

Core Content

Cluster Title: Understand properties of multiplication and the relationship between multiplication and division.

Standard 5: Apply properties of operations as strategies to multiply and divide. Examples: If $6 \times 4 = 24$ is known, then $4 \times 6 = 24$ is also known (commutative property of multiplication). $3 \times 5 \times 2$ can be found by $3 \times 5 = 15$, then $15 \times 2 = 30$, or by $5 \times 2 = 10$, then $3 \times 10 = 30$ (associative property of multiplication). Knowing that $8 \times 5 = 40$ and $8 \times 2 = 16$, one can find 8×7 as $8 \times (5 + 2) = (8 \times 5) + (8 \times 2) = 40 + 16 = 56$ (distributive property).

Note: Students need not use formal terms for these properties at this grade level.

MASTERY Patterns of Reasoning:

Conceptual:

- Students will understand that multiplication is commutative and division is not commutative.
- Students will understand the distributive, associative, and commutative properties of Multiplication.
- Students will understand the multiplicative identity property (i.e., multiplying by 1).
- Students will understand the zero property of multiplication (i.e., multiplying by 0).
- Students will understand that division by 0 is undefined (the zero property).

Procedural:

- Students can multiply two factors in any order.
- Students can multiply by grouping using parentheses and three factors in various ways.
- Students can simplify a multiplication expression into smaller problems to make solving easier
- Students can find the product when multiplying by 1.
- Students can find the product when multiplying by 0.
- Students can find the quotient when dividing by whole numbers.

Representational:

- Students can use an array or grouping to model the commutative property.
- Students can model the associative property using three factors.

Supports for Teachers

Critical Background Knowledge	
<p>Conceptual:</p> <p>Students will understand the commutative and associative properties of addition. Students will understand that numbers can be broken apart into smaller numbers. Students will understand the identity property of addition. Students will understand that three numbers can be multiplied together.</p> <p>Procedural:</p> <p>Students can find the product of a multiplication equation. Students can break addition equations into smaller addition problems to solve.</p> <p>Representational:</p> <p>Students can make an array. Students can show equal groups. Students can model a division equation using pictures, objects, or numbers. Students can draw equal groups of objects. Students can represent repeated subtraction expressions and division expressions pictorially (and write the expressions). Students can represent repeated addition expressions and multiplication expressions pictorially (and write the expressions).</p>	
Academic Vocabulary and Notation	
array, product, factor, (), \times , \div , =, commutative property, associative property, distributive property, zero property, dividend, divisor, quotient	
Instructional Strategies Used	Resources Used
<p>Associative Property:</p> <ul style="list-style-type: none"> Have student lay out 18 counters in 3 groups, which have 2 rows of 3 counters each. Write an expression using parenthesis to represent the model (e.g., $(2 \times 3) \times 3$). Ask students to use 18 more counters and model $2 \times (3 \times 3)$. Have them compare the models. 	<p>Giganti, Paul. <i>Each Orange Had 8 Slices</i>. Greenwillow Books, 1999.</p> <p>Multiplication chart http://illuminations.nctm.org/LessonDetail.aspx?ID=U110</p>

- Explain that the concept of changing the grouping of factor does not change the product.

Commutative property:

- Have the students make 2 groups of 3. Then have them make 3 groups of 2. Compare and discuss the similarities and differences of the two models.
- Connect to the concept that changing the order of the factors does not change the product.
Note: arrays are also good models to teach this concept.

Distributive property:

- Make an array of 6 rows of 8 counters (6×8).
- Break the array into 5 rows of 8 and 1 row of 8 ($5 \times 8 + (1 \times 8)$).
- Add together the products of both arrays ($40 + 8 = 48$).
- Connect the total area of the larger array to the area of the two smaller arrays.
- Have the students draw an array of 4 rows of 9 (4×9) on graph paper.
- Cut into 4 rows of 4 (4×4) and 4 rows of 5 (4×5).
- Add together the product of both arrays ($16 + 20$)
- Connect the total area of the larger array to the area of the two smaller arrays.

Identity Property:

- Bring up 9 students and give them each one object. How many total objects do they have?
- Add $3 + 1$ and then multiply 3×1 and compare what happens between the two.

Zero Property:

- Add $3 + 0$ and then multiply 3×0 and compare what happens between the two.
- Have 9 students come up. Tell them they will each get zero cookies. How many cookies will they have? ($9 \times 0 = 0$)

Interactive Multiplication Chart

<http://www.mathsisfun.com/tables.html>

<http://www.aaamath.com/pro74b-propertiesmult.html>

Assessment Tasks Used	
<p>Skill-Based Task: $5 \times 8 = 40$; so $8 \times 5 =$ $(8 \times 3) \times 2 = \underline{\quad} \times (2 \times 3)$ $8 \times 6 = (8 \times \underline{\quad}) + (8 \times \underline{\quad})$ $6 \times 0 = \underline{\quad}$ $8 \times 1 = \underline{\quad}$</p>	<p>Problem Task: Robin says I can find 8×7 by multiplying 4×7 and then doubling it. Is she correct? How do you know? Use pictures words, and numbers to explain your reasoning.</p> <p>Paul put his toy cars into 8 rows of 2. What is another way he could group his cars? Show Paul's group and then show your grouping.</p> <p>Emily and Sam multiply $5 \times 4 \times 3$. Emily multiplies 5×4 first and then multiplies by 3. Sam multiplies 4×3 first and then multiplies by 5. Which student is correct? Use pictures, words and numbers to explain your thinking.</p>