content areas
Setting	<ul style="list-style-type: none"> General education classroom Before- and/or after-school programs Summer programs
Community and Home	<ul style="list-style-type: none"> Parent training and materials Home practice and support Encouragement of home/school and business/school partnerships Teacher collaboration Community-based volunteers Frequent communication of clear expectations and student progress

K-6 Tier 3 Intensive Supplemental Targeted Instruction

Tier 3 instruction is for students who have not responded successfully to Tier 2 interventions. This small percentage of students has severe deficiencies in specific mathematics skills. Diagnostic and weekly progress monitoring assessments check progress and identify problems. Teachers use this information to provide targeted interventions to support Tier 1 instruction. Tier 3 instruction replaces Tier 2 and is in addition to Tier 1. If progress monitoring and diagnostic assessments show a student is not making progress, he/she may be referred for further evaluation and additional services.



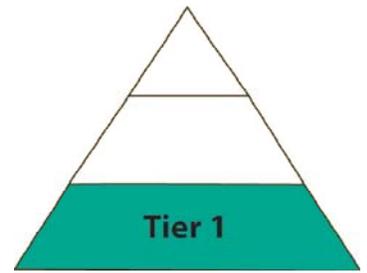
Tier 3 Essential Elements of Instruction

Component	Instruction
Curriculum	<ul style="list-style-type: none"> • Emphasis on student needs within the Utah Mathematics Core Curriculum • Essential knowledge and skills for individual student success • Emphasis on number sense and operations • Development of mathematics vocabulary
Instruction	<ul style="list-style-type: none"> • Explicit, intense, targeted instruction on specific conceptual components for individuals or small groups • Use of manipulatives and technology for accommodations, conceptual development, and individualized skill practice • Guided practice to develop specific skills and strategies • Instructional methods that explicitly link concepts and skills with physical, visual, and abstract representations
Assessment	<ul style="list-style-type: none"> • Formative assessments • Diagnostic assessments • Weekly progress monitoring for making targeted instructional decisions • Communication within the school regarding student progress • Communication with parents regarding student progress and needs
Resources	<ul style="list-style-type: none"> • Evidence-based programs/materials that remediate specific mathematics skills and concepts • Intervention and content materials that support Tier 1 instruction • Differentiated curriculum materials • Manipulatives • Use of technology for accommodations
Teacher Quality	<ul style="list-style-type: none"> • Elementary mathematics intervention specialist, special education teacher, or ESL-endorsed teacher each with a mathematics endorsement or specialization • Ongoing professional development and collaboration in content and pedagogy
Scheduling	<ul style="list-style-type: none"> • Minimum of 20-30 minutes daily in addition to Tier 1 instruction
Setting	<ul style="list-style-type: none"> • Appropriate specialized instructional setting within the school
Community and Home	<ul style="list-style-type: none"> • Parent training and access to materials • Home practice and support • Encouragement of home/school and business/school partnerships • Teacher collaboration • Mathematically trained, community-based tutors • Frequent communication of clear expectations and student progress

**Implementation of
the
3-Tier Model of
Mathematics
Instruction in
Secondary Schools**

7-12 Tier 1 Core Classroom Instruction for All Students

Tier 1 instruction guarantees access to the Mathematics Core Curriculum for **all** students. Teachers use appropriate materials and effective practices to teach the Secondary Mathematics Core Curriculum. Differentiated instruction occurs using various instructional delivery methods and grouping structures. Frequently collected formative assessment data is used to monitor and maintain ongoing success in mathematics by informing instruction. **All** students take formative and summative assessments to show proficiency and to demonstrate student learning over time. Assessments measure student growth/gain over time, and identify students who are experiencing difficulties or who need enrichment.

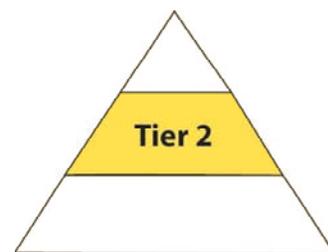


Tier 1 Essential Elements of Instruction

Component	Instruction
Curriculum	<ul style="list-style-type: none"> Utah Secondary Mathematics Core Curriculum Curriculum guides for Applied, Advanced and Supplemental Mathematics Courses
Instruction	<ul style="list-style-type: none"> Whole class conceptual development of mathematics ideas and skills Differentiated instruction for skill acquisition and application, re-teaching, enrichment, and/or additional practice Guided practice that supports independent practice Technology as an instructional tool and for individual skill practice
Assessment	<ul style="list-style-type: none"> Screening assessments Diagnostic assessments Formative assessments Summative assessments
Resources	<ul style="list-style-type: none"> Instructional materials supporting specific core content areas Content materials that support the Intended Learning Outcomes Content materials that support the Five Strands of Mathematical Proficiency Pacing guides Criterion Referenced Test (CRT) Reference Sheets USOE Assessment Accommodation Manual Utah Test Item Pool (UTIPS) Technology Trained paraprofessionals and volunteers providing support under the direction of the classroom teacher
Teacher Quality	<ul style="list-style-type: none"> Level 2, 3, or 4 mathematics-endorsed teacher as appropriate to course Ongoing professional development and collaboration in content and pedagogy
Scheduling	<ul style="list-style-type: none"> Instruction provided in core content class periods Majority of time in conceptual development and skill acquisition
Setting	<ul style="list-style-type: none"> General education classroom
Community and Home	<ul style="list-style-type: none"> Communication of clear expectations and student progress Home practice and support Teacher collaboration Application of skills and strategies in other content areas, at home, and on the job Encouragement of teacher/home, school/business, and secondary/post-secondary partnerships

7-12 Tier 2 Supplemental Targeted Instruction

Tier 2 instruction remediates the specific conceptual and skill deficits of students who fail to meet Tier 1 core objectives in one or more essential areas of mathematics. Tier 2 instruction is more intense and differentiated based on the needs of individual students as determined by assessment data. Tier 2 intervention is systematic, explicit, and aligned with Tier 1 instruction. Tier 2 instruction is given in addition to Tier 1 instruction, by or under the direction of the mathematics teacher.

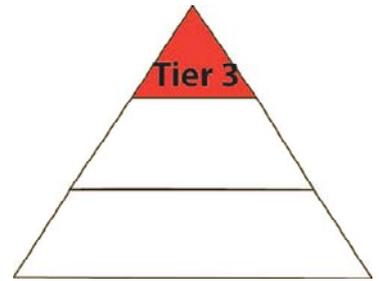


Tier 2 Essential Elements of Instruction

Component	Instruction
Curriculum	<ul style="list-style-type: none"> Emphasis on student needs within the Utah Mathematics Core Curriculum or Curricula for Applied, Advanced and Supplemental Mathematics Courses
Instruction	<ul style="list-style-type: none"> Targeted, systematic, explicit instruction Differentiated instruction in small groups Explicit connections between physical, visual, and abstract representations Additional conceptual development of core mathematics ideas and skills Guided practice to develop skills for independent practice Technology for individual skill practice
Assessment	<ul style="list-style-type: none"> Screening and progress monitoring Diagnostic assessments Formative assessments Communication between regular mathematics teacher and support specialists regarding student progress Communication with parents regarding student progress and needs
Resources	<ul style="list-style-type: none"> Intervention and content materials that support Tier 1 instruction Utah Mathematics Core materials that target one or more of the student's needs Content materials that support the Intended Learning Outcomes Content materials that support the Five Strands of Mathematical Proficiency Pacing guides CRT Reference Sheets Utah Test Item Pool (UTIPS) USOE Assessment Accommodation Manual Technology
Teacher Quality	<ul style="list-style-type: none"> Level 2, 3, or 4 mathematics-endorsed teacher with support from a 504 specialist, special education teacher, or English as a Second Language (ESL) endorsed teacher as needed to provide access to the mathematics curriculum Ongoing professional development and collaboration in content and pedagogy
Scheduling	<ul style="list-style-type: none"> Recommended average of at least 30 minutes daily in addition to the core mathematics class period Majority of time in targeted conceptual development and skill acquisition Application of skills and strategies in other content areas
Setting	<ul style="list-style-type: none"> Before and/or after school programs Extra periods in the regular school day Summer programs
Community and Home	<ul style="list-style-type: none"> Home practice and support Teacher collaboration Frequent communication of clear expectations and student progress Encouragement of teacher/home, school/business, and secondary/post-secondary partnerships

7-12 Tier 3 Intensive Supplemental Targeted Instruction

Tier 3 instruction provides more intense, targeted intervention to students with mathematical deficits that prohibit access to the Mathematics Core. Diagnostic and weekly progress monitoring assessments check progress and identify problems. Teachers use this information to supply targeted interventions to support Tier 1 instruction. Tier 3 instruction replaces Tier 2 intervention and provides extended time for students to acquire the skills and knowledge necessary to access core mathematics ideas.



Tier 3 Essential Elements of Instruction

Component	Instruction
Curriculum	<ul style="list-style-type: none"> • Emphasis on student needs within the Utah Mathematics Core Curriculum or Curricula for Applied, Advanced and Supplemental Mathematics courses • Essential knowledge and skills for individual student success • Emphasis on specific mathematics skills, vocabulary, and concepts
Instruction	<ul style="list-style-type: none"> • Explicit, intense, targeted instruction on specific conceptual components for individual or small groups • Use of manipulatives and technology for accommodations, conceptual development, and individualized skill practice • Guided practice to develop specific skills and strategies • Instructional methods that explicitly link concepts and skills with physical, visual, and abstract representations
Assessment	<ul style="list-style-type: none"> • Formative assessments • Diagnostic assessments • Weekly progress monitoring for making targeted instructional decisions • Communication within the school regarding student progress • Communication with parents regarding student progress and needs
Resources	<ul style="list-style-type: none"> • Evidence-based programs/materials that remediate specific mathematics skills and concepts • Intervention and content materials that support Tier 1 instruction • Differentiated curriculum materials • Manipulatives • Technology
Teacher Quality	<ul style="list-style-type: none"> • Level 2, 3, or 4 mathematics-endorsed regular education, special education or ESL-endorsed teacher • Ongoing professional development and collaboration in content and pedagogy
Scheduling	<ul style="list-style-type: none"> • Recommended minimum of 60 minutes daily
Setting	<ul style="list-style-type: none"> • Appropriate specialized instructional setting within the school
Community and Home	<ul style="list-style-type: none"> • Parent training and access to materials • Home practice and support • Encouragement of home/school and business/school partnerships • Teacher collaboration • Mathematically trained, community-based tutors • Frequent communication of clear expectations and student progress

Effective Instructional Practices for All Teachers

Effective Instructional Practices

Utah's 3-Tier Model of Mathematics Instruction provides educators with a framework for delivering effective mathematics instruction to all K-12 students. Mathematics consists of concepts, relationships, and conventions. Mathematical concepts and relationships are the ideas of mathematics that can be discovered and deeply understood through exploration. Conventions are the symbolic syntax used to communicate mathematical content, and must be explicitly taught. All three components of mathematics are necessary for mathematical proficiency; thus, mathematics must be taught with direct, explicit instruction as well as exploration and discovery.

Instruction can be either teacher-directed or student-directed. Many mathematics classrooms are primarily teacher-directed, with teachers explaining problem solutions and processes or providing materials to guide students through inquiry lessons. Some teachers employ student-directed activities such as peer tutoring or open ended inquiry on a mathematics topic of the student's choice. Most activities fall somewhere on a continuum and include elements that are both teacher- and student-directed.

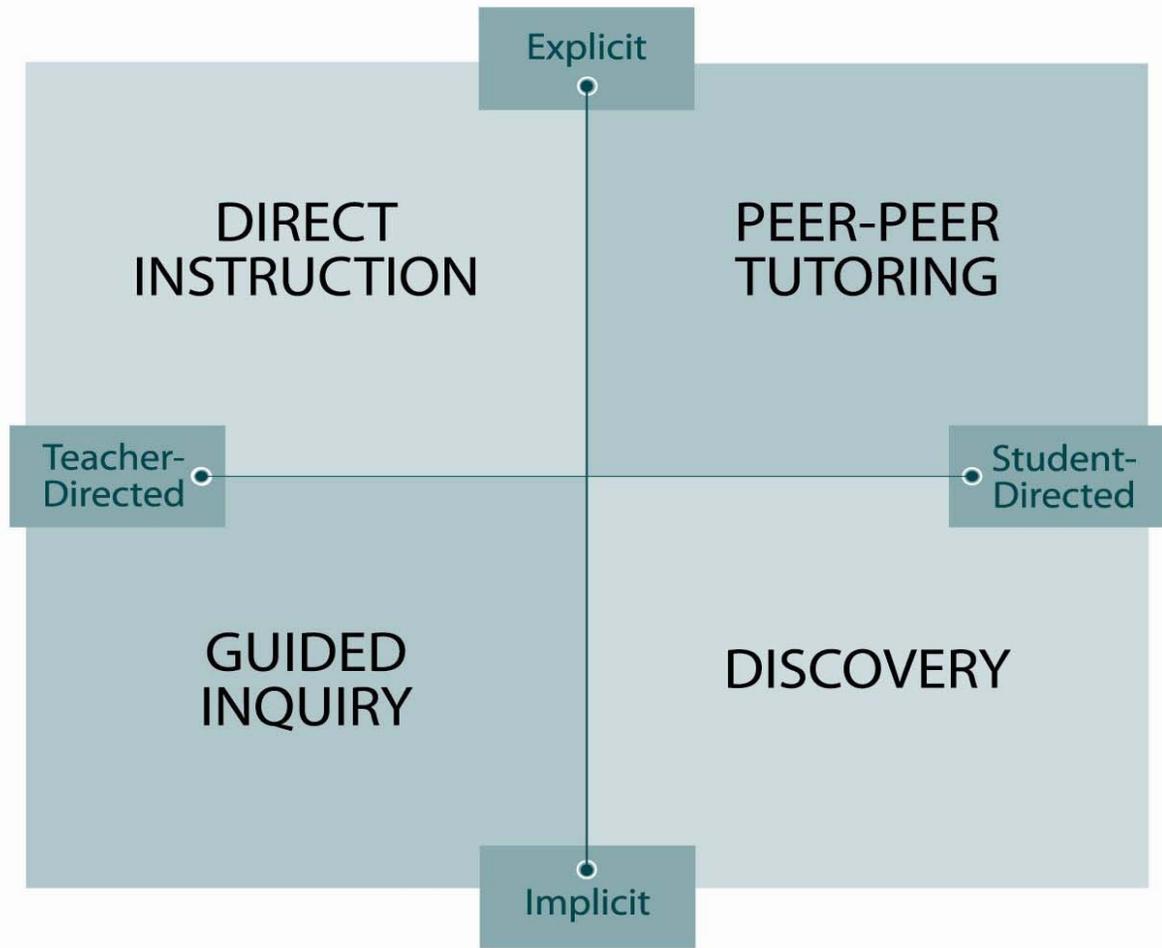
Instruction can also fall on a continuum between implicit and explicit. The National Mathematics Panel defined explicit instruction as "instruction given by the teacher that provides clear models for solving a problem type using an array of examples and then providing students with opportunities to think aloud" (NMP, 2008). Its purpose and process is clear and overt. Implicit lessons leave more for the student to discover. Students may build on prior knowledge to discover new processes or apply strategies to new situations.

Systematic instruction is a purposeful instructional sequence that guides students through progressions of mathematics learning. Systematic instruction has a clear objective and may be either teacher- or student-directed, and inquiry-based or explicit.

The model on the following page gives four examples of instructional activities that are either teacher- or student-directed and implicit or explicit. This model should be seen as a continuum where teachers design lessons and student opportunities, and the four examples are illustrations of where lessons might fall along the continuum.

The National Mathematics Advisory Panel's Final Report emphasizes the need to blend the use of teacher-directed and student-directed approaches to mathematics instruction. As teachers plan instruction, they must continually choose teacher-directed and student-directed tasks at various levels of explicitness based on the abilities of their students. Research supports the effectiveness of instruction that is systematic at all levels and increasingly explicit for students experiencing difficulties. Tier 2 and Tier 3 instruction must be more explicit and Tier 3 instruction requires greater teacher direction. The graphic on the following page will help teachers plan instruction in all four quadrants to meet the various needs of their students.

Examples in the Continuum of Systematic Instruction



Several differentiated instruction techniques should be used in Tier 1 instruction for all students. These techniques include the use of flexible, cooperative groups, manipulatives, technology, discourse, and other tools that lead to student learning.

When designing instruction for all levels, teachers should consider the following elements of instructional practice:

- Choice of appropriate instructional tasks
- Facilitation of classroom discourse
- Organization of classroom groupings
- Choice of tools for student use

Choice of Appropriate Instructional Tasks

Mathematics instruction is composed of carefully chosen instructional tasks that support student learning of concepts and acquisition of skills. Instructional tasks support the five strands of mathematical proficiency-- adaptive reasoning, strategic competence, conceptual understanding, productive disposition, and procedural fluency. Tasks may include algorithmic procedures, technology-based inquiry, question-answer sessions, problem-solving activities, or other tasks that lead to understanding of and proficiency with mathematics.

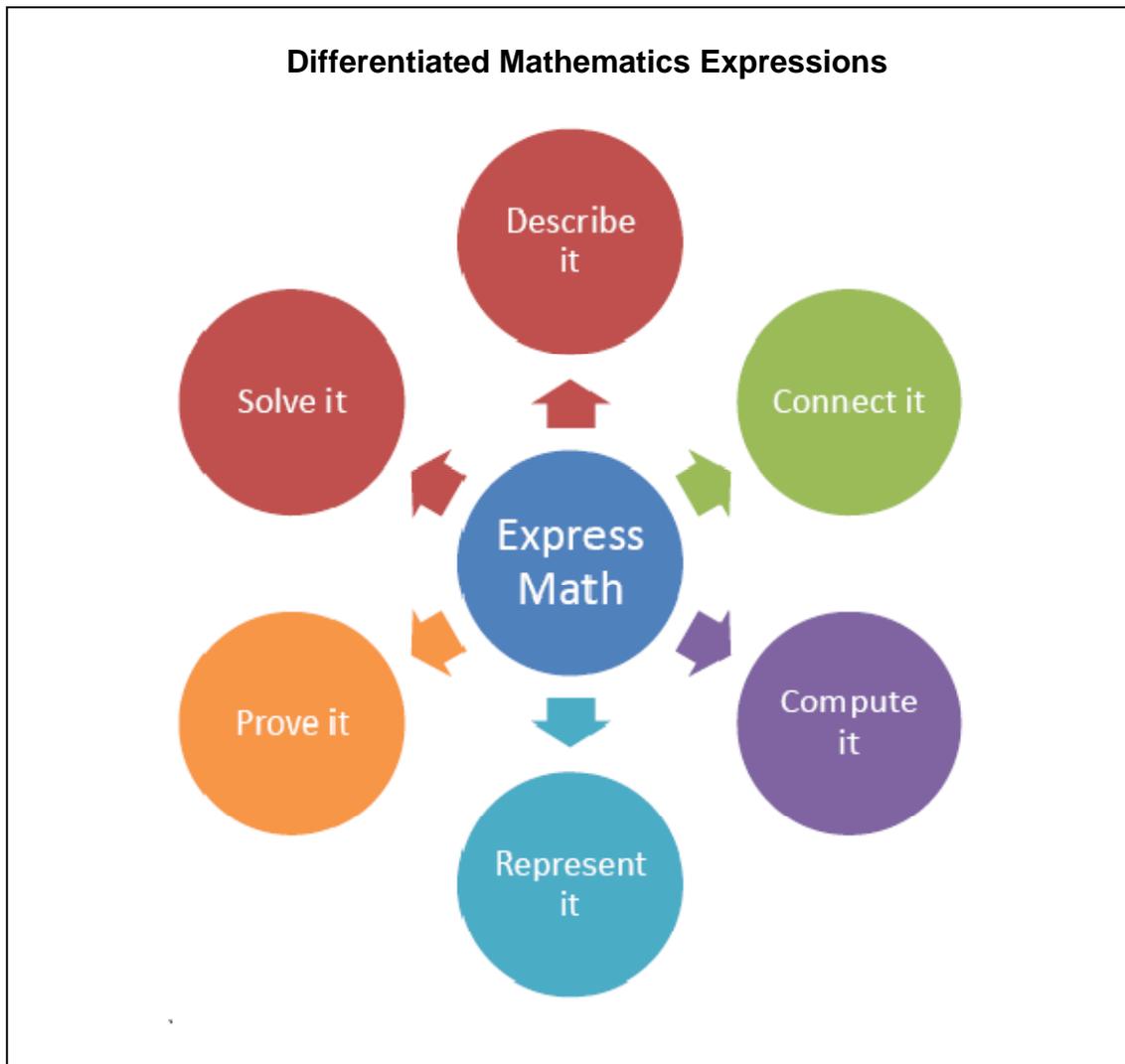
Mathematics instruction includes traditional skill-based tasks, but it should also incorporate strategies that address multiple ways of accessing mathematical knowledge. Students may draw, use graphic organizers, explore mathematics in authentic situations through the Internet or on field trips, decode mathematics humor, explore mathematics using manipulatives and technology, express mathematics in music, rhythm, or rhyme, and write about mathematics.

All students benefit from a Concrete-Representational-Abstract (CRA) teaching sequence that introduces concepts concretely, moves to visual and pictorial representations, and culminates in abstract symbolic reasoning and representation. For example, when students are learning to add, they may begin with concrete cubes, then draw dots, and finally use symbolic notation to find sums. This sequence supports the understanding of concepts before students learn rules, thus cementing learning of algorithms in conceptual understanding.

Instruction in all tiers should emphasize problem-solving strategies and transfer of knowledge to a wide variety of contexts. Instructional tasks should be chosen to develop student persistence in mathematics, which will lead to independent and creative problem solving.

Carefully selected instructional tasks provide differentiated ways of expressing mathematics as well as learning mathematics. Students describe mathematics, connect it to other subjects within and outside of mathematics, compute answers with and without technology, represent mathematics in multiple ways, solve problems, and prove mathematical conjectures. Allowing students differentiated ways of accessing and expressing mathematics increases engagement and successful acquisition of concepts and skills.

Supplemental instruction may be delivered in small groups to make explicit connections between physical, visual, and abstract representations. Clear models for solving a problem type using an array of examples and opportunities to think aloud and talk about mathematics help students understand mathematics more deeply. Skill acquisition can be enhanced through the use of manipulatives and technology. Guided practice is a useful and often essential tool to help students acquire procedural skill.



Facilitation of Classroom Discourse

Teachers must carefully plan for classroom discourse as a part of the instructional process. Discourse begins with questioning that fosters higher-order thinking, resulting in productive classroom discussion. Students must verbalize their understanding of mathematics and engage in discourse with their peers and teachers. Teachers engage students in metacognitive activities such as evaluating their own work and goal setting, leading to increased individual responsibility for learning mathematics. When teachers and students collaboratively explore and discuss mathematics, student understanding of concepts and fluency with skills will increase.

Organization of Classroom Groupings

A variety of grouping models is an important aspect of all mathematics instruction. In the Tier 1 classroom, teachers make effective use of whole group, small group, peer-peer, and individual instruction. As students require more intensive intervention, groupings should become smaller. Teachers should ensure that group size is appropriate to target the skill deficits of students.

Equally important is flexibility for students moving in and out of Tier 2 and Tier 3 instructional interventions. Tier 2 students do not exist; Tier 2 instruction does. Students who achieve learning goals no longer require regular Tier 2 or 3 intervention. Participation in the tiers should constantly be reevaluated through an assessment process.

Choice of Tools for Student Use

As teachers design instruction, they not only choose appropriate tools to build student understanding, but also choose tools for students to use as they practice skills. Visual representations such as pictures and number lines; manipulative tools such as base-ten blocks and algebra tiles; and technology tools such as geometric software, graphing calculators, assessment and practice software, and technology-based communication all have an appropriate place in the mathematics classroom. Their use must be intentional and targeted toward the acquisition of mathematics conceptual understanding and skill.

Special Education

The Individuals with Disabilities Education and Improvement Act of 2004 (IDEIA 04) passed by Congress authorizes and encourages local education agencies to utilize a multi-tier system of instruction and intervention as a means to:

- Integrate education problem-solving across educational levels.
- Match instruction to student need.
- Have a system that encompasses general and special education.

As such, students with disabilities are included in a single, well-integrated tiered system of instruction and interventions that connects general and special education and is guided by student outcome data.

IDEIA 2004 also contains the provision to use scientific, research-based interventions as part of the process for child find and in determining eligibility for students suspected of having a specific learning disability. Utah's 3-Tier Model of Mathematics Instruction, an integrated approach to instruction, is a research-based model that enables educators not only to provide explicit instruction and interventions to all students, including students with disabilities, but also to comply with federal regulations and state rules when referring a student for special education evaluation.

When providing instruction for all students, educators should consider the following data-based questions:

- Is the student receiving quality Tier 1 mathematics instruction?
- Has sufficient data been collected through screening and progress monitoring to inform instruction?
- Have parents been informed on a regular basis of their students' progress or lack of progress?
- Have appropriate education interventions been provided and documented?

Data and information collected during the instructional process will help inform the referral process and any subsequent evaluations.

A referral for consideration for special education may be made at any time during the instruction/intervention process.

For additional information, please refer to the Utah State Office of Education Specific Learning Disabilities Guidelines at www.schools.utah.gov/sars/lawsregs/rules.htm.

English Language Learners

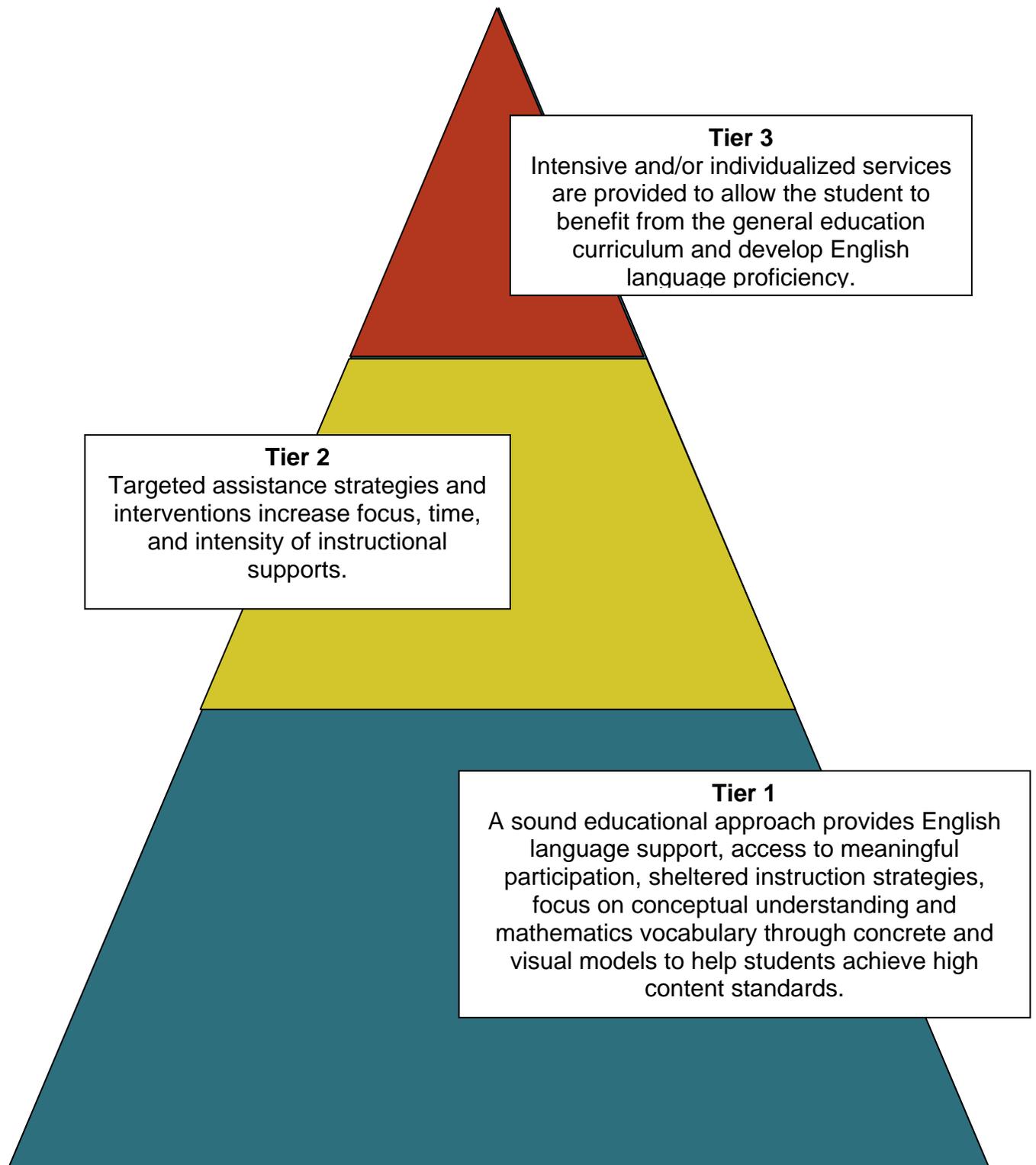
Language can be a barrier for many students when learning mathematics. English language development and knowledge of mathematics academic vocabulary enable access to mathematics for all students. English language learners (ELLs), like all mathematics students, require effective instructional approaches and interventions to prevent difficulties and to augment and support academic development. These interventions must also address possible cultural mismatches, such as differing use of symbols or the use of different algorithms for calculations. Educators must also be aware of cultural influences that appear in application problems. When designing an instructional approach or intervention, educators must consider several factors in addition to the content, such as the format for delivery, the match between the learner's difficulty and selected instructional methods, and the approach for small-group or one-on-one settings.

When providing instruction for students who are English language learners, educators should answer these critical questions:

- Does the student's cultural background serve as a barrier or gateway to what is being taught?
- What are critical vocabulary words that ELL students need to know in order to access the lessons?
- What manipulatives or visuals can enhance this lesson for ELL students?
- Does this student qualify for English language acquisition programs?
- Who will provide these services, and how?
- How are the general education, ELL, and special education staff involved in the decision-making process?
- Does the school have cultural values and strategies that promote access, equity, and academic success for all students?

Some school-aged English language learners must simultaneously acquire two languages while also acquiring skills and knowledge in the K-12 curriculum. Developing mathematics literacy in a second language is not a trivial task. Teachers should take into account the level of the student's content mastery in his/her native language. These learners must quickly begin to develop academic language skills, while simultaneously acquiring English language skills to gain academic knowledge and achieve success in the mathematics classroom. Academic language and vocabulary skills require specific attention, as they are not part of the basic communication skills students learn from their peers.

Three-Tier Model for English Language Learners in Mathematics



Assessment

Assessment is the process of collecting, reviewing, and using information to make decisions about student learning and instructional practices. An effective mathematics program should include both formative and summative assessment. Formative assessments (such as diagnostic assessments and progress monitoring) inform teachers about their own instructional effectiveness, as well as student progress and skill acquisition. Formative assessments may be formal or informal, and are not usually included in a student's reported grade. Teachers use formative assessment to assess learning as it happens and adjust instruction as needed. While there is a large body of research supporting the practice of formative assessment, there are currently few research-based formative assessment products. Thus, local educators must shoulder the responsibility of choosing or developing assessments that meet their needs, based on the best available professional thinking and research.

Summative assessments take place at the conclusion of an instructional cycle and inform stakeholders of student achievement in mathematics. Summative assessments such as chapter and quarter tests are usually formal in nature and are included in a students' reported grade. Summative assessments may provide an evaluation of the effectiveness of instruction and compare student achievement to Utah Core grade and course performance standards. Summative assessments such as the Utah Criterion-Referenced Tests (CRTs), the National Assessment of Educational Progress (NAEP), the Utah Basic Skills Competency Test (UBSCT), and the Iowa Test of Basic Skills are also frequently used for school, district, or state reporting purposes.

When choosing commercial assessment tools, educators should look for those that are aligned to the Utah Mathematics Core Curriculum, easy to administer, and developmentally appropriate. Assessments should be reliable and valid for the purpose for which they were developed. Administrators must also consider any training or technology requirements that may be necessary for effective implementation of an assessment tool.

Types of Assessments

	Definition	Examples	Purpose
Screening	Screening assessments are quick and efficient measures of mathematics ability to identify students who may be at risk for developing difficulties in certain areas of mathematics.	Placement tests Pre-tests Teacher and district developed tests	To determine student need for supplemental instruction or to determine placement in courses.
Diagnostic	Diagnostic assessments provide in-depth information about a student's performance in particular conceptual or skill areas.	No widely available, valid, reliable diagnostic assessments have been identified at this time.	To help teachers plan interventions by providing in-depth information regarding students' skills and instructional needs.
Progress Monitoring	Progress monitoring assessments are formative assessments used to determine whether students are making adequate progress developing skills in mathematics core content. These assessments should be administered as part of the instructional routine: weekly, bi-weekly, or monthly, depending on student need. Progress monitoring occurs more often in Tiers 2 and 3 than in Tier 1. Progress monitoring ensures that all students continue to make adequate progress in core areas of mathematics.	Curriculum-based measurement Commercial progress monitoring tools	To inform teachers of student progress, plan instruction, and determine needed interventions. To regroup students for instruction and evaluate the results of instructional changes.

School Collaborative Teams

Collaborative teams support teachers in improving educational outcomes for all students by meeting together to make decisions leading to improved student achievement. Teaming structures vary and could consist of teachers, administrators, and other educators. Teams plan for student success through data analysis, curriculum planning, sharing of best practices, and discussion of intervention strategies.

To be effective, teams should use a data-driven problem-solving process such as the following:

- Define the problem
- Analyze why it is occurring
- Develop and implement an action plan
- Monitor student progress
- Evaluate plan effectiveness
- Revise plan as needed

Qualified team members should have knowledge and skills in the following areas:

- Group processing (reflection on the group's work and interactions; focus on refining and improving efforts to achieve the group's goals; ensure positive, effective working relationships)
- Effective classroom organization and management
- Problem solving and collaboration
- Selection and use of multiple sources of assessment data for analysis of student learning
- Expertise in effective, research-based differentiation strategies
- Instructional adaptations and interventions based on data
- The Utah Mathematics Core Curriculum
- Content materials that support the NCTM content strands as expressed in the Utah Core Curriculum
- Content materials that support the Intended Learning Outcomes (from the NCTM Process Standards)
- Cultural sensitivity and use of ELL support techniques
- USOE Mathematics Pacing Guides and/or District Curriculum Mappings
- CRT Reference Sheets
- USOE Assessment Accommodation Manual
- Strategies for choosing and using technology effectively

Team members should also have access to support personnel and ongoing professional development

Research supports the collaborative team approach, which is most effective when the team addresses both prevention and intervention of difficulties in mathematics. Use of collaborative teams will result in:

- An increased rate of student achievement.
- Increased capacity of teachers to meet specific needs of struggling students.
- Decreased numbers of special education referrals and placements.

Examples of School Collaborative Teams

<i>Team</i>	<i>Potential Members</i>	<i>Purpose</i>
Grade Level or Mathematics Course Team	<ul style="list-style-type: none"> • All grade level/mathematics course teachers • Instructional coach • ELL teacher or specialist • Special education teacher • School counselor • Administrator(s) • Other school support personnel 	<ul style="list-style-type: none"> • Ensure consistency of effective instructional practices within and across grade levels • Share responsibility for student achievement, using student data as a base for instructional decision making • Act as a problem-solving organization whose members assist each other in meeting the needs of students in mathematics
Mathematics Department Team	<ul style="list-style-type: none"> • Mathematics department chair • Mathematics teachers • Instructional or mathematics coach • ELL teacher or specialist • Special education teacher(s) • School counselor • Administrator(s) • Other school support personnel 	<ul style="list-style-type: none"> • Ensure consistency of instructional practices within the content area • Ensure vertical alignment across grade levels/subject areas • Share responsibility for student achievement, using student data as a base for instructional decision making • Act as a problem-solving organization whose members assist each other in meeting mathematics goals for students
Intervention Team	<ul style="list-style-type: none"> • Core classroom teacher • Intervention specialist • School mathematics coach and/or district mathematics specialist • School counselor • School psychologist • Special educator • ELL teacher or specialist, • Administrator(s) • Other school support personnel 	<ul style="list-style-type: none"> • Analyze data to determine appropriate intervention strategies • Identify appropriate interventions for individual students • Identify appropriate services for or needed evaluations of individual students • Systematize and coordinate school-wide interventions • Schedule and provide access to intervention services • Make recommendations for professional development

School Readiness Guide

Using the School Readiness Evaluation Tool, administrators should examine readiness for participation in a tiered model of instruction. Administrators should work with faculties to make decisions regarding possible scheduling needs, teacher assignments, professional development, or other strategies that will support student learning based on this self-assessment.

Prior to implementing the 3-Tier Model of Mathematics Instruction, school administrators should use the tool provided to examine each of the following areas:

- Curriculum
- Instruction
- Assessment
- Resources
- Teacher quality
- Leadership
- Scheduling
- Setting
- Community and home

Elementary School Readiness Evaluation Tool

Elementary Tier 1

Leadership	Not Evident	Beginning	Intermediate	Advanced
Administrators are committed to mathematics leadership and make decisions to create a quality mathematics program.				
Administrators recognize and support quality mathematics instruction.				
Administrators commit financial and other resources to mathematics.				
Administrators attend to equity issues in mathematics placement.				

Curriculum	Not Evident	Beginning	Intermediate	Advanced
Teachers know and use the Utah Mathematics Core Curriculum, teaching the appropriate grade- or course-level standards and objectives.				
Teachers know and use the <i>NCTM Principles and Standards for School Mathematics</i> and <i>Focal Points</i> .				

Instruction	Not Evident	Beginning	Intermediate	Advanced
A school-wide, seamless instructional program is aligned with materials, assessment, instructional strategies, and procedures.				
Teachers know and use instructional frameworks that support student learning, such as <i>Mathematics Teaching Today</i> .				
Teachers provide whole class, small group, differentiated, and individual instruction.				
Teachers are trained in the 3-Tier Model of Mathematics Instruction.				
Classroom schedules are arranged to accommodate whole class, small group, and individual instruction.				
Students have opportunities to apply skills and strategies throughout the day across other content areas.				

Assessment	Not Evident	Beginning	Intermediate	Advanced
Educators analyze overall demographic performance and trends in mathematics core testing.				
Administrators and grade-level teams use assessments to identify trends, strengths, and weaknesses on core objectives for each course.				
Teachers use assessments to monitor the students' mathematics progress during the current school year.				
Teachers use a variety of data to monitor student progress and achievement.				

Resources	Not Evident	Beginning	Intermediate	Advanced
Teachers use core program materials that are aligned to State Mathematics Core objectives and intended learning outcomes.				
Manipulatives and technology support high- quality mathematics instruction				
Teachers appropriately use classroom assistants and volunteers.				
Classroom assistants and volunteers receive appropriate training.				
Collaborative teams support teachers and review student progress.				

Teacher Quality	Not Evident	Beginning	Intermediate	Advanced
Teachers are highly qualified to be teaching mathematics.				
Teachers are highly skilled and able to provide high-quality mathematics instruction.				
Teachers use appropriate management techniques and procedures to facilitate quality instruction.				
Professional development opportunities focus on mathematics content and pedagogy.				
Highly skilled instructors are assigned to the students of greatest need.				

Scheduling	Not Evident	Beginning	Intermediate	Advanced
Tier 1 mathematics instruction is 60-90 minutes daily.				
Administrators attend to equity issues in scheduling students and teachers.				

Setting	Not Evident	Beginning	Intermediate	Advanced
All students participate in Tier 1 mathematics instruction in the regular classroom.				
The classroom is a mathematically rich environment with technology and applications available for everyday use.				
Course loads are appropriately distributed so that struggling learners are placed in the smallest possible classes with the highest quality teachers.				
The classroom environment is arranged to accommodate whole class, small group, and individual instruction.				

Community and Home	Not Evident	Beginning	Intermediate	Advanced
Parents receive regular communication about their child's mathematical progress.				
Materials and training are provided for parents to help children with mathematics at home.				
After-school parent programs support student progress in mathematics.				
Educators collaborate with business and industry to improve mathematics education.				

Elementary Tier 2

Curriculum	Not Evident	Beginning	Intermediate	Advanced
Teachers know and use the Utah Mathematics Core Curriculum, teaching the appropriate grade- or course-level standards and objectives.				
Teachers use the <i>NCTM Principles and Standards for School Mathematics</i> and <i>Focal Points</i> to identify the foundational concepts in the core curriculum.				
Teachers have received training in the 3-Tier Model of Mathematics Instruction.				

Instruction	Not Evident	Beginning	Intermediate	Advanced
The targeted instruction is systematic and explicit and includes guided practice.				
Instruction builds both conceptual understanding and procedural fluency.				
Instruction teaches concepts, moving from manipulatives and concrete models to pictorial representations, and finally to abstract symbols (CRA).				
Mathematical connections are explicit.				
Students are engaged in the learning process.				
Students ask and answer questions.				
Students receive appropriate feedback.				
Students receive supplemental instruction in small, homogeneous groups (3 to 5).				
Instruction includes vocabulary activities, higher-order thinking and problem-solving strategies.				

Assessment	Not Evident	Beginning	Intermediate	Advanced
Teachers conduct screening assessments to identify struggling mathematics students.				
Diagnostic assessments identify specific mathematical deficits for struggling students.				
Student progress is monitored at least every two weeks.				

Resources	Not Evident	Beginning	Intermediate	Advanced
Teachers use core-based materials to target specific diagnosed needs and support Tier 1 instruction.				
Locations are available to accommodate small instructional groups.				
Classroom assistants/volunteers receive training to support struggling students.				
Assistants/volunteers provide additional practice, but do not provide instruction.				
The collaborative team supports the classroom teacher.				
Supplemental materials support high-quality mathematics instruction and practice.				

Teacher Quality	Not Evident	Beginning	Intermediate	Advanced
All mathematics teachers are highly qualified.				
Mathematics teachers are highly skilled at instructional techniques that target student difficulties.				
Management techniques and procedures facilitate quality instruction.				
Professional development activities support Tier 2 instruction.				

Scheduling	Not Evident	Beginning	Intermediate	Advanced
An additional 15-30 minutes beyond Tier 1 instruction is provided for Tier 2 interventions.				
Schedules ensure that students do not miss Tier 1 Core Mathematics instruction.				

Setting	Not Evident	Beginning	Intermediate	Advanced
Tier 2 interventions occur in the regular classroom.				
Tier 2 interventions are available before/after school.				
Summer school is provided for struggling students.				

Community and Home	Not Evident	Beginning	Intermediate	Advanced
Parents receive regular communication about their child's mathematical progress.				
Materials and training are provided for parents to help children with mathematics at home.				

Elementary Tier 3

Curriculum	Not Evident	Beginning	Intermediate	Advanced
The curriculum provides intense, targeted work on essential mathematics understanding and skills.				
The curriculum emphasizes number and operation.				
The curriculum provides opportunities for learning mathematics vocabulary.				

Instruction	Not Evident	Beginning	Intermediate	Advanced
Instruction is systematic, explicit, and highly intensive.				
Students are actively engaged in their learning.				
Students ask and answer questions.				
Students receiving specific feedback.				
Students receive Tier 3 instruction in small, homogeneous groups (3 to 5).				

Assessment	Not Evident	Beginning	Intermediate	Advanced
Diagnostic assessments identify specific deficits.				
Progress monitoring occurs at least weekly.				

Resources	Not Evident	Beginning	Intermediate	Advanced
Materials and methods provide the intensity of instruction needed.				
Tier 3 mathematics materials support Tier 1 instruction.				
Locations are available to accommodate small instructional groups.				
Classroom assistants/volunteers receive training to support struggling students.				
Assistants/volunteers provide additional practice, but do not provide instruction.				
A collaborative team designs and implements the Tier 3 intervention plan.				

Teacher Quality	Not Evident	Beginning	Intermediate	Advanced
Highly qualified specialists deliver interventions.				
Management techniques and procedures facilitate quality instruction.				
Professional development activities support Tier 3 instruction.				

Scheduling	Not Evident	Beginning	Intermediate	Advanced
At least 30 minutes a day is allocated to Tier 3 instruction, in addition to Tier 1 instructional time.				
Scheduling ensures that students do not miss Tier 1 Core Mathematics instruction.				

Setting	Not Evident	Beginning	Intermediate	Advanced
Instruction occurs in an appropriate specialized instructional setting within the school.				

Community and Home	Not Evident	Beginning	Intermediate	Advanced
Parents receive regular communication about their child's mathematical progress.				
Materials and training are provided for parents to help children with mathematics at home.				

Secondary School Readiness Evaluation Tool

Secondary Tier 1

Leadership	Not Evident	Beginning	Intermediate	Advanced
Administrators are committed to mathematics leadership and make decisions to create a quality mathematics program.				
Administrators recognize and support quality mathematics instruction.				
Administrators commit financial and other resources to mathematics.				
Administrators attend to equity issues in mathematics placement.				

Curriculum	Not Evident	Beginning	Intermediate	Advanced
Teachers know and use the Utah Mathematics Core Curriculum, teaching the appropriate grade- or course-level standards and objectives.				
Teachers know and use the <i>NCTM Principles and Standards for School Mathematics</i> and <i>Focal Points</i> .				

Instruction	Not Evident	Beginning	Intermediate	Advanced
A school-wide, seamless instructional program is aligned with materials, instructional strategies, assessment, and procedures.				
Teachers know and use instructional frameworks that support student learning, such as <i>Mathematics Teaching Today</i> .				
Teachers provide whole class, small group, differentiated, and individual instruction.				
Teachers are trained in the 3-Tier Model of Mathematics Instruction.				
Students have opportunities to apply skills and strategies throughout the day across other content areas.				

Assessment	Not Evident	Beginning	Intermediate	Advanced
Educators analyze overall demographic performance and trends in mathematics core testing.				
Administrators and departments use assessments to identify trends, strengths, and weaknesses on core objectives for each course.				
Teachers use assessments to monitor the students' mathematics progress during the current school year.				
Teachers use a variety of data to monitor student progress and achievement.				

Resources	Not Evident	Beginning	Intermediate	Advanced
Teachers use core program materials that are aligned to State Mathematics Core objectives and intended learning outcomes.				
Manipulatives and technology support high- quality mathematics instruction				
Teachers appropriately use classroom assistants and volunteers.				
Classroom assistants and volunteers receive appropriate training.				
Collaborative teams support teachers and review student progress.				

Teacher Quality	Not Evident	Beginning	Intermediate	Advanced
Teachers are highly qualified and endorsed to teach assigned mathematics courses.				
Teachers are highly skilled and able to provide high-quality mathematics instruction.				
Teachers use appropriate classroom management techniques and procedures to facilitate quality instruction.				
Professional development opportunities focus on mathematics content and pedagogy.				
Highly skilled instructors are assigned to the students with the greatest need.				

Scheduling	Not Evident	Beginning	Intermediate	Advanced
Administrators attend to equity issues in scheduling students and teachers.				
Students receive additional support before or after school.				

Setting	Not Evident	Beginning	Intermediate	Advanced
The classroom is a mathematically rich environment with technology and applications available for everyday use.				
Course loads are appropriately distributed so that struggling learners are placed in the smallest possible classes with the highest quality teachers.				
Teachers are provided with adequate preparation time for the number of courses assigned to them.				
The classroom environment is arranged to accommodate whole class, small group, and individual instruction.				

Community and Home	Not Evident	Beginning	Intermediate	Advanced
Parents receive regular communication about their child's mathematical progress.				
Educators collaborate with business and industry to improve mathematics education.				

Secondary Tier 2

Curriculum	Not Evident	Beginning	Intermediate	Advanced
Teachers know and use the Utah Mathematics Core Curriculum, teaching the appropriate grade- or course-level standards and objectives.				
Teachers use the <i>NCTM Principles and Standards for School Mathematics</i> and <i>Focal Points</i> to identify the foundational concepts in the core curriculum.				
Teachers have received training in the 3-Tier Model of Mathematics Instruction.				

Instruction	Not Evident	Beginning	Intermediate	Advanced
Targeted instruction is systematic and explicit and includes guided practice.				
Instruction builds both conceptual understanding and procedural fluency.				
Instruction teaches concepts, moving from manipulatives and concrete models to pictorial representations, and finally to abstract symbols (CRA).				
Mathematical connections are explicit.				
Students are engaged in the learning process.				
Students ask and answer questions.				
Students receive appropriate feedback.				
Students receive supplemental instruction in small, homogeneous groups (3 to 5).				
Instruction includes vocabulary activities, higher-order thinking and problem-solving strategies.				

Assessment	Not Evident	Beginning	Intermediate	Advanced
Teachers conduct screening assessments to identify struggling mathematics students.				
Diagnostic assessments identify specific mathematical deficits for struggling students.				
Student progress is monitored at least every two weeks.				

Resources	Not Evident	Beginning	Intermediate	Advanced
Teachers use core-based materials to target specific diagnosed needs and support Tier 1 instruction.				
Locations are available to accommodate small instructional groups.				
Classroom assistants receive training to support struggling students.				
Assistants provide additional practice, but do not provide instruction.				
The collaborative team supports the classroom teacher.				
Supplemental materials support high-quality mathematics instruction and practice.				

Teacher Quality	Not Evident	Beginning	Intermediate	Advanced
Teachers are appropriately endorsed to teach assigned mathematics courses.				
Mathematics teachers are highly skilled at instructional techniques that target student difficulties.				
Management techniques and procedures facilitate quality instruction.				
Professional development activities support Tier 2 instruction.				

Scheduling	Not Evident	Beginning	Intermediate	Advanced
Students have at least 30 minutes daily mathematics instruction in addition to the core mathematics period.				

Setting	Not Evident	Beginning	Intermediate	Advanced
Additional contact time has been provided for struggling learners of mathematics				
Tier 2 interventions are available before/after school.				
Summer school is provided for struggling students.				

Community and Home	Not Evident	Beginning	Intermediate	Advanced
Parents receive regular communication about their child's mathematical progress.				

Secondary Tier 3

Curriculum	Not Evident	Beginning	Intermediate	Advanced
The curriculum provides intense, targeted work on essential mathematics understanding and skills.				
The curriculum provides opportunities for learning mathematics vocabulary.				

Instruction	Not Evident	Beginning	Intermediate	Advanced
Instruction is systematic, explicit, and highly intensive.				
Students are actively engaged in their learning.				
Students ask and answer questions.				
Students receive specific feedback.				
Students receive Tier 3 instruction in small, homogeneous groups (3 to 5).				

Assessment	Not Evident	Beginning	Intermediate	Advanced
Diagnostic assessments identify specific deficits.				
Progress monitoring occurs at least weekly				

Resources	Not Evident	Beginning	Intermediate	Advanced
Materials and methods provide the intensity of instruction needed.				
Tier 3 mathematics materials support Tier 1 instruction.				
Locations are available to accommodate small instructional groups.				
A collaborative team designs and implements the Tier 3 intervention plan.				

Teacher Quality	Not Evident	Beginning	Intermediate	Advanced
Highly qualified specialists deliver interventions.				
Management techniques and procedures facilitate quality instruction.				
Professional development activities support Tier 3 instruction.				

Scheduling	Not Evident	Beginning	Intermediate	Advanced
A class period is allocated to Tier 3 instruction, in addition to the Tier 1 class period.				
Scheduling ensures that students do not miss Tier 1 Core Mathematics instruction.				

Setting	Not Evident	Beginning	Intermediate	Advanced
Instruction occurs in an appropriate specialized instructional setting within the school.				

Community and Home	Not Evident	Beginning	Intermediate	Advanced
Parents receive regular communication about their child's mathematical progress.				

Survey of Teacher Professional Development Needs

This survey assists principals, teachers, mathematics specialists and coaches in prioritizing professional development. In the first survey, consider teacher confidence in each topic on the left and rate the priority for professional development on the right. In the second survey, consider the degree to which each pedagogical practice is implemented in the classroom on the left and the priority for professional development on the right. Consider both assessed deficits and mathematical importance to student success to prioritize professional development needs.

Assessment of Expertise in Mathematics Content

Fully Confident	Somewhat Confident	Not Confident	Mathematics Content	High Priority	Medium Priority	Low Priority
			Number sense			
			Mathematical procedures			
			Creating mathematics concepts			
			Discovering mathematical relationships			
			Problem solving			
			Mathematics vocabulary			
			Symbolic reasoning			
			Logical thinking			
			Making connections			
			Mathematical habits of mind (persistence, enjoyment, confidence, creativity)			
			Appropriate use of technology			
			Mathematical models			
			Applications			
			Data analysis			
			Algebra			
			Geometry			
			Other:			

Assessment of Expertise in Mathematics Pedagogy

Fully Implemented	Partially Implemented	No Evidence of Implementation	Mathematics Pedagogy	High Priority	Medium Priority	Low Priority
			Research-based interventions			
			Selecting appropriate materials			
			Appropriate use of technology			
			Communication and vocabulary strategies			
			Classroom management			
			Classroom environment			
			Large/small group instruction			
			Selecting appropriate assessments			
			Using assessment results to inform instruction			
			Monitoring student progress			
			Providing differentiated instruction			
			Facilitating mathematical discourse			
			Facilitating the construction of concepts			
			Facilitation of the discovery of relationships			
			Developing creative thinking			
			Comprehension and communication strategies			
			Providing tutoring and supplemental practice			
			Providing parent support and instruction			
			Other:			

Online Resources

Progress Monitoring and Tutoring Programs

Aimswab	http://www.aimswab.com/
Apangea Learning	http://apangea.com/
CompassLearning	http://www.compasslearning.com
Curriculum-Based Measurement Warehouse	http://www.interventioncentral.org/htmldocs/interventions/cbmwarehouse.php
LearnKey	http://www.learnkey.com
Mathematics Dynamic Assessment	http://fcit.usf.edu/mathvids/strategies/da.html
National Center on Student Progress Monitoring	http://www.studentprogress.org/
Plato Learning	http://www.plato.com
Pro-Ed Inc.	http://www.proedinc.com
Star Math	http://www.renlearn.com/sm/
Yearly Progress Pro	http://www2.ctb.com/products_services/ypp/index.html

Resources for Teaching Struggling Learners

Imagine Learning (for English Language Acquisition)	http://www.imaginelearning.com/
MathVIDS (Video Instructional Development Source)	http://www.coedu.usf.edu/main/departments/sped/mathvids/index.html

Instructional Resources

Utah Education Network <http://www.uen.org/>

Utah State Office of Education (USOE)

Assessment <http://www.schools.utah.gov/assessment/>

Elementary Mathematics <http://www.schools.utah.gov/curr/Math/elem/>

Secondary Mathematics <http://www.schools.utah.gov/curr/math/sec/>

Special Education Rules

<http://www.schools.utah.gov/sars/lawsregs/rules.htm>

Utah State University's National Library of Virtual Manipulatives

<http://nlvm.usu.edu/>

For Recent Information on Research and Interventions

What Works Clearinghouse <http://ies.ed.gov/ncee/wwc/>

Glossary

Accommodations—Accommodations refer to the teaching supports and services made available to students with special needs for accessing or demonstrating competency in the defined curriculum. Accommodations meet the needs of individual learners by reducing or eliminating the effects of a student’s disability. Accommodations may alter instruction or assessment, but do not change desired learning outcomes.

Adaptive reasoning—Adaptive reasoning is the capacity for logical thought, reflection, explanation, and justification.

Assessment—Mathematics assessment is the process of determining a student’s understanding of mathematics and measuring his or her ability to perform procedures.

Collaborative team—The collaborative team is a group of two or more people who meet on a scheduled basis to support teachers in improving educational outcomes for all students by meeting together to make decisions leading to improved student achievement. Teaming structures and membership vary to plan for student success through data analysis, curriculum planning, sharing of best practices, and discussion of intervention strategies.

Conceptual understanding—Conceptual understanding is comprehension of mathematical concepts, operations, and relations.

Curriculum-based measurement (CBM)—CBM is a reliable and valid assessment system that is used to screen students or to monitor student progress in mathematics, reading, writing, and spelling. With CBM, teachers and schools can assess individual responsiveness to instruction.

Differentiated instruction—Differentiated instruction tailors the learning environment and teaching practices to create appropriately different learning experiences for students in order to meet each student’s needs. Teachers recognize students’ varying interests, readiness levels, and levels of responsiveness to the standard Core Curriculum, and plan responsively to address these individual differences. There are four elements of the curriculum that can be differentiated: content, process, products, and learning environment.

English language learners (ELLs)—English language learners are those students whose first language is not English and who are in the process of learning English.

Explicit instruction—Explicit instruction is that which provides clear models for solving a problem type using an array of examples. Its purpose and process is clear and overt.

Five Strands of Mathematical Proficiency—The five strands of mathematical proficiency, as defined by the National Research Council in *Adding it Up*, are adaptive reasoning, strategic competence, conceptual understanding, productive disposition, and procedural fluency.

Formative assessment—Formative assessment is used to plan instruction in a recursive way through systematic and continuous feedback to both the student and the teacher concerning successes and failures. Formative assessments such as progress monitoring inform teachers about their own instructional effectiveness, student progress, and skill acquisition. Formative assessments may be formal or informal and are not usually included in a student's reported grade. Teachers use formative assessment to assess learning as it happens and adjust instruction as needed.

Individualized Education Program (IEP)—An IEP is a written statement for a student with a disability that is developed, reviewed, and revised in accordance with IDEA (Individuals with Disabilities Education Act).

Intervention—Interventions are specific services, activities, or products developed and implemented to change or improve student knowledge, attitudes, behavior, or awareness.

Peer tutoring—Peer tutoring is an instructional arrangement where students work in pairs or small groups to help one another learn material or practice an academic task.

Procedural fluency—Students with procedural fluency have skill in carrying out procedures flexibly, accurately, efficiently, and appropriately.

Process standards—The National Council of Teachers of Mathematics Standards describes the processes students use to conceptualize and do mathematics: reasoning and proof, representation, communication, connections, and problem solving.

Productive disposition—A productive disposition is the habitual inclination to see mathematics as sensible, useful, and worthwhile, coupled with a belief that hard work and diligence lead to success.

Progress monitoring—Progress monitoring is the process of collecting and evaluating data to make decisions about student progress toward a goal. Progress monitoring evaluates the student's rate of change as compared to the anticipated progress and informs instructional planning.

Scaffolding—Scaffolding is a carefully designed learning progression that provides assistance to students learning new material through explicit instruction, modeling, questioning, and feedback. Scaffolding breaks learning into small steps, allowing students to grow gradually in independence as learners.

Specialist—Specialists in mathematics education have particular skills to help struggling learners. They may have deeper content or pedagogy knowledge necessary to provide access to the mathematics curriculum. Special education teachers, ESL-endorsed teachers, and mathematics coaches are all examples of specialists.

Strategic competence—Strategic competence is the ability to formulate, represent, and solve mathematical problems.

Summative assessment—Summative assessments take place at the conclusion of an instructional cycle and inform stakeholders of student achievement in mathematics. Summative assessments such as chapter and quarter tests are usually formal in nature and are included in a students' reported grade. Summative assessments provide an evaluation of the effectiveness of instruction and compare student achievement to core grade and course performance standards.

Systematic instruction—Systematic instruction is an approach in which teachers guide students through a defined instructional sequence. Systematic instruction may be either teacher- or student- directed, and inquiry-based or explicit.

Targeted instruction—Targeted instruction focuses instruction on the specific skill deficits among particular struggling learners.

Tutoring—Tutoring is one-on-one or small group instruction and guided practice.

Utah Core Curriculum—The Utah Core Curriculum is standards of learning defined by the Utah State Board of Education as essential for all students, which define teacher instruction.

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