

**Utah State Office of Education**  
**Elementary Mathematics Endorsement Course Framework**  
**Mathematics for Teaching K-8 - Rational Numbers and Proportional Reasoning**

**Course Description:** To provide practicing teachers a deeper understanding of rational numbers, operations with rational numbers, and proportionality, and instructional strategies to facilitate the instruction of this content for elementary students.

**Course Objectives:**

During this course students will:

- Develop and apply concepts of rational numbers using various models and representations including: area, linear, sets, numbers, and division.
- Analyze, compare, and order rational numbers using multiple representations.
- Understand, communicate, perform, and reflect on the basic operations with rational numbers.
- Use proportional reasoning and strategies to solve a variety of real world problems using multiple representations.
- Orchestrate discourse related to rational numbers and proportions in an elementary classroom.
- Develop problem solving skills using rational numbers and proportion.
- Reason, justify, and analyze using rational numbers and ideas of proportionality.
- Use a variety of tools including technology to enhance classroom instruction and increase student understanding of rational numbers and proportions.

**Topics:**

**Concepts of Rational Numbers**

- Develop understanding of fractions as parts of unit wholes, as parts of collections, as location on number lines, and as division of whole numbers.
- Recognize and generate equivalent forms of commonly used fractions, decimals, and percents.
  - Partitioning and quotients.
  - Rational numbers as operators.
  - Rational numbers as measures.
  - Representations for rational numbers: Fractions, decimals, percents.
  - Various models for rational numbers: area, linear (measure), sets, number, and division.

**Comparison and Equivalency**

- Use models, benchmarks, and equivalent forms to judge the size of fractions.
- Compare and order fractions, decimals and percents and find their approximate locations on a number line.

- Fractions, decimals, percents: Comparing and ordering fractions, using benchmarks.
- Equivalencies among representations of rational numbers including fractions, decimals and percents.
- There are infinite numbers between any two numbers.

### **Operations**

- Understand the meaning and effects of arithmetic operations with fractions and decimals.
- Understand and use the inverse relationships of addition and subtraction, multiplication and division.
- Work flexibly with fractions, decimals and percents to solve problems.
- Develop and analyze algorithms for computing with fractions and decimals.
- Develop and use strategies to estimate the results of rational-number computations and judge the reasonableness of the results.
  - Concrete, representational, and abstract representations of addition, subtraction, multiplication, division.
  - Contexts for operations with fractions.

### **Proportional Reasoning**

- Understand and use ratios and proportions to represent quantitative relations.
- Develop, analyze, and explain methods for solving problems involving proportions, such as scaling and finding equivalent ratios.
  - Ratio
  - Scale
  - Algebraic proportions
  - Percents
  - Rates and speed

## **Pedagogy**

The purpose of the Elementary Mathematics Endorsement courses is to ensure that practicing teachers gain the mathematical content knowledge needed to teach mathematical concepts to students in the elementary grades. Teachers must also, however, know how to transfer that content knowledge and the conceptual understandings inherent to the content to students. An understanding of sound pedagogical practice is essential to that transfer. Following are suggested pedagogical concepts and strategies that should be infused into the courses to aid teachers in student instruction. These concepts should never be taught in isolation, but should be modeled throughout the courses. It is not necessary nor intended that all the following concepts would be infused in every course, but they should be covered in their entirety in the series.

### **Knowledge of mathematics learners:**

- Employ prior knowledge, skills, dispositions, beliefs, conceptions, misconceptions and confusions around particular topics;
- Employ knowledge of cognition, child development, and learning theory;
- Use knowledge of learners culturally and socially to know what motivates and inspires them.

### **Knowledge of effective mathematics teaching practices for student and teacher learning:**

- Design, select and/or adapt worthwhile tasks and sequences of examples that support a particular learning goal, including tasks to be solved by teachers themselves, and others for which they try to anticipate children's solutions, and design responses.
- Develop sensitivity with the careful use of technical language, attending to both mathematical integrity, and usability by learners.
- Construct and evaluate multiple representations of a mathematical idea or process, and establish correspondences between representations.
- Use questions to effectively probe mathematical understanding and make productive use of responses.
- Develop learners' skills at clear and coherent public mathematical communication in a classroom setting, using a variety of resources and media.
- Model mathematical practices – questioning, representing, communicating, problem solving, inquiring and conjecturing, making connections, reasoning and proving, self-assessing and cultivate the learning of such practices.
- Use various instructional applications of technology and be able to make judicious, mathematically and pedagogically grounded use of them.
- Construct and mathematically evaluate contexts or story problems for various mathematical situations and processes.

- Analyze and evaluate student work, and design appropriate responses.
- Develop skillful and flexible use of different instructional formats – whole group, small group, partner, and individual – in support of learning goals.
- Know the resources for managing the diversities – cultural, linguistic, gender, socioeconomic, developmental – of the classroom, and ways to skillfully deploy them in support of the learning of all students.
- Understand and address diversity in planning and implementing instruction and in helping other teachers foster mathematics classroom environments that support the learning of all students.
- Understand and utilize various cultures' approaches to learning mathematics.

### **Knowledge of mathematics curriculum and assessment:**

- Know common developmental paths (learning trajectories) related to foundational mathematical topics (e.g., place value, rational numbers and equivalence); understand these developmental paths, and use them to sequence activities to build learning environments that are developmentally appropriate and effective.
- Select, use, adapt and determine the suitability of mathematics curricula and teaching materials (e.g., textbooks, technology, manipulatives) for particular learning goals.
- Be cognizant of a formative assessment cycle (administering a formative assessment task, analyzing student work from the task, using that analysis to enhance teacher knowledge, and designing and teaching reengagement lessons) and resources.
- Provide appropriate interpretations of assessment results, and communicate results (in context) to specific individuals and groups (e.g., students, parents, caregivers, colleagues, administrators, policymakers, community members).

(Standards for Elementary Mathematics Specialists, Association of Mathematics Teacher Educators, 2009)

### **Possible Assignments:**

- Mathematical problem solving tasks: Strategically selected mathematical tasks focused on developing problem solving strategies and exposing important ideas about rational numbers and proportions.
- Clinical interview: Design an interview to determine student understanding of rational numbers and/or operations, including interview questions with extensions to press for student understanding. Videotape and conduct the interview. Analyze the video using a rubric that assesses both student understanding and the interview process.
- Differentiated lesson: Select one of the course objectives. Develop a lesson to meet the objective for the whole group and addition differentiation activities for at least

three populations that could include gifted students, English Language Learners, Special Education, and/or other groups.

- Action research project: Action research is a particular approach to research that aims to improve practice by addressing real-world situations and needs through focused, reflective practice and the application of interventions or procedures that are designed for improving performance. Develop a classroom research project about teaching concepts of mathematics for your own classroom. Consult the mathematics education literature, citing at least three sources from peer-reviewed journals. Develop a research question which could be a new teaching, assessment, or curriculum strategy for rational numbers in your class. Collect data, and analyze the results.
- Case study: Write a case presenting the mathematical thinking of a student or group of students. Be sure to include details in your narrative, such as student dialogue, your questions, and what you were thinking as you listened to the students. Analyze the student thinking and discuss the questions that are raised for you in the students' mathematical thinking.
- Lesson study cycle: Work with a group to develop a task-based lesson including anticipated student responses, questions for classroom discourse, and formative assessment. One person in the group will teach the lesson while other members of the group observe student thinking. Group members refine the lesson in response to the observation data and then teach and analyze the lesson in their own classes.
- Reflections on your own mathematical understanding: Keep a reflections log that discusses: a mathematical idea from the session that is new or important for you, a question you have, and an application of the mathematical idea for your classroom.
- Content exams, performance tasks, and homework for determining participants' mathematical knowledge for teaching on the topic of rational numbers and proportionality.
- Reading research – choose from possible articles and do an assignment based on the article; discussions, agreement/disagreement, impact in classrooms.
- Lesson study via video tools such as Teaching Channel Teacher Teams
- Use technology to enhance and explore e.g., Geometer's Sketchpad, Geogebra, smartboard tools.

### **Suggested Texts:**

Parker, Thomas H. and Baldrige, Scott J. (2005). *Elementary Mathematics for Teachers*. Portland, OR: Sefton-Ash Publishers.

Van De Walle, John A. et. al., (2009). *Elementary and Middle School Mathematics, Teaching Developmentally (7<sup>th</sup> Edition)*. Boston, MA: Allyn & Bacon.

Beckman, Sybilla. (2008). *Mathematics for Elementary Teachers (2<sup>nd</sup> Edition)*. Reading, MA: Addison Wesley.

Lamon, Susan J. (2009). *Teaching Fractions and Ratios for Understanding: Essential Content Knowledge and Instructional Strategies for Teachers*. Florence, KY: Routledge.

Carpenter, Thomas P. et. al., (2003). *Thinking Mathematically: Integrating Arithmetic and Algebra in Elementary School*. Chicago, IL: Heinemann.

Schifter, Deborah, et. al., (2007). *Making Meaning of Operations: A Collaborative Project by the Staff and Participants of Teaching to the Big Ideas (Developing Mathematical Ideas)*. Palo Alto, CA: Dale Seymour Publications.

### **Supplementary Texts/Readings:**

Ministry of Education, Singapore. *Singapore Mathematics Singapore Primary Mathematics Textbooks 3A, 4A, 5A, and 6A*. Oregon City, OR: Marshall Cavendish Education.

Ministry of Education, Singapore. *Singapore Primary Mathematics Workbook 5A*. Oregon City, OR: Marshall Cavendish Education.

Schifter, Deborah, et. al., (2000). *Making Meaning of Operations: Casebook (Development Mathematical Ideas)*. Palo Alto, CA: Dale Seymour Publications.

Fosnot, Catherine Twomey and Dolk, Maarten (2002). *Young Mathematicians at Work, Constructing Fractions, Decimals and Percents*. Chicago, IL: Heinemann.

[Illustrativemathematics.org](http://Illustrativemathematics.org), Fractions Progressions Modules