TARGETED INVERVENTION: PRACTICE and STRATEGIES
MATHEMATICS TUTORING
TIPS

Grade

3

MATH

UTAH STATE OFFICE OF EDUCATION

K 1 2 3 4 5 6

MATHEMATICS TUTORING TIPS
Targeted Intervention: Practice and Strategies

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Third Grade Math Tutoring TIPS

Table of Contents

Domain: Operations and Algebraic Thinking

Standard 1   **Lesson 1**

Interpret products of whole numbers, e.g., interpret 5 × 7 as the total number of objects in 5 groups of 7 objects each. For example, describe a context in which a total number of objects can be expressed as 5 × 7.

Standard 2   **Lesson 2**

Interpret whole-number quotients of whole numbers, e.g., interpret 56 ÷ 8 as the number of objects in each share when 56 objects are partitioned equally into 8 shares, or as a number of shares when 56 objects are partitioned into equal shares of 8 objects each. (For example, describe a context in which a number of shares or a number of groups can be expressed as 56 ÷ 8.)

Standard 3   **Lesson 3**

Use multiplication and division within 100 to solve word problems in situations involving equal groups, arrays, and measurement quantities, e.g., by using drawings and equations with a symbol for the unknown number to represent the problem.

Standard 3   **Lesson 4**

Use multiplication and division within 100 to solve word problems in situations involving equal groups, arrays, and measurement quantities, e.g., by using drawings and equations with a symbol for the unknown number to represent the problem.

Standard 4   **Lesson 5**

Determine the unknown whole number in a multiplication or division equation relating three whole numbers. For example, determine the unknown number that makes the equation true in each of the following equations? 8 × ? = 48, 5 = ? ÷ 3, 6 × 6 = ?

Standard 5   Lesson 6

Apply properties of operations as strategies to multiply and divide. Examples: If 6 x 4 = 24 is known, then 4 x 6 = 24 is also known. (Commutative property of multiplication.) 3 x 5 x 2 can be found by 3 x 5 = 15, then 15 x 2 = 30, or by 5 x 2 = 10, then 3 x 10 = 30. (Associative property of multiplication.) Knowing that 8 x 5 = 40 and 8 x 2 = 16, one can find 8 x 7 as 8 x (8 + 2) = (8 x 2) = 40 + 16 = 56. (Distributive property.)

Standard 6   Lesson 7

Understand division as an unknown-factor problem. For example, find 32 ÷ 8 by finding the number that makes 32 when multiplied by 8.

Standard 7   Lesson 8

Fluently multiply and divide within 100, using strategies such as the relationship between multiplication and division (e.g., knowing that 8 x 5 = 40, one knows 40 ÷ 5 = 8) or properties of operations. By the end of Grade 3, know from memory all products of two one-digit numbers.

Standard 8   Lesson 9

Solve two-step word problems using the four operations. Represent the problems using equations with a letter standing for the unknown quantity. Assess the reasonableness of answers using mental computation and estimation strategies, including rounding.

Standard 9   Lesson 10

Identify arithmetic patterns (including patterns in the addition table or multiplication table), and explain them using properties of operations. For example, observe that 4 times a number is always even, and explain why 4 times a number can be decomposed into two equal addends.

Domain: Number and Operations in Base ten

Standard 1   **Lesson 11**

Use place value understanding to round whole numbers to the nearest 10 or 100.

Standard 2   **Lesson 12**

Fluently add and subtract within 1,000 using strategies and algorithms based on place value, properties of operations, and/or the relationship between addition and subtraction.

Standard 3   **Lesson 13**

Multiply one-digit whole numbers by multiples of 10 in the range 10-90 (e.g., 9 x 80, 5 x 60) using strategies based on place value and properties of operations.

Standard 3   **Lesson 13**

Multiply one-digit whole numbers by multiples of 10 in the range 10-90 (e.g., 9 x 80, 5 x 60) using strategies based on place value and properties of operations.

Domain: Number and Operations – Fractions

Standard 1   **Lesson 14**

Understand a fraction 1/b as the quantity formed by one part when a whole is partitioned into b equal parts; understand a fraction a/b as the quantity formed by a parts of size 1/b.

Standard 2a   **Lesson 15**

Represent a fraction a/b on a number line by defining the interval from 0 to 1 as the whole and partitioning it into b equal pieces. Recognize that each part has size 1/b and that the endpoint of the part based at 0 locates the number 1/b on the number line.

Standard 2b   **Lesson 16**

Represent a fraction a/b on a number line diagram by marking off a lengths 1/b from 0. Recognize that the resulting interval has size a/b and that its end point locates the number a/b on the number line.

Standard 3a   **Lesson 17**

Understand two fractions as equivalent (equal) if they are the same size, or the same point on a number line.

Standard 3b   **Lesson 18**

Recognize and generate simple equivalent fractions, e.g., 1/2= 2/4, 4/6 =2/3). Explain why the fractions are equivalent, e.g., by using a visual fraction model.

Standard 3c   **Lesson 19**

Express whole numbers as fractions, and recognize fractions that are equivalent to whole numbers. Examples: Express 3 in the form 3 = 3/1; recognize that 6/1 = 6; locate 4/4 and 1 at the same point on a number line diagram.

Standard 3d   **Lesson 20**

Compare two fractions with the same numerator or the same denominator by reasoning about their size. Recognize that comparisons are valid only when the two fractions refer to the same whole. Record the results of comparisons with the symbols >, =, or <, and justify the conclusions, e.g., by using a visual fraction model.

Domain: Measurement and Data

Standard 1   **Lesson 21**

Tell and write time to the nearest minute and measure time intervals in minutes.

Standard 1   **Lesson 22**

Solve word problems involving addition and subtraction of time intervals in minutes, e.g., by representing the problem on a number line diagram.

Standard 2   **Lesson 23**

Measure and estimate liquid volumes and masses of objects using standard units of grams (g), kilograms (kg), and liters (l).

Standard 2   Lesson 24

Measure and estimate liquid volumes and masses of objects using standard units of grams (g), kilograms (kg), and liters (L). Add, subtract, multiply, or divide to solve one-step word problems involving masses or volumes that are given in the same units, e.g., by using drawings (such as a beaker with a measurement scale) to represent the problem.

Standard 3   Lesson 25

Draw a scaled picture graph and a scaled bar graph to represent a data set with several categories. Solve one-and two-step "how many more" and "how many less" problems using information presented in scaled bar graphs. For example, draw a bar graph in which each square in the bar graph might represent 5 pets.

Standard 4   Lesson 26

Generate measurement data by measuring lengths using rulers marked with halves and fourths of an inch. Show the data by making a line plot, where the horizontal scale is marked off in appropriate units— whole numbers, halves, or quarters.

Standard 5a   Lesson 27

A square with side length 1 unit, called "a unit square," is said to have "one square unit" of area, and can be used to measure area. A plane figure which can be covered without gaps or overlaps by *n* unit squares is said to have an area of *n* square units.

Standard 5b   Lesson 28

A plane figure which can be covered without gaps or overlaps by *n* unit squares is said to have an area of *n* square units.

Standard 6   Lesson 29

Measure areas by counting unit squares (square cm, square m, square in., square ft., and improvised units).

Standard 7a   Lesson 30

Find the area of a rectangle with whole-number side lengths by tiling it, and show that the area is the same as would be found by multiplying the side lengths.

Standard 7b   Lesson 31

Multiply side lengths to find areas of rectangles with whole-number side lengths in the context of solving real-world and mathematical problems, and represent whole-number products as rectangular areas in mathematical reasoning.

Standard 7c   Lesson 32

Use tiling to show in a concrete case that the area of a rectangle with whole-number side lengths a and $b + c$ is the sum of $a \times b$ and $a \times c$. Use area models to represent the distributive property in mathematical reasoning.

Standard 7d   Lesson 33

Recognize area as additive. Find areas of rectilinear figures by decomposing them into non-overlapping rectangles and adding the areas of the non-overlapping parts, applying this technique to solve real-world problems.

Standard 8   Lesson 34

Solve real world and mathematical problems involving perimeters of polygons, including finding the perimeter given the side lengths, finding an unknown side length, and exhibiting rectangles with the same perimeter and different areas or with the same area and different perimeters.

Domain: Geometry

Standard 1   Lesson 35

Understand that shapes in different categories (e.g., rhombuses, rectangles, and others) may share attributes (e.g., having four sides), and that the shared attributes can define a larger category (e.g., quadrilaterals). Recognize rhombuses, rectangles, and squares as examples of quadrilaterals, and draw examples of quadrilaterals that do not belong to any of these subcategories.

Standard 2   Lesson 36

Partition shapes into parts with equal areas. Express the area of each part as a unit fraction of the whole. For example, partition a shape into 4 parts with equal area, and describe the area of each part as 1/4 of the area of the shape.

STAR Mathematics Tutoring Session Template

Before You Begin

Utah State Core Identification

Domain: Operations and Algebraic Thinking	Cluster: Represent and solve problems involving multiplication and division.

Standard 1: Interpret products of whole numbers, e.g., interpret 5 × 7 as the total number of objects in 5 groups of 7 objects each. For example, describe a context in which a total number of objects can be expressed as 5 × 7.

Key Academic Mathematics Vocabulary (3 or fewer)

product (the answer to a multiplication problem)

Notes and Materials

Notes and suggestions for an effective lesson: In this standard, multiplication is interpreted as finding the total number of objects in *n* groups. Work with the student to group objects into a certain number of groups. Begin with the student counting the objects and then, as he/she gains facility, moving to skip counting (e.g., count 4 objects in three groups—4, 8, 12).

Materials you will need:
- large rubber bands
- counters (marbles, beans, etc.)
- white board/marker

2 Minutes

LAUNCH—Assess and Provide Background Knowledge

1. **Connect to prior learning. Use a "hook" to gain the student's attention.** If three people went to a Chinese restaurant, how many chopsticks would they need? (Chopsticks come in pairs, or twos.)

2. **Introduce and review Key Academic Mathematics Vocabulary.** Three people would need six chopsticks. Three groups of two is six. Six is the product in this problem.

8 Minutes

INSTRUCT—Provide Explicit Interactive Instruction

3. **State the objective.** The student will learn that a multiplication problem has three parts:
   1. The number of groups,
   2. The size of groups, and
   3. The product (total number).

4/5. **Provide explicit step-by-step instructions. Model.** Place three rubber bands on the desk, (this represents "groups of"). Put two counters inside each band. Ask the student how many groups of counters are shown. Ask how many counters are in each. Record the equation: 3 x 2 = 6. The total number of counters is 6. Six is the product of three groups of two.

6. **Check for understanding (work problem with student).** How would you use the bands and counters to show two groups of five? (Student should place two rubber bands on the desk and place five counters in each. As time permits, continue with these equations: 1 x 2 = 2, 3 x 1 = 3, 2 x 4 = 8, etc.)

   If you had three toy trucks, how many wheels would you have? (Have student model using bands/counters.) How many ears are on five bunnies? (Model.)

5 Minutes

GUIDED PRACTICE—Monitor Student Work

7. **Student works problems independently while tutor watches and coaches.** How many fingers are on three hands? How many tires are on six bicycles? What is the product of three pairs of shoes? (Have student write the corresponding equation including the product.)

	ASSESS—Evaluate Student Demonstration
5 Minutes	**8. Student orally defines at least one Key Academic Mathematics Vocabulary word and skill or concept.** What does the "x" mean in 3 x 2 = 6?
	9. Student works a problem while explaining EVERY step orally. Have the student model and record an equation that shows this information: Billy bought 3 bags of marbles. Each bag contains 5 marbles. How many marbles does he have?

STAR Mathematics Tutoring Session Template

Before You Begin

Utah State Core Identification

Domain:Operations and Algebraic Thinking	Cluster: Represent and solve problems involving multiplication and division.

Standard 2: Interpret whole-number quotients of whole numbers, e.g., interpret 56 ÷ 8 as the number of objects in each share when 56 objects are partitioned equally into 8 shares, or as a number of shares when 56 objects are partitioned into equal shares of 8 objects each. (For example, describe a context in which a number of shares or a number of groups can be expressed as 56 ÷ 8.)

Key Academic Mathematics Vocabulary (3 or fewer)

quotient, rows, columns

Notes and Materials

Notes and suggestions for an effective lesson: This lesson deals with the equal shares model of division. Students should be able to divide groups of objects into equal groups, given a division problem.

Materials you will need:
- counters
- rubber bands

2 Minutes

LAUNCH—Assess and Provide Background Knowledge

1. **Connect to prior learning. Use a "hook" to gain the student's attention.** Mom brought home a dozen (12) donuts. Three children have to share them equally. How many will each child get?

2. **Introduce and review Key Academic Mathematics Vocabulary.** A quotient is the answer when you divide. The quotient in the donut problem is four, because twelve donuts divided equally among three children is four.

8 Minutes

INSTRUCT—Provide Explicit Interactive Instruction

3. **State the objective.** Today we will look at division by deciding how many equal groups can be made from a whole number.

4. **Provide explicit step-by-step instructions.** Count out 12 counters. Explain that we need to have the 12 divided evenly among 3 groups. Place three rubber bands on the desk. In the donut question, what would the rubber bands represent? (the 3 children) Divide the 12 counters **equally** among the 3 rubber bands. Each group ends up with 4 counters. These 4 counters are the "quotient" in this division equation.

5. **Model.** Do another division problem with the rubber bands and the counters, "thinking aloud" as you do it. Tell the student each step you are taking and why you are taking it.

6. **Check for understanding (work problem with student).** Have the student show the number of equal groups of five you can get from fifteen. What would the equation look like? Say: "15 divided by 5 equals 3." Which number is the quotient? (3)

5 Minutes

GUIDED PRACTICE—Monitor Student Work

7. **Student works problems independently while tutor watches and coaches.** Write a division equation that represents these word problems, solving to find the quotient. Have the student voice aloud the steps as they are taken. Ask him/her why he/she is taking that particular step.
- Suzy has 18 dolls. She divides them into 2 piles. How many dolls are in each pile?
  $18 \div 2 = 9$ (What is the quotient?)
- Ben bought 48 baseball cards. They come in packs of 8. How many packs did he buy?
  $48 \div 8 = 6$ packs (What is the quotient?)
- Pam sold 14 candy bars. They came in packages of 2. How many packages are there? $14 \div 2 = 7$ (What is the quotient?)

5 Minutes	**ASSESS—Evaluate Student Demonstration**
	8. **Student orally defines at least one Key Academic Mathematics Vocabulary word and skill or concept.** What is a quotient? In the equation 10 ÷ 2 = 5, which number is the quotient?
	9. **Student works a problem while explaining EVERY step orally.** Have the student think up a problem where he/she must divide something into equal groups. Have him/her draw a picture and write an equation. Ask him/her which number is the quotient.

STAR Mathematics Tutoring Session Template

Before You Begin

Utah State Core Identification

Domain: Operations and Algebraic Thinking	Cluster: Represent and solve problems involving multiplication and division.

Standard 3: Use multiplication and division within 100 to solve word problems in situations involving equal groups, arrays, and measurement quantities, e.g., by using drawings and equations with a symbol for the unknown number to represent the problem.

Key Academic Vocabulary (3 or fewer)

array (an arrangement of objects in equal rows)

Notes and Materials

Notes and suggestions for an effective lesson: This lesson only focuses on multiplication. Students need to understand that there is more than one way to solve a multiplication problem.

Materials you will need:
- 15 pennies
- whiteboard or scratch paper

2 Minutes

LAUNCH—Assess and Provide Background Knowledge

1. **Connect to prior learning. Use a "hook" to gain the student's attention.** Let's say your teacher is going to give you 3 pennies every day this week, Monday-Friday. What could you do to figure out how many pennies you would have by the end of the week? (There are a number of strategies that can be used. Students may use whichever strategy works best for them.)

2. **Introduce and review Key Academic Mathematics Vocabulary.** array (an arrangement of objects in equal rows)

8 Minutes

INSTRUCT—Provide Explicit Interactive Instruction

3. **State the objective.** Today we are going to solve multiplication word problems in a variety of ways, including pictures with equal groups, arrays, and number lines.

4. **Provide explicit step-by-step instructions.** Go back to the penny problem of 5 x 3 and model the solution using different strategies. Help the students see that there are at least three strategies, including:

- Drawing a picture showing 5 groups of 3 pennies, or forming groups with actual pennies, with each circle representing a day of the week.

- Drawing (or forming) an array showing 5 rows of 3 pennies, with each row representing a day of the week.
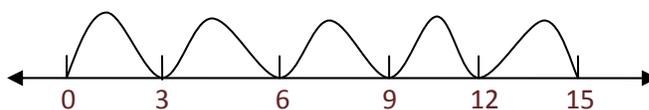
- Skip counting using a number line, with each jump representing a day of the week.

Point out that each strategy can be used to solve any multiplication problem. (The student may have already used one of these strategies to solve the problem in the introduction.)

5. **Model.** Solve another word problem modeling the same three strategies, e.g., There are 6 basketball teams playing in a tournament. There are 5 players on each team. How many players are there in all? (6 x 5 = 30) Model each strategy with the student's help.

6. **Check for understanding (work problem with student).** Have the student solve 2 x 3 for you using each strategy. Have him/her identify which strategy he/she prefers and why. Point out that some strategies work better than others for different people. Students should use whichever strategy works best for them.

GUIDED PRACTICE—Monitor Student Work

7. **Student works problems independently while tutor watches and coaches.** Have the student solve the following problems using a least two different strategies, one strategy for each problem. He/she should orally explain how he/she is solving each problem.
   a. Mom bought 5 packages of cookies at the store. Each package had 4 cookies. How many cookies in all?
   b. We planted 4 rows of flowers in the yard. Each row had 3 flowers. How many flowers did we plant in all?

ASSESS—Evaluate Student Demonstration

8. **Student orally defines at least one Key Academic Vocabulary word.** Ask the student to explain what an array is and how it can be helpful in solving a multiplication problem.

9. **Student works a problem while explaining EVERY step orally.** Use a Guided Practice activity as assessment. If time permits, you can ask the student to come up with his/her own multiplication problem. Have him/her solve it using at least 2 different strategies, orally explaining each step as it is being done.

5 Minutes

5 Minutes

STAR Mathematics Tutoring Session Template

Utah State Core Identification

Domain: Operations and Algebraic Thinking	Cluster: Represent and solve problems involving multiplication and division.

Standard 3: Use multiplication and division within 100 to solve word problems in situations involving equal groups, arrays, and measurement quantities, e.g., by using drawings and equations with a symbol for the unknown number to represent the problem.

Key Academic Vocabulary (3 or fewer)

array (an arrangement of objects in equal rows)

Notes and Materials

Notes and suggestions for an effective lesson: This lesson only focuses on division. Students need to understand that there is more than one way to solve a division problem.

Materials you will need:
- counters
- 10 pennies (optional)

LAUNCH—Assess and Provide Background Knowledge

1. **Connect to prior learning. Use a "hook" to gain the student's attention.** Let's say your teacher gave you the same number of pennies each day, Monday-Friday. By the end of the week you had 10 pennies. Ask the student what he/she could do to figure out how many pennies he/she would be given each day. He/she may use the counters or pennies if needed. (There are a number of strategies that can be used. Students may use what works best for them.)

2. **Introduce and review Key Academic Mathematics Vocabulary.**
array (an arrangement of objects in equal rows)

INSTRUCT—Provide Explicit Interactive Instruction

3. **State the objective.** "Today we are going to solve division word problems in a variety of ways, including pictures with equal groups, arrays, and number lines."

4. **Provide explicit step-by-step instructions.** Go back to the penny problem and explain that the number sentence 10 ÷ 5 (10 pennies divided into 5 days) can be used to solve the word problem. Model how to solve 10 ÷ 5 using different strategies. Help the student see that there are at least three strategies, including:

   a. Drawing a picture, or using counters, that shows the 10 pennies divided into 5 groups—one for each day—with each group having 2 pennies once they are all sorted.

   b. Drawing an array, or making one with counters, showing 10 pennies divided into 5 rows of 2 pennies, with each row representing a day.
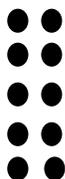
c. Skip counting backwards using a number line – start at ten jump back 5 spaces (representing 5 days of the week). You will notice it takes 2 jumps back to get to zero.

Point out that each strategy can be used to solve any division problem. (The student may have already used one of these strategies to solve the problem in the introduction.)

5. **Model.** Solve another word problem modeling the same three strategies, e.g., There are 12 cookies to be divided between kids. How many cookies will they each get? ($12 \div 3 = ?$). Model each strategy with the student's help using either counters or paper and pencil to form the groups and array.

6. **Check for understanding (work problem with student).** Have the student solve $8 \div 2$ using each strategy. Have him/her identify which strategy he/she prefers and why. Point out that some strategies work better than others for different people. Students should use whichever strategy works best for them.

GUIDED PRACTICE—Monitor Student Work

7. **Student works problems independently while tutor watches and coaches.** Have the student solve the following problems using two different strategies (one strategy for each problem). He/she should orally explain how he/she is solving each word problem.
   a. Mom bought 5 packages of cookies at the store. There were 20 cookies total. How many cookies were in each package?
   b. There were 27 muffins in the display case at the bakery. There were 3 different flavors— blueberry, chocolate, and poppy seed. If each flavor has the same number, how many of each flavor are there?
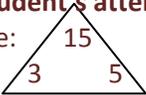
ASSESS—Evaluate Student Demonstration

8. **Student orally defines at least one Key Academic Mathematics Vocabulary word.** Ask the student to explain what an array is and how it can be helpful in solving a multiplication problem.

9. **Student works a problem while explaining EVERY step orally.** Guided Practice can be used as an assessment. If time permits, you may ask the student to come up with his/her own division problem. Have him/her solve it using at least two different strategies, orally explaining each step as it is being done.

5 Minutes

5 Minutes

STAR Mathematics Tutoring Session Template

<table>
<tr><td rowspan="8">**Before You Begin**</td><td colspan="2">**Utah State Core Identification**</td></tr>
<tr><td>Domain: Operations and Algebraic Thinking</td><td>Cluster: Represent and solve problems involving multiplication and division.</td></tr>
<tr><td colspan="2">Standard 4: Determine the unknown whole number in a multiplication or division equation relating three whole numbers. For example, determine the unknown number that makes the equation true in each of the following equations? 8 × ? = 48, 5 = ? ÷ 3, 6 × 6 = ?</td></tr>
<tr><td colspan="2">**Key Academic Mathematics Vocabulary (3 or fewer)**</td></tr>
<tr><td colspan="2">missing factor (numbers that are multiplied to give a product)</td></tr>
<tr><td colspan="2">**Notes and Materials**</td></tr>
<tr><td colspan="2">**Notes and suggestions for an effective lesson:** Students need to be aware that multiplication and division problems are similar and can be grouped as fact families or similar facts.</td></tr>
<tr><td colspan="2">**Materials you will need:**
• triangular flash cards for multiplication and division
• cups
• counters</td></tr>
</table>

2 Minutes	**LAUNCH—Assess and Provide Background Knowledge**

1. **Connect to prior learning. Use a "hook" to gain the student's attention.**

Show a △ flashcard and how all number relate. For example:

15
3   5

3 x 5 = 15   5 x 3 = 15   15 ÷ 3 = 5   15 ÷ 5 = 3

Do one more problem together.

2. **Introduce and review Key Academic Mathematics Vocabulary.** Factors are numbers multiplied to give a product. We get missing factors if we are given the products. For example:

2 x ? = 18

Review (number of groups (factor 1)) x (size of group (factor 2)) = total (product)

INSTRUCT—Provide Explicit Interactive Instruction

8 Minutes	

3. **State the objective.** Today we will be solving for unknown numbers in multiplication and division problems.

4. **Provide explicit step-by-step instructions.**
Write the problem 8 x ? = 48 The ? in this problem is an unknown number (missing factor). We know we have 8 groups, but we need to find how many are in each group when there are a total of 48.

5. **Model.**
•   Put out 8 cups or groups.
•   Count out 48 counters.
•   Put one counter in each cup.
•   Continue to add counters to cups until all are gone and so each cup has an equal number of counters.
•   Count the number inside of one cup.
•   We see that there are 8 groups of 6 that make 48.
•   Show student △ flashcard relating 8 x 6 = 48 and discuss other related problems.

6. **Check for understanding (work problem with student).**
•   Demonstrate ☐ x 4 = 32.
•   ☐ is the missing factor or unknown.
•   Now we are looking for the number of groups of 4 to get 32 total.
•   Count out 32 and put into groups of 4.

- How many groups can you make? (8)
- Show student the △ flashcard that relates 8 x 4 = 32
- Demonstrate □ ÷ 3 = 4.
- □ is the unknown.
- What's the total number if I have 3 groups with 4 in each group?
- 3 cups and 4 inside each cup for a total of 12.

GUIDED PRACTICE—Monitor Student Work

7. **Student works problems independently while tutor watches and coaches.**
- Do 7 x □ =42.
- Solve with student.
- □ Is the missing factor.
- Have student list other facts, ? x 5 = 30. 5 x 6 = 30, 30 ÷ 5 = 6, 30 ÷ 6 = 5
- Solve □÷6=18.

ASSESS—Evaluate Student Demonstration

8. **Student orally defines at least one Key Academic Mathematics Vocabulary word and skill or concept.**
Define a missing factor.

9. **Student works a problem while explaining EVERY step orally.** Solve for the unknown number in x 7 = 28.

STAR Mathematics Tutoring Session Template

Utah State Core Identification	
Domain: Operations and Algebraic Thinking	Cluster: Understand properties of multiplication and the relationship between multiplication and division.

Standard 5: Apply properties of operations as strategies to multiply and divide. Examples: If 6 x 4 = 24 is known, then 4 x 6 = 24 is also known. (Commutative property of multiplication.) 3 x 5 x 2 can be found by 3 x 5 = 15, then 15 x 2 = 30, or by 5 x 2 = 10, then 3 x 10 = 30. (Associative property of multiplication.) Knowing that 8 x 5 = 40 and 8 x 2 = 16, one can find 8 x 7 as 8 x (8 + 2) = (8 x 2) = 40 + 16 = 56. (Distributive property.)

Key Academic Mathematics Vocabulary (3 or fewer)

Distributive Property

Notes and Materials

Notes and suggestions for an effective lesson: When applying the Distributive Property to single-digit multiplication facts, make sure the student "splits" one of the factors into two "friendly" or known factors, making the facts easier to work with.

Materials you will need:
- Unifix cubes or counters.

LAUNCH—Assess and Provide Background Knowledge

1. **Connect to prior learning. Use a "hook" to gain the student's attention.** Think about when you have played with your set of Legos and you had just built the coolest fortress ever. Your little brother came along and destroyed it. You have to rebuild it but can't remember where all the pieces go. Finally at the end of the day and the last piece placed, the fortress is rebuilt. This is something similar to what we will be talking about today with numbers.

2. **Introduce and review Key Academic Mathematics Vocabulary.** Distributive Property (break numbers into easier pieces to solve for the whole)

INSTRUCT—Provide Explicit Interactive Instruction

3. **State the objective. "**Today we are going to work with the Distributive Property for multiplication. You can find 6 x 8 because you know 6 x 3 = 18 and 6 x 5 = 30; 18 + 30 = 48.

4. **Provide explicit step-by-step instructions.** The tutor will show the student how to build a 6 x 8 array with unifix cubes (6 x 8 cube). Write 6 x 8. Split the array into two separate columns to show 6 x 3 (6 x 3 cube) and 6 x 5 (6 x 5 cube). Does the array still have the same number of cubes? (yes)

5. **Model.** Another way to write this equation is 6 x (3 + 5). 6 x 3 (6 x 3 cube) + 6 x 5 (6 x 5 cube) = 18 + 30 = 48. Notice that the array could have been split into two separate sections of rows. You can find 6 x 8 because you know 3 x 8 (3 x 8 cube) + 3 x 8 (3 x 8 cube). Does the array still have the same number of cubes? (yes)

6. **Check for understanding (work problem with student).** The student will need to model 7 x 4 (7 x 4 cube). He/she may split the rows or columns into whatever product he/she is comfortable working with or has memorized e.g., 7 x 2 + 7 x 2 = 14 + 14 = 28 (show cubes) or 2 x 4 + 5 x 4 = 8 + 20 = 2.

GUIDED PRACTICE—Monitor Student Work

7. **Student works problems independently while tutor watches and coaches.** Use the Distributive Property to solve 4 x 6.

ASSESS—Evaluate Student Demonstration

8. **Student orally defines at least one Key Academic Mathematics Vocabulary word and skill or concept.** What is the Distributive Property?

Before You Begin

2 Minutes

8 Minutes

5 Minutes

5 Minutes

9. **Student works a problem while explaining EVERY step orally.** Using the Distributive Property solve 6 x 7. One way to solve by 6 x (5 x 2) = (6 x 5) + (6 x 2) = 30 + 12 = 42.

STAR Mathematics Tutoring Session Template

Before You Begin

Utah State Core Identification

Domain: Operations and Algebraic Thinking	Cluster:

Standard 6: Understand division as an unknown-factor problem. For example, find 32 ÷ 8 by finding the number that makes 32 when multiplied by 8.

Key Academic Mathematics Vocabulary (3 or fewer)

divisor, quotient, dividend, factor, partition

Notes and Materials

Notes and suggestions for an effective lesson: Student needs to be able to model division using linking cubes or other manipulatives to show equal groups. The student will then connect the groupings with multiplication and then with division.

Materials you will need:
- linking cubes or other manipulatives

2 Minutes

LAUNCH—Assess and Provide Background Knowledge

1. **Connect to prior learning. Use a "hook" to gain the student's attention.** You have 32 cookies to share equally among 8 people. How many cookies will each person get?

2. **Introduce and review Key Academic Mathematics Vocabulary.** Partitioning is sharing in equal groups.

8 Minutes

INSTRUCT—Provide Explicit Interactive Instruction

3. **State the objective.** "Today we are going to solve division problems with unknown factors, such as 32 ÷ 8 = ?"

4. **Provide explicit step-by-step instructions.**
   Step 1: Get a group of 32 manipulatives (whatever you have on hand). Place them into 8 equal groups. How many are in each group?
   Step 2: Ask the student, "Eight times what equals 32?" (8 x ? = 32)
   Step 3: If 8 x 4 = 32, then 32 ÷ 8 = ?

5. **Model.** Step 4: Do the same thing with other division problems (see below).

6. **Check for understanding (work problem with student).** Provide another problem and have the student model it for you.

5 Minutes

GUIDED PRACTICE—Monitor Student Work

7. **Student works problems independently while tutor watches and coaches.** Give the student at least two problems (see below). Each problem will have a context.

5 Minutes

ASSESS—Evaluate Student Demonstration

8. **Student orally defines at least one Key Academic Mathematics Vocabulary word and skill or concept.** What is division?

Instruct Problem 2:

There are 40 tires on cars in the parking lot. Each car has 4 tires. How many cars are in the parking lot?

Guided Practice Problems:

There are 45 buttons in a jar. You divide them into 5 equal groups. How many are in each group?

The teacher had 36 pencils. She shared the pencils equally among 6 students. How many pencils did each student receive?

Assessment problem:

There are 56 pieces of candy in a bag. Seven friends share the candy equally. How many pieces of candy will each person get?

STAR Mathematics Tutoring Session Template

Before You Begin

Utah State Core Identification

Domain: Operations and Algebraic Thinking	Cluster: Multiply and divide within 100.

Standard 7: Fluently multiply and divide within 100, using strategies such as the relationship between multiplication and division (e.g., knowing that 8 x 5 = 40, one knows 40 ÷ 5 = 8) or properties of operations. By the end of Grade 3, know from memory all products of two one-digit numbers.

Key Academic Mathematics Vocabulary (3 or fewer)

multiplication, division

Notes and Materials

Notes and suggestions for an effective lesson: The student needs to be able to solve multiplication and division problems within 100 in a variety of ways, which could include doubles, half and double, adding on a set, multiplying by 0 and 1, and fact families. The problems in this section lead to fact families/number bonds.

Materials you will need:
- paper
- pencil

LAUNCH—Assess and Provide Background Knowledge

1. **Connect to prior learning. Use a "hook" to gain the student's attention.** 2 x 3 = ?   3 x 2 = ?   6 ÷ 3 = ? 6 ÷ 2 = ?

2. **Introduce and review Key Academic Mathematics Vocabulary.** Define multiplication and division.

INSTRUCT—Provide Explicit Interactive Instruction

3. **State the objective.** "Today we are going to practice multiplication and division facts."

4. **Provide explicit step-by-step instructions.**
   Step 1: What is 5 x 4?
   Step 2: What is 20 ÷ 4?

5. **Model.** Do the same thing with six other multiplication and division problems. (See below.)

6. **Check for understanding (work problem with student).** As the student is solving the problems, ask him/her about the strategies he/she used.

GUIDED PRACTICE—Monitor Student Work

7. **Student works problems independently while tutor watches and coaches.** Give the student at least 10 problems to solve independently. (See below.)

ASSESS—Evaluate Student Demonstration

8. **Student orally defines at least one Key Academic Mathematics Vocabulary word and skill or concept.** Explain the relationship between multiplication and division.

9. **Student works a problem while explaining EVERY step orally.** 9 x 8 = ?   72 ÷ 8 = ?

Left margin labels: Before You Begin · 2 Minutes · 8 Minutes · 5 Minutes · 5 Minutes

Instructed Problems:

6 x 5 = ?   30 ÷ 5 = ?   4 x 4 = ?   16 ÷ 4 = ?   3 x 4 = ?   12 ÷ 3 = ?

Guided Problems:

4 x 6 = ?   24 ÷ 6 = ?   7 x 9 = ?   72 ÷ 8 = ?   9 x 10 = ?   60 ÷ 5 = ?   6 x 6 = ?   36 ÷ 4 = ?   5 x 8 = ?   25 ÷ 5 = ?

STAR Mathematics Tutoring Session Template

Before You Begin

Utah State Core Identification

Domain: Operations and Algebraic Thinking	Cluster: Solve problems involving the four operations, and identify and explain patterns in arithmetic.

Standard 8: Solve two-step word problems using the four operations. Represent the problems using equations with a letter standing for the unknown quantity. Assess the reasonableness of answers using mental computation and estimation strategies, including rounding.

Key Academic Mathematics Vocabulary (3 or fewer)

operations, equations, rounding, estimation, unknown quantity

Notes and Materials

Notes and suggestions for an effective lesson:
- The student needs to be able to read the word problem and find out what information they have been given and what they need to solve for.
- Encourage the student to use strategies to help them figure out the problem (draw a picture, look for clue words, etc.)

Materials you will need:
- paper
- pencil

LAUNCH—Assess and Provide Background Knowledge

2 Minutes

1. **Connect to prior learning. Use a "hook" to gain the student's attention.** Say: "Your dad will pay you 10¢ for each wheel you clean in the garage. There are 2 cars and 3 bicycles. How much money can you earn?"

2. **Introduce and review Key Academic Mathematics Vocabulary.** An unknown quantity is a letter that represents a missing part.

INSTRUCT—Provide Explicit Interactive Instruction

8 Minutes

3. **State the objective.** "Today we will solve word problems that require more than one step."

4/5. Provide explicit step-by-step instructions/Model.

- When solving a word problem, first read it carefully. Underline information that may help you find the answer. Pay special attention to the last sentence, because it usually tells what you are solving for. Example: Betty has a photo album with 10 pages. Each page contains 2 photos. She needs 25 photos for her poster. How many more photos does she need to complete her poster?
- In the problem above, underline "10 pages," "2 photos," "25 pictures," and the words "How many more."
- What information do we need to find out? (How many more photos does she need?)
- Make a plan for how to solve the problem. First we can figure out how many photos she already has (10 x 2).
- We can write the number she needs for the poster on the other side of the equation (10 x 2 + ? =25)
- Let's call the amount we need to find "n." So, (10 x 2) + n = 25.
- The next step is to try your plan. 10 x 2 = 20 and 20+ 5 = 25. So, n = 5.

6. **Check for understanding (work problem with student).**
The last step is to make sure it makes sense. (Yes, if we have 20, we will need 5 more to have the 25 we need for the poster.)

GUIDED PRACTICE—Monitor Student Work

7. **Student works problems independently while tutor watches and coaches.**
- Here is another two-step problem: Betty needs $5.50 to go to the movie. She has 10 quarters in her piggy bank. How much more money does she need?
- Underline important data and what the problems asks the student to find ("$5.50," "10 quarters," "how much more money").
- Have the student talk through what he/she has to find first (how much money is 10 quarters?)
- Ask him/her how he/she will find out how much more money is needed.
- Have the student solve the problem.

ASSESS—Evaluate Student Demonstration

8. **Student orally defines at least one Key Academic Mathematics Vocabulary word and skill or concept.**
What is an unknown quantity?

9. **Student works a problem while explaining EVERY step orally.** Tami blew up 24 red balloons and 15 blue balloons for her party. Seven balloons popped. How many balloons were left?

5 Minutes

5 Minutes

STAR Mathematics Tutoring Session Template

Before You Begin

Utah State Core Identification

Domain: Operations and Algebraic Thinking	Cluster: Solve problems involving the four operations, and identify and explain patterns in arithmetic.

Standard 9: Identify arithmetic patterns (including patterns in the addition table or multiplication table), and explain them using properties of operations. For example, observe that 4 times a number is always even, and explain why 4 times a number can be decomposed into two equal addends.

Key Academic Vocabulary (3 or fewer)

product, multiple, sum, pattern

Notes and Materials

Notes and suggestions for an effective lesson: A 10 x 10 multiplication chart is provided with lesson. Since this is a 20-minute lesson, have the student color enough multiples to generate a pattern.

Materials you will need:
- multiplication chart (3 copies)
- crayon

2 Minutes

LAUNCH—Assess and Provide Background Knowledge

1. **Connect to prior learning. Use a "hook" to gain the student's attention.** Say, "Have you noticed patterns around you?" (Examples: bricks in the wall, ceiling tiles, lights on ceiling, possibly patterns on clothing.) Have the student find patterns around him/her. Multiplication can create patterns as well.

2. **Introduce and review Key Academic Mathematics Vocabulary.** What is a pattern?

8 Minutes

INSTRUCT—Provide Explicit Interactive Instruction

3. **State the objective.** "We will be indentifying patterns on a multiplication chart and using the properties of operations to help us explain the patterns."

4. **Provide explicit step-by-step instructions.** Use the multiplication chart. Have the student count by twos.

5. **Model.** Color all the multiples of 2, e.g., 2, 4, 6, 8... the patterns of the products are even.

6. **Check for understanding (work problem with student).** Discuss patterns that he/she observed.

5 Minutes

GUIDED PRACTICE—Monitor Student Work

7. **Student works problems independently while tutor watches and coaches.** Use a multiplication chart to color the multiples of 3 (the student should color 3, 6, 9, 12, 15...). Find patterns for products of 3. Have the student describe the patterns he/she sees.

5 Minutes

ASSESS—Evaluate Student Demonstration

8. **Student orally defines at least one Key Academic Vocabulary word.** Have the student define what a pattern is.

9. **Student works a problem while explaining EVERY step orally.** Use a multiplication chart to color the multiples of 5 (the student should color 5, 10, 15, 20, 25…). Find patterns for products of 5. Have the student describe the patterns he/she sees.

Product or Multiple

$6 \times 7 = 42$

Product or Multiple

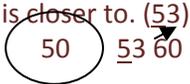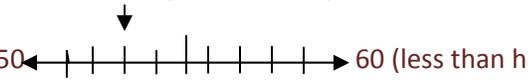The result when 2 or more factors are multiplied.

Sum

$13 = 6 + 3 + 4$

Sum

The result of when 2 or more addends are added together.

1	2	3	4	5	6	7	8	9	10
2	4	6	8	10	12	14	16	18	20
3	6	9	12	15	18	21	24	27	20
4	8	12	16	20	24	28	32	36	40
5	10	15	20	25	30	35	40	45	50
6	12	18	24	30	36	42	48	54	60
7	15	21	28	35	42	49	56	63	70
8	16	24	32	40	48	56	64	72	80
9	18	27	36	45	54	63	72	81	90
10	20	30	40	50	60	70	80	90	100

STAR Mathematics Tutoring Session Template

Before You Begin

Utah State Core Identification	
Domain: Number and Operations in Base ten	Cluster: Use place value understanding and properties of operations to perform multi-digit arithmetic.
Standard 1: Use place value understanding to round whole numbers to the nearest 10 or 100.	

Key Academic Mathematics Vocabulary (3 or fewer)

rounding (the process of finding about how many or about how much by expressing a number to its nearest unit)

Notes and Materials

Notes and suggestions for an effective lesson: A firm grasp of place value is required to accurately round. Student should also be able to count by tens and hundreds.

Materials you will need:
- number line
- meter stick
- counters

2 Minutes

LAUNCH—Assess and Provide Background Knowledge

1. **Connect to prior learning. Use a "hook" to gain the student's attention.**
- Use the meter stick. Ask, "How many centimeters are on the meter stick?" (100)
- Ask, "Can you find the numbers you say when you count by tens?"
- Put a counter on the 38.
- Which two tens is it between? (30 and 40)
- Which ten is it closest to? (40)
- When we round 38 to the closest ten, we get 40.

2. **Introduce and review Key Academic Mathematics Vocabulary.** Rounding is a tool we use to help us find the closest ten or hundred a number is closest to.

8 Minutes

INSTRUCT—Provide Explicit Interactive Instruction

3. **State the objective.** "Today you will place value and counting by tens to help you decide which number is closest in value."

4. **Provide explicit step-by-step instructions.**
- This is the "noisy" way to learn rounding. Underline the digit in the place you are rounding: 53 to the nearest ten, you'd underline the 53.
- Say the number, being "noisy" on the underlined digit ("FIFTY-three"), then say the tens that number is between by first repeating the "noisy" number ("FIFTY-three is between FIFTY and sixty").
- Write down the ten on each side of the number.
- Draw an arrow to the digit to the right to help you decide which ten it is closer to. (53) If that digit is less than 5, round back. If it is five or higher, round up to the next ten.   50   53 60
- Check on a number line. 50 ←–|–|–|–|–|–|–|–|–|–→ 60 (less than halfway, closer to fifty)

5. **Model.** Check using a number line or the meter stick.

6. **Check for understanding (work problem with student).** Write 78. Have the student:
   1. Underline.
   2. Say it "noisy."

3. Write down the ten on each side.
4. Draw an arrow to the number in the ones place and decide whether it is closer to the ten before or after it.

5 Minutes

GUIDED PRACTICE—Monitor Student Work

7. **Student works problems independently while tutor watches and coaches.** 23 , 47, 88, etc. (as time permits)

5 Minutes

ASSESS—Evaluate Student Demonstration

8. **Student orally defines at least one Key Academic Mathematics Vocabulary word and skill or concept.** What does it mean to round to the nearest ten? (See what ten the number is closest to.)

9. **Student works a problem while explaining EVERY step orally.** Give the student a number to round, and have him/her explain the process as he/she goes.

STAR Mathematics Tutoring Session Template

<table>
<tr><td colspan="2">Utah State Core Identification</td></tr>
<tr><td>Domain: Number and Operations in Base ten</td><td>Cluster: Use place value understanding and properties of operations to perform multi-digit arithmetic.</td></tr>
<tr><td colspan="2">Standard 2: Fluently add and subtract within 1,000 using strategies and algorithms based on place value, properties of operations, and/or the relationship between addition and subtraction.</td></tr>
</table>

Key Academic Mathematics Vocabulary (3 or fewer)

addition, subtraction, algorithms, place value, operations

Notes and Materials

Notes and suggestions for an effective lesson: Students need to understand that there are a variety of ways to solve a problem, using a variety of materials. Students also need to understand place value when solving three- and four-digit addition and subtraction problems in order to re-group.

Materials you will need:
Materials could include:
- number lines
- linking cubes
- base ten blocks
- place value mat
- paper and pencil

LAUNCH—Assess and Provide Background Knowledge

1. **Connect to prior learning. Use a "hook" to gain the student's attention.** There were 254 students in fifth grade at West Elementary, and 135 third grade students. How many students are there altogether?

2. **Introduce and review Key Academic Mathematics Vocabulary.** Review algorithm (series of steps followed to solve an expression).

INSTRUCT—Provide Explicit Interactive Instruction

3. **State the objective.** "Today we are going to solve addition and subtraction problems within 1,000 using a variety of ways."

4. **Provide explicit step-by-step instructions.**
   Step 1: Pose the problem: If Heather had collected 456 pennies, her friend Lisa had 125 pennies. How many pennies did they have altogether?
   Step 2: Write an equation that matches this problem.

5. **Model.**
   Step 3: Have the student solve the equation from Step 2.

6. **Check for understanding (work problem with student).** Have the student explain how he/she solved the equation from Step 2.

GUIDED PRACTICE—Monitor Student Work

7. **Student works problems independently while tutor watches and coaches.** Solve 2-3 additional problems (see below).

Before You Begin

2 Minutes

8 Minutes

5 Minutes

	ASSESS—Evaluate Student Demonstration
5 Minutes	8. **Student orally defines at least one Key Academic Mathematics Vocabulary word and skill or concept.** What is an algorithm?
	9. **Student works a problem while explaining EVERY step orally.** Have the student solve a 3-digit by 3-digit problem (see below).

Guided Practice:

McKenna had 236 pieces of candy. Her mother gave her 199 more pieces of candy. How many pieces of candy does she have in all?

I had 236 pieces of candy. I gave away 147 to my friend. How many pieces of candy do I have left?

There were 838 kids at the water park. 269 kids left for lunch. How many kids were left at the water park?

Assessment Practice:

On Monday, the movie theater sold 623 tickets. On Tuesday, they sold 377 tickets. How many tickets did they sell altogether on both days?

STAR Mathematics Tutoring Session Template

Before You Begin

Utah State Core Identification

Domain: Number and Operations in Base ten	Cluster: Use place value understanding and properties of operations to perform multi-digit arithmetic.

Standard 3: Multiply one-digit whole numbers by multiples of 10 in the range 10-90 (e.g., 9 x 80, 5 x 60) using strategies based on place value and properties of operations.

Key Academic Mathematics Vocabulary (3 or fewer)

multiples (the product of a number and any whole number; what you say when you skip count by a number)

Notes and Materials

Notes and suggestions for an effective lesson: The best way to teach this concept is through discovery. You may need to provide more examples than are included in this lesson to clarify the concept for the student you are tutoring.

Materials you will need:
- base ten models (ones and tens)
- pennies and dimes
- manipulatives

LAUNCH—Assess and Provide Background Knowledge

2 Minutes

1. **Connect to prior learning. Use a "hook" to gain the student's attention.**
   - How much money do I have if I have 5 pennies? (5¢)
   - How much money do I have if I have 5 dimes? (50¢)
   - How much money do I have if I have 3 pennies? (3¢)
   - How much money do I have if I have 3 dimes? (30¢)

2. **Introduce and review Key Academic Mathematics Vocabulary.**
   10, 20, 30, 40, 50, etc. are all multiples of ten. When we multiply by ten, the number in the ones place is a zero.

INSTRUCT—Provide Explicit Interactive Instruction

8 Minutes

3. **State the objective.** "Today we will multiply by multiples of ten and will discover that it is like putting a zero in the ones place as a place marker."

4. **Provide explicit step-by-step instructions.** Say, "Let's take a look at some examples of multiplication."
   - Talk through the following problems.

     5 x 1 = 5   3 x 1 = 3
     5 x 10 = 50   3 x 10 = 30

     2 x 2 = 4   3 x 6 = 18
     2 x 20 = 40   [2 x (2 x 10)]   3 x 60 = 180   [3 x (6 x 10)]

   - Show the student that multiplying by a multiple of ten is like multiplying the basic fact he/she knows, then multiplying that product by ten.
   - Say, "We know that when multiplying by ten, it's like adding a zero in the ones place as a place marker."
   - Use the examples to discuss the process (2 x 2 = 4, 2 x 20 = 40; and 3 x 6 = 18, 3 x 60 = 180)

5. **Model.** Have the student build the model with blocks or other manipulatives (example: 1 x 5 = 5, 1 x 10 = 10) to see the pattern building.

6. **Check for understanding (work problem with student).** Work through the two problems listed below, making sure the student sees the pattern and the reasoning.

4 x 3 = ___   7 x 2 = ___
4 x 30 = ___   7 x 20 = ___

___ x (___ x ___)   ___ x (___ x ___)

7. **Student works problems independently while tutor watches and coaches.** Have the student show you he/she understands by working the problems below:

8 x 8 = ___   9 x 3 = ___
8 x 80 = ___   9 x 30 = ___

___ x (___ x ___)   ___ x (___ x ___)

ASSESS—Evaluate Student Demonstration

8. **Student orally defines at least one Key Academic Mathematics Vocabulary word and skill or concept.** 10, 20, 30, 40, 50, etc. are multiples of ten. What is a multiple?

9. **Student works a problem while explaining EVERY step orally.** Have the student solve these problems. Ask him/her to explain how he/she found the product.

3 x 40 = ___   6 x 60 = ___
80 x 3 = ___   70 x 5 = ___
20 x 9 = ___   8 x 70 = ___

5 Minutes

5 Minutes

STAR Mathematics Tutoring Session Template

<table>
<tr><td rowspan="7">**Before You Begin**</td><td colspan="2">**Utah State Core Identification**</td></tr>
<tr><td>Domain: Number and Operations—Fractions</td><td>Cluster: Develop understanding of fractions as numbers.</td></tr>
<tr><td colspan="2">Standard 1: Understand a fraction 1/b as the quantity formed by one part when a whole is partitioned into b equal parts; understand a fraction a/b as the quantity formed by a parts of size 1/b.</td></tr>
</table>

Key Academic Vocabulary (3 or fewer)

fraction (a number expressible in the form a/b where *a* is a whole number and *b* is a positive whole number)
numerator (the number written above the line of a fraction. It tells how many equal parts are described by the fraction)
denominator (the quantity below the line in a fraction. It tells how many equal parts are in the whole)

Notes and Materials

Notes and suggestions for an effective lesson: Be sure to emphasize the need to divide each object into equal parts. This lesson focuses on fractions with a one as the numerator.

Materials you will need:
- paper cut into circles representing cookies (about 10)
- scissors

LAUNCH—Assess and Provide Background Knowledge

1. **Connect to prior learning. Use a "hook" to gain the student's attention.** Show the student a paper circle and say, "Pretend this is a cookie that you and I were going to share. How many pieces would we need to divide it into? Will you do that for me?" (Give him/her the paper "cookie" and have him/her divide it using scissors.) Record the number 2 on the white board and explain that this is the denominator, or how many equal parts we will have.

   Ask the student whether both parts are equal. Discuss the importance of their being equal. If the pieces are not equal, have the student try again. Then ask how many pieces you will each get (1). Say, "One will be our numerator, or how many parts of the whole cookie we will each get." Record the number 1 on the whiteboard above the denominator and show the student the fraction you just made. 1/2 means you each get one out of the two pieces of cookie.

2. **Introduce and review Key Academic Mathematics Vocabulary.** Say, "We just made a *fraction*. A fraction is a way to describe a part of a whole or a part of a group by using equal parts. It is a number expressible in the form a/b where *a* is a whole number and *b* is a positive whole number. The number 1 is the *numerator*. The numerator always goes on the top of the line and tells you how many equal parts you are using. The number 2 is the *denominator*. The denominator always goes below the line in a fraction and tells how many total equal parts there are."

Instruct—Provide Explicit Interactive Instruction

3. **State the objective.** "Today we are going to discover how a fraction is formed when you divide a whole object into equal groups…just like we formed a fraction by dividing a whole cookie into 2 equal parts."

4. **Provide explicit step-by-step instructions.** Say, "Let's take another cookie and divide it into 4 equal parts or groups now." After the cookie has been cut, pick up one of the four pieces. Point out that this is *one* of the *four* pieces, or one-fourth of the cookie. Write that as a fraction (1/4) so the student can see what the fraction looks like. Then pick up two pieces. Say and write the fraction 2/4. Continue with 3/4 and 4/4 (or 1 whole).

(Left margin time markers: 2 Minutes, 8 Minutes)

5. **Model.** Give the student a cookie and have him/her divide it into 2 equal pieces. Explain that one piece equals 1/2 and the other equals 1/2. When you add them together you get 2/2 or 1 whole.

6. **Check for understanding (work problem with student).** Give the student a new cookie. Tell him/her you want 1/3 of the cookie. Have him/her cut the cookie and show you 1/3, providing help if needed.

GUIDED PRACTICE—Monitor Student Work

7. **Student works problems independently while tutor watches and coaches.** On the whiteboard, quickly draw the following shapes and shade in one or more sections, depending on student's ability. Have the student identify the shaded portion as a fraction. He/she should record his/her answer on a whiteboard.
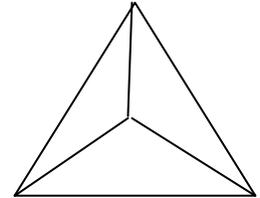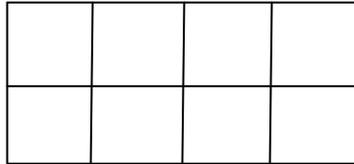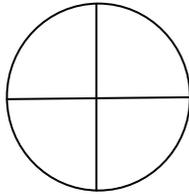
ASSESS—Evaluate Student Demonstration

8. **Student orally defines at least one Key Academic Vocabulary word.** Using the last fraction the student wrote in the above activity, have him/her identify and describe the numerator and denominator.

9. **Student works a problem while explaining EVERY step orally.** On the whiteboard, have student draw a picture that shows 1/6, explaining the steps as he/she goes along.

STAR Mathematics Tutoring Session Template

Before You Begin

Utah State Core Identification

Domain: Number and Operations—Fractions	Cluster: Develop understanding of fractions as numbers.

Standard 2a: Represent a fraction a/b on a number line by defining the interval from 0 to 1 as the whole and partitioning it into b equal pieces. Recognize that each part has size 1/b and that the endpoint of the part based at 0 locates the number 1/b on the number line.

Key Academic Mathematics Vocabulary (3 or fewer)

numerator, denominator, fraction, equivalent fractions, equal parts

Notes and Materials

Notes and suggestions for an effective lesson: Students need to know halves, thirds, fourths, etc. Students also need to know about a number line and how to place equal parts on the line.

Materials you will need:
- paper
- pencil
- paper for folding

LAUNCH—Assess and Provide Background Knowledge

1. **Connect to prior learning. Use a "hook" to gain the student's attention.** Say, "We have a piece of string that we need to share with four people. How long would each person's string be?"

2. **Introduce and review Key Academic Mathematics Vocabulary.** Partitioning (or cutting and sharing into equal parts)

2 Minutes

INSTRUCT—Provide Explicit Interactive Instruction

3. **State the objective.** "Today we will be placing fractions on a number line."

4. **Provide explicit step-by-step instructions.**
   Step 1: Show student a blank number line. Ask him/her how we would place whole numbers on the line.
   Step 2: Ask the student if he/she knows how to place fractions on the number line.
   Step 3: Give the student a piece of paper that represents a number line and have him/her fold the paper into two pieces.
   Step 4: Ask what fraction one share would be.

5. **Model.** Have the student show how they would cut the number line into two equal pieces.

6. **Check for understanding (work problem with student).** Have the student do the same thing with fourths to make four equal pieces.

8 Minutes

GUIDED PRACTICE—Monitor Student Work

7. **Student works problems independently while tutor watches and coaches.** Have the student do the same thing for 1/8.

5 Minutes

5 Minutes	**ASSESS—Evaluate Student Demonstration**
	8. **Student orally defines at least one Key Academic Mathematics Vocabulary word and skill or concept.** What are equal parts?
	9. **Student works a problem while explaining EVERY step orally.** Have the student complete the same task with 1/3 and 1/6.

STAR Mathematics Tutoring Session Template

Before You Begin

Utah State Core Identification

Domain: Number and Operations—Fractions	Cluster: Develop understanding of fractions as numbers.

Standard 2b: Represent a fraction a/b on a number line diagram by marking off a lengths 1/b from 0. Recognize that the resulting interval has size a/b and that its end point locates the number a/b on the number line.

Key Academic Mathematics Vocabulary (3 or fewer)

number line, unit whole, fraction, equal parts, denominator, numerator

Notes and Materials

Notes and suggestions for an effective lesson: Students should have an understanding of a number line from earlier experiences. The number line previously included only whole numbers; now it will include fractions between 0 and 1.

Materials you will need:
- paper and pencil
- foldable paper

2 Minutes

LAUNCH—Assess and Provide Background Knowledge

1. **Connect to prior learning. Use a "hook" to gain the student's attention.** Have the student place the following numbers in sequential order on the number line: 4, 16, 8, 12, 10, 22, 20.

   ←---→

2. **Introduce and review Key Academic Mathematics Vocabulary.** equal parts

8 Minutes

INSTRUCT—Provide Explicit Interactive Instruction

3. **State the objective.** "Today we are going to be using a number line to order fractions from smallest to largest."

4. **Provide explicit step-by-step instructions.**
   Step 1: Draw a number line and label 0 (on the left) and 1 (on the right). Explain that there are numbers that fit in between each of the whole numbers. These are called fractions. They have a numerator (the top number) and a denominator (the bottom number).
   Step 2: Give the student paper to fold and cut strip for the fractions.
   Step 3: Have him/her fold the strip into halves. Continue with additional strips to create 1/3, 1/4, 1/6 and 1/8.
   Step 4: Have the student place 1/2 on the number line. Ask, "Is 1/2 bigger or smaller than 1? Than 0?"
   Step 5: Continue doing the same thing with 1/4, 1/3, 1/8.

5. **Model.** Have the student compare his/her other fractional pieces to the size of one half, using the paper strips as he/she places them on the number line.

6. **Check for understanding (work problem with student).** Have student place 1/6 on the number line.

5 Minutes

GUIDED PRACTICE—Monitor Student Work

7. **Student works problems independently while tutor watches and coaches.** Ask, "If we folded 2/4, where would it belong on the number line?"

5 Minutes	**ASSESS—Evaluate Student Demonstration**
	8. **Student orally defines at least one Key Academic Mathematics Vocabulary word and skill or concept.** Have student retell the definitions of a numerator and a denominator.
	9. **Student works a problem while explaining EVERY step orally.** Give the student a picture of each of the fractions 1/8, 1/4, 1/6, 1/3. Have the student draw a number line on his/her paper and explain where he/she would place these fractions.

STAR Mathematics Tutoring Session Template

Before You Begin

Utah State Core Identification

Domain: Number and Operations—Fractions	Cluster: Develop understanding of fractions as numbers.

Standard 3a: Understand two fractions as equivalent (equal) if they are the same size, or the same point on a number line.

Key Academic Mathematics Vocabulary (3 or fewer)

equivalent, same size, proportion

Notes and Materials

Notes and suggestions for an effective lesson: Students need to know what fractions are, including numerators and denominators. They need to be familiar with how number lines work. They also need to be familiar with different models of fractions/ shapes.

Materials you will need:
- paper
- pencils
- pattern block shapes

2 Minutes

LAUNCH—Assess and Provide Background Knowledge

1. **Connect to prior learning. Use a "hook" to gain the student's attention.** Say, "I had two pans of brownies. I cut one pan into two equal pieces. The second pan, I cut into four equal pieces. Do I still have the same amount of brownies?"

2. **Introduce and review Key Academic Mathematics Vocabulary**. equivalent fractions (fractions that are equal)

8 Minutes

INSTRUCT—Provide Explicit Interactive Instruction

3. **State the objective.** "Today we are going to learn how to identify equivalent fractions."

4. **Provide explicit step-by-step instructions.**
   Step 1: Have the student take out a yellow hexagon block.
   Step 2: Explain that this is a *unit whole*.
   Step 3: "We can cut it into equal pieces, using other shapes."
   Step 4: "If we use the trapezoid, how many would it take to make the whole hexagon?"
   Step 5: Explain that each piece equals one half.

5. **Model.** Repeat with the rhombus block.

6. **Check for understanding (work problem with student).** Have the student find out how many triangles it would take to make the hexagon.

5 Minutes

GUIDED PRACTICE—Monitor Student Work

7. **Student works problems independently while tutor watches and coaches.** Start with the trapezoid as the unit whole, and ask the student to find out how many triangles it would take to create the unit whole.

5 Minutes

ASSESS—Evaluate Student Demonstration

8. **Student orally defines at least one Key Academic Mathematics Vocabulary word and skill or concept.** What is a unit whole?

9. **Student works a problem while explaining EVERY step orally.** "Using a combination of all three shapes, how many different ways can you make a yellow hexagon?" Have the student label the fractional amount for each piece. Example: 1/2, + 1/3 + 1/6 = 1 whole.

STAR Mathematics Tutoring Session Template

Before You Begin

Utah State Core Identification

Domain: Number and Operations—Fractions	Cluster: Develop understanding of fractions as numbers.

Standard 3b: Recognize and generate simple equivalent fractions, e.g., 1/2= 2/4, 4/6 =2/3). Explain why the fractions are equivalent, e.g., by using a visual fraction model.

Key Academic Mathematics Vocabulary (3 or fewer)

equivalent (two or more different fractions that name the same amount)

Notes and Materials

Notes and suggestions for an effective lesson: Students will understand that two fractions are equivalent if they are the same size, or at the same point on a number line.

Materials you will need:
- fraction bars
- paper
- pencil

2 Minutes

LAUNCH—Assess and Provide Background Knowledge

1. **Connect to prior learning. Use a "hook" to gain the student's attention.** Jaime ate 1/4 of the bread. Darin ate 2/8 of the bread. If I use fraction bars, did they both eat the same amount?

2. **Introduce and review Key Academic Mathematics Vocabulary.** Equivalent fractions are two or more different fractions that name the same amount.

8 Minutes

INSTRUCT—Provide Explicit Interactive Instruction

3. **State the objective.** "Today we will be looking at equivalent fractions and what makes them the same."

4. **Provide explicit step-by-step instructions.**
   Step 1: Put the fraction bar ½ on the desk.
   Step 2: Show student that there are other fraction bars that are the same length as 1/2 (e.g., 2/4, 3/6, 4/8).

1/2			
1/4		1/4	
1/8	1/8	1/8	1/8

   Show that they are the same size.
   Step 3: Give student 1/3. Work together to find equivalent fractions.
   Step 4: Show that the fractions are equal in size.
   Step 5: Try the same exercise with 3/8.

5. **Model.** Have the student put the fractions on a number line to show that they are the same.

6. **Check for understanding (work problem with student).** Have the student find the equivalent fractions for 3/4 and place them on a number line.

5 Minutes

GUIDED PRACTICE—Monitor Student Work

7. **Student works problems independently while tutor watches and coaches.** Show whether 1/3 and 2/6 are equivalent and whether 2/8 or 1/6 are equivalent. Find equivalent fractions for 1/4.

	ASSESS—Evaluate Student Demonstration
5 Minutes	8. **Student orally defines at least one Key Academic Mathematics Vocabulary word and skill or concept.** What does *equivalent* mean?
	9. **Student works a problem while explaining EVERY step orally.** Explain whether 4/6 and 2/3 are equivalent.

STAR Mathematics Tutoring Session Template

<table>
<tr><td rowspan="7">**Before You Begin**</td><td colspan="2">**Utah State Core Identification**</td></tr>
<tr><td>Domain: Number and Operations—Fractions</td><td>Cluster: Develop understanding of fractions as numbers.</td></tr>
<tr><td colspan="2">Standard 3c: Express whole numbers as fractions, and recognize fractions that are equivalent to whole numbers. Examples: Express 3 in the form 3 = 3/1; recognize that 6/1 = 6; locate 4/4 and 1 at the same point on a number line diagram.</td></tr>
<tr><td colspan="2">**Key Academic Mathematics Vocabulary (3 or fewer)**</td></tr>
<tr><td colspan="2">whole number, equivalent, numerator, denominator</td></tr>
<tr><td colspan="2">**Notes and Materials**</td></tr>
<tr><td colspan="2">**Notes and suggestions for an effective lesson.** Students need to know the unit whole is equal to 1/1. Students need to recognize that the fraction 1/1 and the equivalent whole number are located on the same point on a number line.</td></tr>
</table>

Materials you will need:
- fraction models (fraction strips or pattern blocks)

LAUNCH—Assess and Provide Background Knowledge

2 Minutes

1. **Connect to prior learning. Use a "hook" to gain the student's attention.** Say, "I had one pizza. If I were to cut the pieces into eighths, how many slices would it take to make the whole pizza?"

2. **Introduce and review Key Academic Mathematics Vocabulary.** equivalent (equal)

INSTRUCT—Provide Explicit Interactive Instruction

8 Minutes

3. **State the objective.** "Today we will find how many equivalent pieces it takes to make a unit whole."

4. **Provide explicit step-by-step instructions.**
   Step 1: "Let's take a yellow hexagon pattern block."
   Step 2: "How many red trapezoids does it take to make the unit whole?" (2)
   Step 3: "If one trapezoid equals one half, what fraction does it take to make the whole?" (2/2)
   Step 4: Follow the same pattern with the blue rhombi.
   Step 5: Demonstrate that 2/2 = 3/3 = 1. Explain that the fractions are all the same size.

5. **Model.** Have the student model 1/6. How many 1/6 pieces does it take to make the whole?

6. **Check for understanding (work problem with student).** Using fraction strips, have the student find how many 1/8 pieces it takes to make the whole.

GUIDED PRACTICE—Monitor Student Work

5 Minutes

7. **Student works problems independently while tutor watches and coaches.** Have the student use fraction strips to find the equivalent to 1 whole. How many 1/4 pieces does it take to make the whole?

ASSESS—Evaluate Student Demonstration

5 Minutes

8. **Student orally defines at least one Key Academic Mathematics Vocabulary word and skill or concept.** What is a unit whole?

9. **Student works a problem while explaining EVERY step orally.** Play the following game for assessment: http://www.explorelearning.com/index.cfm?method=cResource.dspDetail&ResourceID=1012

STAR Mathematics Tutoring Session Template

Before You Begin

Utah State Core Identification

Domain: Number and Operations—Fractions	Cluster: Develop understanding of fractions as numbers.

Standard 3d: Compare two fractions with the same numerator or the same denominator by reasoning about their size. Recognize that comparisons are valid only when the two fractions refer to the same whole. Record the results of comparisons with the symbols >, =, or <, and justify the conclusions, e.g., by using a visual fraction model.

Key Academic Mathematics Vocabulary (3 or fewer)

compare, >, <, =, equivalent fraction

Notes and Materials

Notes and suggestions for an effective lesson: Students need to be able to compare whole numbers using >, <, and =. Students need to be able to verbally explain how they compared the two numbers together.

Materials you will need:
- paper
- pencil

2 Minutes

LAUNCH—Assess and Provide Background Knowledge

1. **Connect to prior learning. Use a "hook" to gain the student's attention.** Katie had 2/3 of a candy bar. Melanie had 1/4 of the same kind of candy bar. Who had the biggest share?

2. **Introduce and review Key Academic Mathematics Vocabulary.** Comparisons (deciding whether one fraction is larger, smaller, or equal to another fraction)

8 Minutes

INSTRUCT—Provide Explicit Interactive Instruction

3. **State the objective.** "Today we will be learning how to compare fractions."

4. **Provide explicit step-by-step instructions.**
   Step 1: Have the student draw a picture of a rectangle that has 1/6 shaded in.
   Step 2: Have the student draw a second rectangle that is the same size as the first.
   Step 3: Have him/her shade in 1/8 of the rectangle.
   Step 4: Ask, "Which rectangle has the biggest share shaded in?"
   Step 5: Say, "We can compare these fractions using *greater than*, *less than*, or *equal to*."
   Step 6: Have the student compare the two fractions using the symbols >, <, and =.

5. **Model.** Have the student do the same thing with another set of fractions (1/4 and 1/3).

6. **Check for understanding (work problem with student).** Have the student explain how he/she knows 1/3 is larger than 1/4.

5 Minutes

GUIDED PRACTICE—Monitor Student Work

7. **Student works problems independently while tutor watches and coaches.** Compare the following fractions: 1/4 and 2/4, 3/8 and 5/8, 2/6 and 2/4.

5 Minutes

ASSESS—Evaluate Student Demonstration

8. **Student orally defines at least one Key Academic Mathematics Vocabulary word and skill or concept.** What does *equal* mean?

9. **Student works a problem while explaining EVERY step orally.** Have the student draw a picture of each of the following fractions and compare them using >, <, and =.
4/5 and 2/5
3/6 and 1/4
2/4 and 2/8

STAR Mathematics Tutoring Session Template

Utah State Core Identification	
Domain: Measurement and Data	Cluster: Solve problems involving measurement and estimation of intervals of time, liquid volumes, and masses of objects.
Standard 1: Tell and write time to the nearest minute and measure time intervals in minutes.	

Before You Begin

Key Academic Mathematics Vocabulary (3 or fewer)

second (1/60 of a minute), minute (1/60 of an hour), colon (double dots that separate the hour and minutes (:))

Notes and Materials

Notes and suggestions for an effective lesson: Make sure the student knows:
- That the bold numbers on the clock represent five minutes.
- That there are 60 minutes in an hour.
- That the short hand is the hour hand and the long hand is the minute hand.

Materials you will need:
- clocks
- stop watch or timer
- clock page

LAUNCH—Assess and Provide Background Knowledge

2 Minutes

1. **Connect to prior learning. Use a "hook" to gain the student's attention.** Have the student close his/her eyes and open them when he/she thinks it has been a minute. Time the student and tell him/her how close he/she was.

2. **Introduce and review Key Academic Mathematics Vocabulary.** Ask the student how he/she decided how long to keep his/her eyes shut. Discuss that a minute is made up of 60 seconds and an hour is 60 minutes.

INSTRUCT—Provide Explicit Interactive Instruction

8 Minutes

3. **State the objective.** "Today we will tell and write time to the nearest minute and measure time intervals."

4. **Provide explicit step-by-step instructions.**
   Step 1: The first thing to look at when telling time is the hour hand (short).
   Step 2: Follow the hour hand out to the edge and travel back to the hour (bold number) on the clock.

Out and back
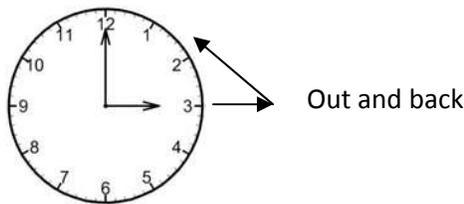
   Step 3: Now look at the minute (long hand). If it is pointing at a bold number, start at the bold number one and count by fives until you reach where the long hand is pointing.
   Step 4: Write the time with hour first, a colon, then the minutes. (3:00)
   Step 5: If the minute hand is between two bold numbers, extend the hand out to the minute marks. Count by fives to the bold number, then count on by ones for each minute mark past the bold number.

5. **Model.** Have the student use a clock model to represent the following times: 12:00, 6:00, 5:00, 9:00.

5 Minutes	6. **Check for understanding (work problem with student).** Give the student a clock with 12:40 on it and have him/her tell you what time it is.
	GUIDED PRACTICE—Monitor Student Work
	7. **Student works problems independently while tutor watches and coaches. "**Talk to me about how to find the time on these clocks. How would you write this time down for someone to read?"
5 Minutes	**ASSESS—Evaluate Student Demonstration**
	8. **Student orally defines at least one Key Academic Mathematics Vocabulary word and skill or concept.** • What is a minute? • What makes up a minutes? • What makes up an hour?
	9. **Student works a problem while explaining EVERY step orally.** Provide the student with a sheet of printed clocks and have him/her identify the time. Use this link: http://www.superteachersworksheets.com/time.

STAR Mathematics Tutoring Session Template

Before You Begin

Utah State Core Identification	
Domain: Measurement and Data	Cluster: Solve problems involving measurement and estimation of intervals of time, liquid volumes, and masses of objects.

Standard 1: Solve word problems involving addition and subtraction of time intervals in minutes, e.g., by representing the problem on a number line diagram.

Key Academic Mathematics Vocabulary (3 or fewer)

interval (amount of time between two events), minute, hour, elapsed time

Notes and Materials

Notes and suggestions for an effective lesson: Students will need to know how to tell time and what the minute hand and hour hand are on the clock. Students will understand that the clock is divided into intervals. They need to understand the words *before*, *after*, *ago*, *from now*, *start*, *end*, *begin*, *half past*, *o'clock*, and how to write time and use the colon correctly.

Materials you will need:
- clock (analog, preferably geared)
- digital clock

2 Minutes

LAUNCH—Assess and Provide Background Knowledge

1. **Connect to prior learning. Use a "hook" to gain the student's attention.** You left your house at 3:00. Your mom tells you to be home in an hour. How do you know when to come home?

2. **Introduce and review Key Academic Mathematics Vocabulary.** When we talk about the time between two events, we are talking about an *interval*. For example, the interval between the start of recess and the end of recess is 15 minutes.

8 Minutes

INSTRUCT—Provide Explicit Interactive Instruction

3. **State the objective.** "Today we will solve word problems by adding and subtracting to find time intervals."

4. **Provide explicit step-by-step instructions.** Say, "Let's start by finding an hour interval. If I left home at 8:05, at what time would I complete a one-hour trip?"

    First, put the time <u>8:05</u> on your clock. When we add an hour, the minute hand makes one complete round of the clock, returning to the same position. Meanwhile, the hour hand shifts ahead one hour. So 8:05 plus one hour makes 9:05.

5. **Model.** Give the student a fifteen-minute interval and have him/her tell you the elapsed time. For example, "We started reading at 9:30 and read for 15 minutes. What time did we finish reading?" Have the student use his/her clock to solve this problem.

6. **Check for understanding (work problem with student).** "It is 4:10. If I spent 20 minutes to get my homework done, what time would it be?" Have the student explain how he/she got the answer.

5 Minutes

GUIDED PRACTICE—Monitor Student Work

7. **Student works problems independently while tutor watches and coaches.** Say, "Now you try." (Work as many of the following problems as time permits.)
    - You woke up at 5:45. What time will it be in an hour?
    - You get home at 4:00. It took you 15 minutes to get there. When did you leave?
    - Your mom called at 6:30 and said she'd be home in an hour. When will she be home?

5 Minutes	**ASSESS—Evaluate Student Demonstration**
	8. **Student orally defines at least one Key Academic Mathematics Vocabulary word and skill or concept.** What do we call the time between two events? (interval)
	9. **Student works a problem while explaining EVERY step orally.** Have the student solve the problems from this free website printable: http://www.superteacherworksheets.com/elapsed-time/elapsed-movies_TZTWD.pdf

STAR Mathematics Tutoring Session Template

Before You Begin

Utah State Core Identification

Domain: Measurement and Data	Cluster: Solve problems involving measurement and estimation of intervals of time, liquid volumes, and masses of objects.

Standard 2: Measure and estimate liquid volumes and masses of objects using standard units of grams (g), kilograms (kg), and liters (l).

Key Academic Mathematics Vocabulary (3 or fewer)

mass (the amount of matter in an object/how heavy it is), volume (liquid measurement)

Notes and Materials

Notes and suggestions for an effective lesson: It may help to assess that the student has a general idea how big/heavy each of the measurement units are (e.g., gram (g)—about the weight/mass of a marshmallow; kilogram (kg)—about the weight/mass of a heavy book, equals 1,000 grams; liter (L)—measures volume, about the size of a mayonnaise jar).

Materials you will need:
- common items (paperclip, eraser, ball, book, pencil, crayon, box, etc.)
- scale
- liter container
- gram/kilogram weights
- measuring cups
- water

2 Minutes

LAUNCH—Assess and Provide Background Knowledge

1. **Connect to prior learning. Use a "hook" to gain the student's attention.** Take out measuring cups and a scale. Ask, "Have you ever used these before? If so, what did you use them for?"

2. **Introduce and review Key Academic Mathematics Vocabulary.** Review mass and volume.

8 Minutes

INSTRUCT—Provide Explicit Interactive Instruction

3. **State the objective.** "Today we will work with mass and volume."

4. **Provide explicit step-by-step instructions.**
   Step 1: Have a variety of tools that measure volume (such as a measuring cup, 2-liter bottle, teaspoon, water bottle, etc.) and mass (such as a scale, cereal box, mail package/ box, etc.).
   Step 2: Have the student sort the objects into categories for mass and volume.

5. **Model.** Ask, "Which one do you think hold the most? Which one holds the least?" (within each category)

6. **Check for understanding (work problem with student).** Have the student work to sort from least to greatest. Check for accuracy.

5 Minutes

GUIDED PRACTICE—Monitor Student Work

7. **Student works problems independently while tutor watches and coaches.** Hold a scavenger hunt for items that weigh close to a gram or kilogram, and for vessels that are close to a liter.

ASSESS—Evaluate Student Demonstration

8. **Student orally defines at least one Key Academic Mathematics Vocabulary word and skill or concept.** What is mass? (amount of matter in an object/how heavy it is)

9. **Student works a problem while explaining EVERY step orally.** The student will precisely measure and label a required amount of liquid using measuring tools accurately.

STAR Mathematics Tutoring Session Template

<table>
<tr><td colspan="2">Utah State Core Identification</td></tr>
<tr>
<td>Domain: Measurement and Data</td>
<td>Cluster: Solve problems involving measurement and estimation of intervals of time, liquid volumes, and masses of objects.</td>
</tr>
<tr>
<td colspan="2">Standard 2: Measure and estimate liquid volumes and masses of objects using standard units of grams (g), kilograms (kg), and liters (L). Add, subtract, multiply, or divide to solve one-step word problems involving masses or volumes that are given in the same units, e.g., by using drawings (such as a beaker with a measurement scale) to represent the problem.</td>
</tr>
<tr><td colspan="2">Key Academic Mathematics Vocabulary (3 or fewer)</td></tr>
<tr><td colspan="2">mass (the amount of matter in an object/how heavy it is), volume (liquid measurement), metric units</td></tr>
<tr><td colspan="2">Notes and Materials</td></tr>
<tr><td colspan="2">Notes and suggestions for an effective lesson: It may help to assess that the student has a general idea how big/heavy each of the measurement units are (e.g., gram (g)—about the weight/mass of a marshmallow; kilogram (kg)—about the weight/mass of a heavy book, equals 1,000 grams; liter (L)—measures volume, about the size of a mayonnaise jar).</td></tr>
<tr><td colspan="2">Materials you will need:

common items (paperclip, eraser, ball, book, pencil, crayon, box, etc.)
scale
liter container
gram/kilogram weights
measuring cups
water

</td></tr>
</table>

Before You Begin

LAUNCH—Assess and Provide Background Knowledge

1. **Connect to prior learning. Use a "hook" to gain the student's attention.** Say, "Joy has 3 books. The first weighs 5 grams. The second weighs 8 grams. The third weighs 11 grams. How many grams do all three books weigh together?"

2. **Introduce and review Key Academic Mathematics Vocabulary.** Metric units of measurement for mass and volume.

2 Minutes

INSTRUCT—Provide Explicit Interactive Instruction

3. **State the objective.** "Today we will solve problems using mass and volume."

4. **Provide explicit step-by-step instructions.** Pose the following problem to the student:
   Step 1: Charlie bought 12 two-liter bottles of soda. How many liters of soda did he buy?
   Step 2: Have the student solve the problem.

5. **Model.** Student writes out the equation.

6. **Check for understanding (work problem with student).** Have the student explain how he/she solved the problem step by step.

8 Minutes

GUIDED PRACTICE—Monitor Student Work

7. **Student works problems independently while tutor watches and coaches.** Student will solve two more problems (see below).

5 Minutes

5 Minutes	**ASSESS—Evaluate Student Demonstration**
	8. **Student orally defines at least one Key Academic Mathematics Vocabulary word and skill or concept.** Define metric units.
	9. **Student works a problem while explaining EVERY step orally.** Have the student solve this problem: Angela is doing an experiment with paperclips. She discovered that 90 paperclips have a mass of 450 grams. What is the mass of each paperclip?

Guided Practice:

Lisa's pool will hold 120 liters of water. Right now it has 50 liters of water in it. How many more liters of water can she put in the pool?

Lisa has 320 grams of marshmallows. There are 16 marshmallows. How much does each marshmallow weigh?

STAR Mathematics Tutoring Session Template

Before You Begin

Utah State Core Identification

Domain: Measurement and Data	Cluster: Represent and interpret data.

Standard 3: Draw a scaled picture graph and a scaled bar graph to represent a data set with several categories. Solve one-and two-step "how many more" and "how many less" problems using information presented in scaled bar graphs. For example, draw a bar graph in which each square in the bar graph might represent 5 pets.

Key Academic Mathematics Vocabulary (3 or fewer)

data, picture graph, symbol, key, scale, bar graph

Notes and Materials

Notes and suggestions for an effective lesson: Students need to understand the different types of graphs or understand the difference between a picture graph and a bar graph. The students also need to understand that a graph must have a title, labeled categories, and picture graphs must have a key.

Materials you will need:
- computer access
- paper
- pencil

2 Minutes

LAUNCH—Assess and Provide Background Knowledge

1. **Connect to prior learning. Use a "hook" to gain the student's attention.** Show the student the following graph: http://www.kidslearningstation.com/graphing/images/learn-graphing-sports.jpg Have the student explain what he/she learn from the graph.

2. **Introduce and review Key Academic Mathematics Vocabulary.** Use the graph to discuss key, bar graph, and scale.

8 Minutes

INSTRUCT—Provide Explicit Interactive Instruction

3. **State the objective.** "Today we will be making bar graphs and picture graphs to represent data."

4. **Provide explicit step-by-step instructions.**
   Step 1: Provide the student with the following data for a bar graph:
   Title: Favorite Fruit; Categories/Quantity: Apples—5; Bananas—4; Oranges—8; Strawberries—7
   Step 2: Have the student label the categories and the scale for the data.
   Step 3: The student will represent the data on the bar graph.

5. **Model.**
   Step 5: Have the student complete the graph.

6. **Check for understanding (work problem with student).** Interpret the data. How many more said they like apples than bananas? How many like strawberries or oranges?

5 Minutes

GUIDED PRACTICE—Monitor Student Work

7. **Student works problems independently while tutor watches and coaches.** Have the student use the same set of data to create a picture graph.

ASSESS—Evaluate Student Demonstration
8. **Student orally defines at least one Key Academic Mathematics Vocabulary word and skill or concept.** What is scale (in relation to size)? Explain the difference between a bar graph and a picture graph.
9. **Student works a problem while explaining EVERY step orally.** Given a piece of graph paper and data, the student will create both a picture graph and bar graph to represent the data with several categories using a scale of 2, 5, or 10. (See below for data.)

Assessment Practice:

Joey wanted to know how many picture books certain authors have written. He found the following data:

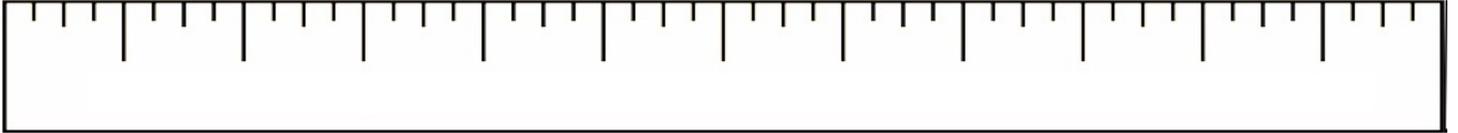Dr. Suess—55

Mercer Mayer—300

Robert Munsch—40

Patricia Pollacco—50

STAR Mathematics Tutoring Session Template

<table>
<tr><td rowspan="7">**Before You Begin**</td><td colspan="2">**Utah State Core Identification**</td></tr>
<tr><td>Domain: Measurement and Data</td><td>Cluster: Represent and interpret data.</td></tr>
<tr><td colspan="2">Standard 4: Generate measurement data by measuring lengths using rulers marked with halves and fourths of an inch. Show the data by making a line plot, where the horizontal scale is marked off in appropriate units—whole numbers, halves, or quarters.</td></tr>
<tr><td colspan="2">**Key Academic Mathematics Vocabulary (3 or fewer)**</td></tr>
<tr><td colspan="2">line plot (a way to show data using a number line)</td></tr>
<tr><td colspan="2">**Notes and Materials**</td></tr>
<tr><td colspan="2">**Notes and suggestions for an effective lesson:** Remind the student to start at "0" when measuring with a ruler, even though some have a gap and don't start right at the left of the wood/plastic.

Materials you will need:
• ruler (with halves and fourths of an inch)
• ruler template
• objects to measure (eraser, crayon, pencil)</td></tr>
</table>

<table>
<tr><td rowspan="3">**2 Minutes**</td><td>**LAUNCH—Assess and Provide Background Knowledge**</td></tr>
<tr><td>1. **Connect to prior learning. Use a "hook" to gain the student's attention.** Warm up by naming the points on a ruler: 1/4, 1/2, 3/4, 1, 1¼, 1½, 1¾, 2, etc.</td></tr>
<tr><td>2. **Introduce and review Key Academic Mathematics Vocabulary.** The ruler is an example of a line plot, or a horizontal scale like a number line, used to measure length.</td></tr>
</table>

<table>
<tr><td rowspan="5">**8 Minutes**</td><td>**INSTRUCT—Provide Explicit Interactive Instruction**</td></tr>
<tr><td>3. **State the objective.** "Today we will measure lengths and record them on a line plot that you make." Then have the student measure his/her pencil to the nearest inch and half inch.</td></tr>
<tr><td>4. **Provide explicit step-by-step instructions.** Say, "Let's measure this eraser. First, we need to line up the end of the eraser with the zero line on the ruler. Next, follow the eraser to the tip and mark its length. Now, we'll decide which mark on the ruler it is closest to."</td></tr>
<tr><td>5. **Model.** Say, "Finally, we'll record it on the blank ruler, filling in the lines for ¼", ½", ¾", 1", and keep going until the length of the eraser is reached."</td></tr>
<tr><td>6. **Check for understanding (work problem with student).** Say, "Let's measure a crayon." Use the same steps as above, correcting student as needed). Add on to the line plot started with the eraser.</td></tr>
</table>

<table>
<tr><td rowspan="2">**5 Minutes**</td><td>**GUIDED PRACTICE—Monitor Student Work**</td></tr>
<tr><td>7. **Student works problems independently while tutor watches and coaches.** For the last item, measure a new pencil. Have the student tell you how he/she measures. Record the length on the ruler you have created, adding the lines needed to make your line plot complete.</td></tr>
</table>

<table>
<tr><td rowspan="2">**5 Minutes**</td><td>**ASSESS—Evaluate Student Demonstration**</td></tr>
<tr><td>8. **Student orally defines at least one Key Academic Mathematics Vocabulary word and skill or concept.** The ruler is a line plot. What is a line plot? (A way to show data using a number line.)</td></tr>
</table>

9. **Student works a problem while explaining EVERY step orally.** Say, "Measure this new pencil. Tell me how to measure. Record the length on the ruler you've created, adding the lines needed to make your line plot complete."

STAR Mathematics Tutoring Session Template

Utah State Core Identification

Domain: Measurement and Data	Cluster: Geometric measurement: understand concepts of area and relate area to multiplication and to addition.

Standard 5a: A square with side length 1 unit, called "a unit square," is said to have "one square unit" of area, and can be used to measure area. A plane figure which can be covered without gaps or overlaps by *n* unit squares is said to have an area of *n* square units.

Key Academic Mathematics Vocabulary (3 or fewer)

plane figure (any two-dimensional figure), area (the number of square units needed to cover a region or figure), square (a shape with four equal sides)

Notes and Materials

Notes and suggestions for an effective lesson: Make sure the student is accurate when measuring, lining up the tile from the place left off.

Materials you will need:
- color tiles
- worksheet with rectangles to measure (note: You may need to adjust the size of the rectangles in the worksheet to match the size of your color tiles. Be certain the student will have an exact number of square units to measure with nothing left over.)

LAUNCH—Assess and Provide Background Knowledge

1. **Connect to prior learning. Use a "hook" to gain the student's attention.** Mom says you can get new carpet for your room. How do you figure out how much to buy? Talk about how carpet stores sell their carpet in "square" feet or yards.

2. **Introduce and review Key Academic Mathematics Vocabulary.** Area is the number of "squares" needed to cover a region or figure. Remember, squares have four equal sides.

INSTRUCT—Provide Explicit Interactive Instruction

3. **State the objective.** "Today we will learn what a unit square is and how it is used to measure area."

4. **Provide explicit step-by-step instructions.**
   - A unit square is any square used for measuring area. The important thing to remember is that all four sides of a square are equal.
   - To measure square units, lay the square on top of the object or region to be measured. For every square, there is "one square unit."
   - Area is important for people who lay down sod or grass, for painters to know the area of the walls they will paint, and for carpet layers to know how much floor to cover.
   - Practice measuring square units using a color tile, a "unit square" on rectangle #1. Line up the tile in the corner, matching the edges. Slide the tile over, being careful to line it up with where the first measure ended. Count the total number of "squares" covered.
   - This is recorded as _____ square units.

5. **Model.** Say, "Now you try. Measure rectangle #2."

6. **Check for understanding (work problem with student).** Have the student explain how he/she got the measurements for the rectangle #2.

GUIDED PRACTICE—Monitor Student Work

7. **Student works problems independently while tutor watches and coaches.** Measure the remaining rectangles. Make sure the student names the area using the label "square units."

ASSESS—Evaluate Student Demonstration

8. **Student orally defines at least one Key Academic Mathematics Vocabulary word and skill or concept.** Have the student explain what area is.
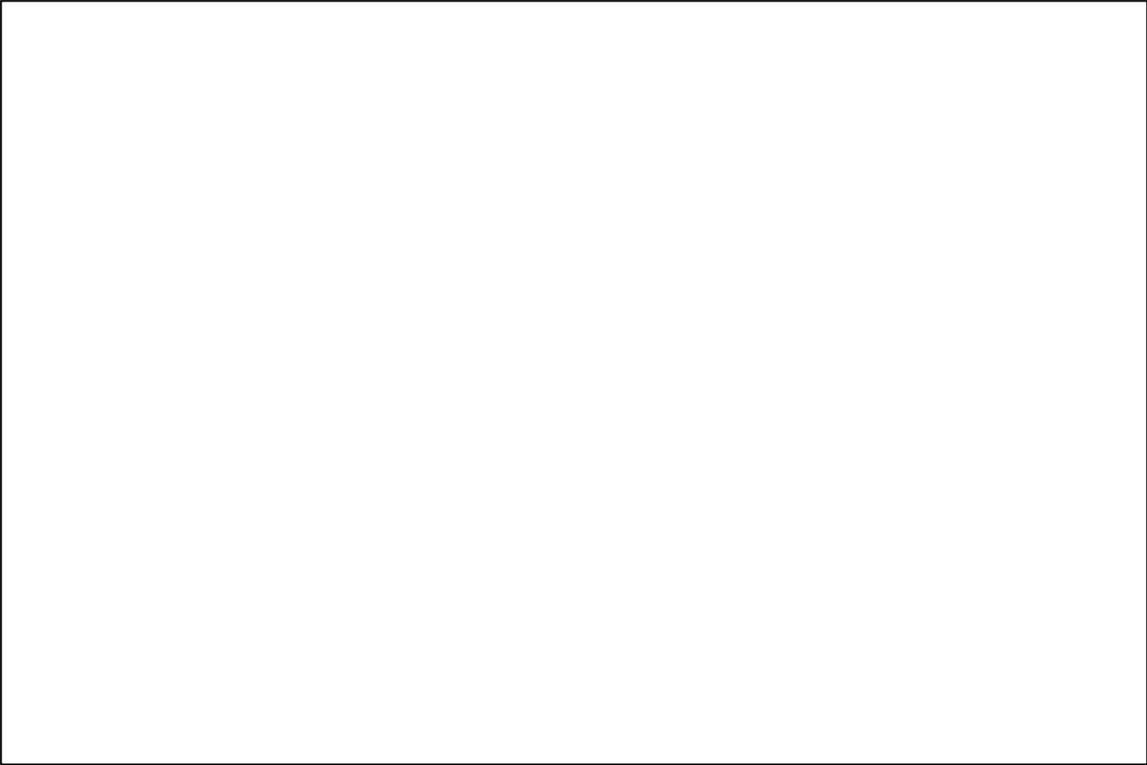
9. **Student works a problem while explaining EVERY step orally.**
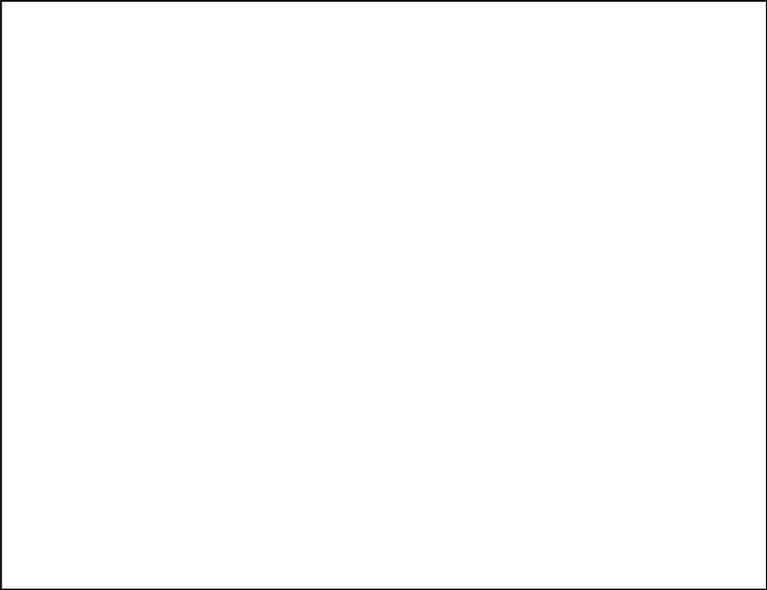
Provide the student with this array:

Have him/her find the area. Write a number sentence to show repeated addition and/or multiplication.
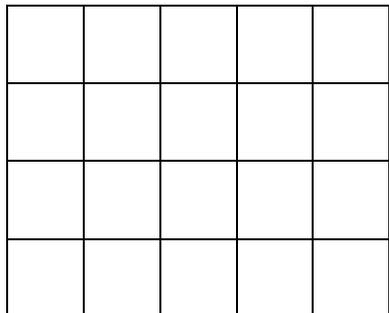
Rectangle Worksheet

#1

#2

STAR Mathematics Tutoring Session Template

<table>
<tr><td colspan="2">Utah State Core Identification</td></tr>
<tr><td>Domain: Measurement and Data</td><td>Cluster: Geometric measurement: understand concepts of area and relate area to multiplication and to addition.</td></tr>
<tr><td colspan="2">Standard 5b: A plane figure which can be covered without gaps or overlaps by n unit squares is said to have an area of n square units.</td></tr>
</table>

Before You Begin

Key Academic Mathematics Vocabulary (3 or fewer)

plane figure (any two-dimensional figure), area (the number of square units needed to cover a region or figure), square (shape with four equal sides; square units)

Notes and Materials

Notes and suggestions for an effective lesson: Students need to understand that a plane figure is a two dimensional shape. To find the area, they are covering with no gaps or overlaps.

Materials you will need:
- computer with Internet access
- paper
- pencil
- crayons
- graph paper

LAUNCH—Assess and Provide Background Knowledge

2 Minutes

1. **Connect to prior learning. Use a "hook" to gain the student's attention.** Give the student an array and have them find the area.

2. **Introduce and review Key Academic Mathematics Vocabulary.** Review square units.

INSTRUCT—Provide Explicit Interactive Instruction

8 Minutes

2. **State the objective.** "Today we are going to cover plane figures with square units and count them to find the area."

4. **Provide explicit step-by-step instructions.**
   Step 1: Provide a plane figure for the student.
   Step 2: Have them cover it with square units.

5. **Model.** Have the student count the square units it took to cover the plane figure.

6. **Check for understanding (work problem with student).** Observe student as he/she is counting the units it took to cover the shape.

GUIDED PRACTICE—Monitor Student Work

5 Minutes

7. **Student works problems independently while tutor watches and coaches.** The student will work on a few problems from the following site:
   http://olc.spsd.sk.ca/de/math1- 3/virtual%20manipulatives/areaGRID.html

5 Minutes

8. **Student orally defines at least one Key Academic Mathematics Vocabulary word and skill or concept.** What is a Plane figure? What is area?

9. **Student works a problem while explaining EVERY step orally.** Give the student the following shape and have him/her color the square units and determine how many there are. Write a number sentence to show repeated addition and/or multiplication.

STAR Mathematics Tutoring Session Template

<table>
<tr><td colspan="2">Utah State Core Identification</td></tr>
<tr><td>Domain: Measurement and Data</td><td>Cluster: Geometric measurement: understand concepts of area and relate area to multiplication and to addition.</td></tr>
<tr><td colspan="2">Standard 6: Measure areas by counting unit squares (square cm, square m, square in., square ft., and improvised units).</td></tr>
</table>

Key Academic Mathematics Vocabulary (3 or fewer)

area (the number of square units needed to cover a figure), square (shape with four equal sides)

Notes and Materials

Notes and suggestions for an effective lesson:
- The big idea for kids to get is that <u>area</u> is measured in <u>square</u> units. In other words, one side of the measuring tool is equal to the other three sides. For example, we could measure the area of a desk in Wheat Thins crackers because each is a "square unit."
- Square centimeters, inches, feet, etc. are simply squares in which each side is a certain unit long (a centimeter, inch, foot, etc.).

Materials you will need:
- color tiles
- variety of materials (see guided practice section below)

LAUNCH—Assess and Provide Background Knowledge

1. **Connect to prior learning. Use a "hook" to gain the student's attention.** Ask, "How many things can you think of that are the shape of a square?" (Wheat Thins, color tiles, game boards, card tables)

2. **Introduce and review Key Academic Mathematics Vocabulary.** Say, "We're going to measure <u>area</u> today using <u>squares</u>. Area measures the number of square units needed to cover a figure."

INSTRUCT—Provide Explicit Interactive Instruction

3. **State the objective.** "Today we will measure area by counting unit squares."

4. **Provide explicit step-by-step instructions.** Say, "When measuring area, we count how many squares it takes to cover the space or shape we're measuring. We choose the unit to measure in by how large the area is we have to measure. I wouldn't want to measure the playground using a centimeter tile—it would take too long! But a square meter might work great. On the other hand, I wouldn't want to measure my desk with a square meter. It would be too big! We choose to measure area in the unit that will give us the closest measurement. If I wanted to measure the area of this paper in square inches, I would count how many tiles it takes to cover it."

5. **Model.** Observe the student as he/she covers his/her paper with square tiles.

6. **Check for understanding (work problem with student).** As the student is covering his/her paper, ask, "How many squares did it take to cover the whole paper?"

GUIDED PRACTICE—Monitor Student Work

7. **Student works problems independently while tutor watches and coaches.** Say, "Measure your math book, using square units. How many squares did it take?"

Before You Begin

2 Minutes

8 Minutes

	ASSESS—Evaluate Student Demonstration
5 Minutes	8. **Student orally defines at least one Key Academic Mathematics Vocabulary word and skill or concept.** Have the student define area. 9. **Student works a problem while explaining EVERY step orally.** Have the student choose an object to measure the area, using square units.

STAR Mathematics Tutoring Session Template

Utah State Core Identification	
Domain: Measurement and Data	Cluster: Geometric measurement: understand concepts of area and relate area to multiplication and to addition.

Standard 7a: Find the area of a rectangle with whole-number side lengths by tiling it, and show that the area is the same as would be found by multiplying the side lengths.

Before You Begin

Key Academic Mathematics Vocabulary (3 or fewer)

area, rectangle, array

Notes and Materials

Notes and suggestions for an effective lesson: Students need to know how to model decomposing shapes, how to find lengths of sides not given, and how to model the distributive property.

Materials you will need:
- graph paper
- pencil
- manipulatives (e.g., tiles)

LAUNCH—Assess and Provide Background Knowledge

2 Minutes

1. **Connect to prior learning. Use a "hook" to gain the student's attention.** Sarah is trying to tile a room in her house, and she needs to know how much tile to buy. She measures the length and width of her room and comes up with the measurement 12' x 10'. How much tile should she purchase?

2. **Introduce and review Key Academic Mathematics Vocabulary.** The number of squares or tiles is called the area of a figure. To find area, multiply one dimension by the opposite side.

INSTRUCT—Provide Explicit Interactive Instruction

8 Minutes

3. **State the objective.** "Today we will find the area of rectangular figures."

4. **Provide explicit step-by-step instructions.**
   Step 1: Give the student square tiles and a rectangle to measure.
   Step 2: Place the tiles on the rectangle. How many squares are there altogether?
   Step 3: Explain that the total area of the rectangle is the number of squares that were placed.
   Step 4: How many tiles are going down? How many are going across?
   Step 5: If we were to multiply these two dimensions, what would you get?
   Step 6: Explain that this is another way to find area.

5. **Model.** Give the student another rectangle and have him/her repeat the process of tiling and multiplying the dimensions (or array) to find the area.

6. **Check for understanding (work problem with student).** Have the student explain how tiling and counting the total and multiplying the dimensions (array) yield the same answer.

GUIDED PRACTICE—Monitor Student Work

5 Minutes

7. **Student works problems independently while tutor watches and coaches.** Give the student another rectangle on his/her graph paper and have him/her tell the dimensions and area.

ASSESS—Evaluate Student Demonstration
8. **Student orally defines at least one Key Academic Mathematics Vocabulary word and skill or concept.** What is an array? (rectangular model of multiplication)
9. **Student works a problem while explaining EVERY step orally.** Tile 18 square units in the grid below. Then show a multiplication equation that shows the same number of tiles.

STAR Mathematics Tutoring Session Template

Utah State Core Identification	
Domain: Measurement and Data	Cluster: Geometric measurement: understand concepts of area and relate area to multiplication and to addition.

Standard 7b: Multiply side lengths to find areas of rectangles with whole-number side lengths in the context of solving real-world and mathematical problems, and represent whole-number products as rectangular areas in mathematical reasoning.

Key Academic Mathematics Vocabulary (3 or fewer)

rectangle, area, array, dimensions

Notes and Materials

Notes and suggestions for an effective lesson: Students will need to know what an array is (a rectangular model of multiplication). They will need to know how to find the dimensions of a shape by tiling or counting the squares down the side.
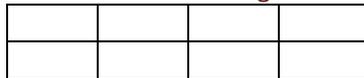
Materials you will need:
- paper
- pencil
- rectangular models with dimensions

LAUNCH—Assess and Provide Background Knowledge

1. **Connect to prior learning. Use a "hook" to gain the student's attention.** Susan and her friends were asked to design a snow fort. Their fort is represented below. Find the area of the walls of the fort.

2. **Introduce and review Key Academic Mathematics Vocabulary.** What is an array?

INSTRUCT—Provide Explicit Interactive Instruction

3. **State the objective.** "Today we will be finding the area by multiplying length times width to find the area of a rectangle."

4. **Provide explicit step-by-step instructions.** Pose a problem such as the following:
   Step 1: Ben lays bricks in his backyard to create a fence. He has 3 rows of 6 bricks.
   Step 2: Draw a rectangle that represents his bricks.
   Step 3: Multiply the rows by each other 3 x 6 = 18.
   Step 4: Explain that you just found the area of the bricks.

5. **Model.** Give the student another problem. Have him/her draw a rectangle that matches the equation and find the area. (For example, 5 x 7 = 35.)

6. **Check for understanding (work problem with student).** The student will find the area of an array with the dimensions 4 x 8 = 32.

GUIDED PRACTICE—Monitor Student Work

7. **Student works problems independently while tutor watches and coaches.** Have student give the area for the following rectangles: 8 x 8 = 64, 9 x 3 = 27, 6 x 5 = 30, 12 x 4 = 48.

Before You Begin

2 Minutes

8 Minutes

5 Minutes

<table>
<tr><td rowspan="3">5 Minutes</td><td colspan="2">ASSESS—Evaluate Student Demonstration</td></tr>
<tr><td>8.</td><td>**Student orally defines at least one Key Academic Mathematics Vocabulary word and skill or concept.** How do you find area?</td></tr>
<tr><td>9.</td><td>**Student works a problem while explaining EVERY step orally.** Have the student solve the following problem and explain how he/she found the area: The school was having an assembly. The principal wanted the chairs to be put in 7 rows of 8. Each chair fills the space of one square foot. How many chairs will fit in this area?</td></tr>
</table>

STAR Mathematics Tutoring Session Template

Before You Begin

Utah State Core Identification

Domain: Measurement and Data	Cluster: Geometric measurement: understand concepts of area and relate area to multiplication and to addition.

Standard 7c: Use tiling to show in a concrete case that the area of a rectangle with whole-number side lengths *a* and *b* + *c* is the sum of *a* × *b* and *a* × *c*. Use area models to represent the distributive property in mathematical reasoning.

Key Academic Mathematics Vocabulary (3 or fewer)

distributive property, decompose, product

Notes and Materials

Notes and suggestions for an effective lesson: Students will need to know that you can break a shape into smaller shapes to find the area. Students will be breaking shapes into squares or smaller rectangles to find the area of the whole.

Materials you will need:
- rectangles
- graph paper
- pencils

2 Minutes

LAUNCH—Assess and Provide Background Knowledge

1. **Connect to prior learning. Use a "hook" to gain the student's attention.** We want to measure the area of our classroom, but we only have 2 yardsticks. How can we find the area of the room using this tool?

2. **Introduce and review Key Academic Mathematics Vocabulary.** To *decompose* is to break into smaller pieces and add them together to find the whole.

INSTRUCT—Provide Explicit Interactive Instruction

3. **State the objective.** "Today we will find the area of rectangles by decomposing shapes into smaller shapes and then adding the area of all the parts together to find the total area."

8 Minutes

4. **Provide explicit step-by-step instructions.**
   Step 1: Say, "If I have a rectangle that measures 7 x 9, how can I break it into two rectangles to find the area?"
   Step 2: Provide the student with a drawing of the rectangle 7 x 9. Say, "I can find the area of a 7 x 5 rectangle and a 7 x 4 rectangle, because I know that 7 x 5 +7 x 4 = 7 x 9."
   Step 3: Explain that when decomposing into smaller shapes, you take one of the dimensions and find the numbers that you would add together to make the total dimension.
   Step 4: Multiply 7 x 5 and 7 x 4. Add the two products together to get the total area of the rectangle.

5. **Model.** Give the student a model of a rectangle that has been decomposed into smaller rectangles and have him/her find the dimensions of the total rectangle. Then have him/her multiply the dimensions together to find the area.

Total 4 x 2 = 8. (Could be broken or decomposed into 2 x 2 +2 x 2= 8, 1 x 2 + 3 x 2 = 8, etc.)

6. **Check for understanding (work problem with student).** Give the student a different rectangle and have him/her decompose the shape to find the area.

GUIDED PRACTICE—Monitor Student Work

7. **Student works problems independently while tutor watches and coaches.** Give him/her more models of rectangles and have him/her repeat the process. (See below.)

ASSESS—Evaluate Student Demonstration

8. **Student orally defines at least one Key Academic Mathematics Vocabulary word and skill or concept.** What is the distributive property?

9. **Student works a problem while explaining EVERY step orally.** Provide an additional rectangle and have the student explain how to find the area.

Guided Practice:

STAR Mathematics Tutoring Session Template

<table>
<tr><td colspan="2">Utah State Core Identification</td></tr>
<tr><td>Domain: Measurement and Data</td><td>Cluster: Geometric measurement: understand concepts of area and relate area to multiplication and to addition.</td></tr>
<tr><td colspan="2">Standard 7d: Recognize area as additive. Find areas of rectilinear figures by decomposing them into non-overlapping rectangles and adding the areas of the non-overlapping parts, applying this technique to solve real-world problems.</td></tr>
<tr><td colspan="2">Key Academic Mathematics Vocabulary (3 or fewer)</td></tr>
<tr><td colspan="2">distributive property, decomposing, rectilinear, additive</td></tr>
<tr><td colspan="2">Notes and Materials</td></tr>
<tr><td colspan="2">Notes and suggestions for an effective lesson: Students need to understand that they can break rectangles into smaller rectangles to find the area. Students also need to understand why we break shapes into smaller pieces.</td></tr>
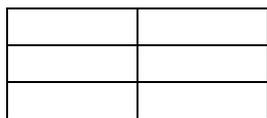<tr><td colspan="2">Materials you will need:
• graph paper
• pencil</td></tr>
<tr><td colspan="2">LAUNCH—Assess and Provide Background Knowledge</td></tr>
<tr><td colspan="2">1. Connect to prior learning. Use a "hook" to gain the student's attention. Joe and John made a poster that was 4 ft. x 3 ft. Mary and Ann made a poster that was 4 ft. x 2 ft. They placed them side by side so there was not space between them. How much area will the two posters cover?</td></tr>
<tr><td colspan="2">2. Introduce and review Key Academic Mathematics Vocabulary. Rectilinear (decomposing a shape into different rectangles)</td></tr>
<tr><td colspan="2">INSTRUCT—Provide Explicit Interactive Instruction</td></tr>
<tr><td colspan="2">3. State the objective. Today we will be finding the area of rectangles by breaking rectangles into smaller rectangles and adding the area of the smaller rectangles together to find the total area of the larger rectangle.</td></tr>
<tr><td colspan="2">4. Provide explicit step-by-step instructions. What is the area of this figure?

You can separate the figure into a square and a rectangle.

Find the area of each part.</td></tr>
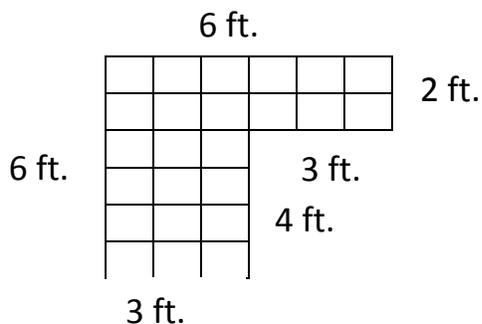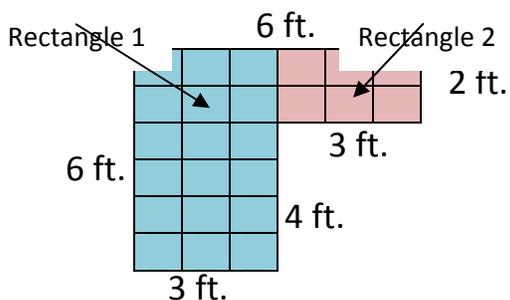</table>

Side labels: Before You Begin · 2 Minutes · 8 Minutes
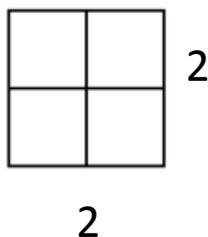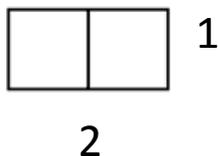
Area of the square: 2 x 2 = 4
Area of the rectangle: 1 x 2 = 2
Find the sum of the areas: 4 + 2 = 6
The area of the figure is 6 square units.

5. **Model.**
   Louise drew the following plan for a garden.

6 ft.

6 ft.   2 ft.

3 ft.

4 ft.

3 ft.

What is the area of the garden?
Separate the figure into two rectangles.

Rectangle 1   6 ft.   Rectangle 2

2 ft.

6 ft.   3 ft.

4 ft.

3 ft.

Find the area of each rectangle.
Area of rectangle 1: 6 x 3 = 18
Area of rectangle 2: 2 x 3 = 6
Find the sum of the two rectangles.

6. **Check for understanding (work problem with student).** Give the student a rectangle and have him/her show how he/she would break the rectangle into smaller rectangles and add them to find the area.
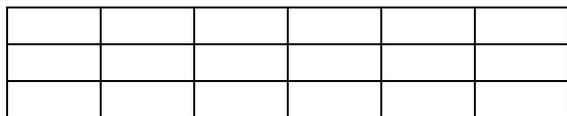
7. **Student works problems independently while tutor watches and coaches.**
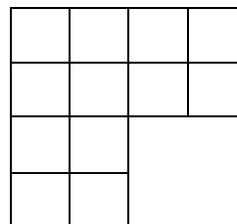
How could this figure be decomposed to help find the area?

5 Minutes

ASSESS—Evaluate Student Demonstration

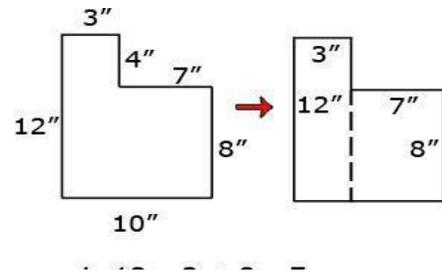8. **Student orally defines at least one Key Academic Mathematics Vocabulary word and skill or concept.**
   What does *additive* mean? (to add together the two parts)

9. **Student works a problem while explaining EVERY step orally.**

Have the student divide the shape into rectangles and find the area.

STAR Mathematics Tutoring Session Template

Before You Begin

Utah State Core Identification

Domain: Measurement and Data	Cluster: Geometric measurement: recognize perimeter as an attribute of plane figures and distinguish between linear and area measures.

Standard 8: Solve real world and mathematical problems involving perimeters of polygons, including finding the perimeter given the side lengths, finding an unknown side length, and exhibiting rectangles with the same perimeter and different areas or with the same area and different perimeters.

Key Academic Mathematics Vocabulary (3 or fewer)

area, perimeter, polygon

Notes and Materials

Notes and suggestions for an effective lesson: Students need to understand what perimeter and area are and how to solve for them.

Materials you will need:
- graph paper
- computer access
- worksheets (see link)

2 Minutes

LAUNCH—Assess and Provide Background Knowledge

1. **Connect to prior learning. Use a "hook" to gain the student's attention.** This is a picture of May's backyard:

   20 ft.

   5 ft.

   What is the area of her backyard?

2. **Introduce and review Key Academic Mathematics Vocabulary.** Using the picture from above, define perimeter as the measurement of the outside edge of a shape. To find the perimeter, you add all the lengths of the sides together.

8 Minutes

INSTRUCT—Provide Explicit Interactive Instruction

3. **State the objective.** "Today we will find the area and the perimeter of shapes."

4. **Provide explicit step-by-step instructions.**
   Step 1: Provide a shape with dimensions.
   Step 2: Have the student find the area of the shape.
   Step 3: Have the student find the perimeter of the shape by adding the all of the lengths of the sides together.

5. **Model.** Have the student find the perimeters for each of the shapes on this link:
   http://ixl.com/math/grade-3/perimeter

6. **Check for understanding (work problem with student).** Have the student explain how he/she found the perimeter for the shapes.

5 Minutes	**GUIDED PRACTICE—Monitor Student Work**
	7. **Student works problems independently while tutor watches and coaches.** Print off the following worksheet for the student to find the perimeter of shapes: http://www.superteacherworksheets.com/geometry/perimeter-3_TZFFD.pdf
5 Minutes	**ASSESS—Evaluate Student Demonstration**
	8. **Student orally defines at least one Key Academic Mathematics Vocabulary word and skill or concept.** What is a polygon? How do you find its area? Its perimeter?
	9. **Student works a problem while explaining EVERY step orally.** Jackie's father told her she could make a garden in her backyard large enough for 100 square feet of flowers. Using graph paper, create a garden that Jackie will like. Find the perimeter and area of that garden.

STAR Mathematics Tutoring Session Template

Before You Begin

Utah State Core Identification

Domain: Geometry	Cluster: Reason with shapes and their attributes.

Standard 1: Understand that shapes in different categories (e.g., rhombuses, rectangles, and others) may share attributes (e.g., having four sides), and that the shared attributes can define a larger category (e.g., quadrilaterals). Recognize rhombuses, rectangles, and squares as examples of quadrilaterals, and draw examples of quadrilaterals that do not belong to any of these subcategories.

Key Academic Mathematics Vocabulary (3 or fewer)

quadrilateral, attributes, rhombus, rectangle, square, parallelogram, trapezoid, sides, angles

Notes and Materials

Notes and suggestions for an effective lesson: Attributes are characteristics of a shape (number of sides, number of angles, type of angle, etc.).

Materials you will need:
- pattern blocks or pictures of shapes

2 Minutes

LAUNCH—Assess and Provide Background Knowledge

1. **Connect to prior learning. Use a "hook" to gain the student's attention.** Show a shape (square or rectangle). Have them identify the shape and tell what they know about it.

2. **Introduce and review Key Academic Mathematics Vocabulary.** Discuss the attributes of a quadrilateral: Four sides, four angles, closed figure

8 Minutes

INSTRUCT—Provide Explicit Interactive Instruction

3. **State the objective.** "Today we will be working with shapes to determine whether or not they are quadrilaterals."

4. **Provide explicit step-by-step instructions.**
   Step 1: Show the student a trapezoid.
   Step 2: Ask, "What do you notice about this shape? How is this shape like the square or rectangle I showed you before?"

5. **Model. Repeat steps 1-2 using a rhombus.**

6. **Check for understanding (work problem with student).** How are all of the shapes we have seen the same?

5 Minutes

GUIDED PRACTICE—Monitor Student Work

7. **Student works problems independently while tutor watches and coaches.** Have the student identify quadrilaterals around the room that have the same attributes.

5 Minutes

ASSESS—Evaluate Student Demonstration

8. **Student orally defines at least one Key Academic Mathematics Vocabulary word and skill or concept.** Identify the attributes of a quadrilateral.

9. **Student works a problem while explaining EVERY step orally.**
   Which of these shapes are quadrilaterals?

STAR Mathematics Tutoring Session Template

<table>
<tr><td rowspan="9">**Before You Begin**</td><td colspan="2">**Utah State Core Identification**</td></tr>
<tr><td>Domain: Geometry</td><td>Cluster: Reason with shapes and their attributes.</td></tr>
<tr><td colspan="2">Standard 2: Partition shapes into parts with equal areas. Express the area of each part as a unit fraction of the whole. For example, partition a shape into 4 parts with equal area, and describe the area of each part as 1/4 of the area of the shape.</td></tr>
<tr><td colspan="2">**Key Academic Mathematics Vocabulary (3 or fewer)**</td></tr>
<tr><td colspan="2">fraction, partition, area, equal parts</td></tr>
<tr><td colspan="2">**Notes and Materials**</td></tr>
<tr><td colspan="2">**Notes and suggestions for an effective lesson:** It is important to remind the student that fractions must be equal parts and that the denominator is how many equal parts the whole is divided into.</td></tr>
<tr><td colspan="2">**Materials you will need:**
• Geoboard
• Elastic Bands
• If the above are not available you can use the virtual geoboard available at http://nlvm.usu.edu/en/nav/frames_asid_172_g_2_t_3.html?open=activities&from=category_g_2_t_3.html</td></tr>
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<table>
<tr><td rowspan="3">**2 Minutes**</td><td>**LAUNCH—Assess and Provide Background Knowledge**</td></tr>
<tr><td>1. **Connect to prior learning. Use a "hook" to gain the student's attention.** What are some things that can be cut up to be shared? (pizza, cake, sandwiches, candy bars, etc.)</td></tr>
<tr><td>2. **Introduce and review Key Academic Mathematics Vocabulary.** When we divide things evenly, we are creating fractions or equal parts of a whole.</td></tr>
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<table>
<tr><td rowspan="5">**8 Minutes**</td><td>**INSTRUCT—Provide Explicit Interactive Instruction**</td></tr>
<tr><td>3. **State the objective.** "Today we will divide shapes into parts with equal areas."</td></tr>
<tr><td>4. **Provide explicit step-by-step instructions.** Geometric shapes may be divided into parts with equal areas. Say, "We can divide the square into four equal parts." Create a square on the geoboard. Create a vertical line and a horizontal line on the square to illustrate. Each part of the square is one part out of four. This can be written as a fraction: 1/4.</td></tr>
<tr><td>5. **Model.** Observe the student as he/she is dividing the square into four equal parts.</td></tr>
<tr><td>6. **Check for understanding (work problem with student).** Create a different rectangle on the geoboard. Say, "Let's divide this rectangle into two equal parts (halves). How would we write this as a fraction?" (1/2)</td></tr>
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<tr><td rowspan="2">**5 Minutes**</td><td>**GUIDED PRACTICE—Monitor Student Work**</td></tr>
<tr><td>7. **Student works problems independently while tutor watches and coaches.** Have the student create shapes on the geoboard. Say, "Can you tell me how many equal parts you'll divide the remaining shapes into? What would the fraction look like? Why?"</td></tr>
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<tr><td rowspan="2">**5 Minutes**</td><td>**ASSESS—Evaluate Student Demonstration**</td></tr>
<tr><td>8. **Student orally defines at least one Key Academic Mathematics Vocabulary word and skill or concept.** Define what a fraction is.</td></tr>
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9. **Student works a problem while explaining EVERY step orally.** Using the worksheet below, ask, "Can you tell me how many equal parts you'll divide the shapes into? What would the fraction look like? Why?"